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A Study on Sum Divisor Cordial Labeling Graphs

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Abstract—Cordial labeling refers to a graph G with p vertices and q edges as a sum divisor if there exists a bijection function $\rho: \mathrm{v} \to 1,2,\ldots,p$ such that for each edge $ab \in E$ assign the label 1 , if $2|(\rho(a)+\rho(b))|$ and 0 otherwise, satisfying the condition $\left|e_\rho(1)-e_\rho(0)\right| \leq 1, e_\rho(1)$ is the number of edges having the label 1 , and $e_\rho(0)$ is the number of edges having the label 0 . A graph with sum-divisor cordial labeling is called a sum-divisor cordial graph. In the paper, we establish this alternate triangular belt graph, twig graph, duplication of the top vertex Alternate triangular snake graph, duplication of top vertex Pentagon snake graph, jellyfish $J(x, y)$ when x and y are even, Spider graph with n spokes $S_{n,2}$.

Keywords- Sum divisor cordial labeling, Twig graph, Pentagon Snake graph, Spider graph.

I. Introduction

A finite undirected graph devoid of loops or numerous edges is referred to as a graph. We recommend Harary for conventional and fundamental graph theory notations and terminology, Graph theory [2]. A graph's labeling is a map connecting the graph's nodes to a collection of numbers, typically a collection of non-negative or positive integers. Vertex labeling is the term for labeling when the domain is a set of vertices. Edge labeling is used when the domain is the set of edges. Total labeling is the labeling process where labels are applied to both vertices and edges. Gallian provides a dynamic overview of various graph labels[1]. Cordial labeling is extended to divisor cordial labeling, prime cordial labeling, total cordial labeling, etc. Varatharajan [10] introduced the concept of divisor cordial labeling. Lourdusamy and Patrick [4] introduced the concept of sum divisor cordial labeling, and they investigated the existence of sum divisor cordial labeling in several families of graphs. Further study on labeling and its behavior is extensively found in many papers by various authors [3], [5], [6], [7], [8], and [9]. In this paper we identify an application using Affline Cipher for Cyber security through encryption and decryption.

II. Preliminaries

Definition 2.1. Divisor cordial graph

Let $G = p, q$ be a simple graph, and $\rho: p \to \{1,2,\ldots,p\}$ be a bijection function. For each edge ab, assign the label 1 if either $\rho(a) \mid \rho(b)$ or $\rho(b) \mid \rho(a)$ and the label 0 otherwise. The function ρ is called a divisor cordial labeling if $\left|e_\rho(0)-e_\rho(1)\right| \leq 1$. A graph that admits a divisor cordial label is called a divisor cordial graph.

Definition 2.2. Sum divisor cordial graph

Let $G = p, q$ be a simple graph, and $\rho: p \to \{1,2,\ldots,p\}$ be a bijection. For each edge ab, assign the label 1 if $2|(\rho(a)+\rho(b))|$ and the label 0 otherwise. The function ρ is called a sum divisor cordial labeling if $\left|e_\rho(0)-e_\rho(1)\right| \leq 1$. A graph that admits sum divisor cordial labeling is called a sum divisor cordial graph.

Definition 2.3. Twig graph

The graph obtained from a path by attaching exactly two pendant edges to each internal vertex of the path is called a Twig graph, and it is denoted by $T(n)$.

Definition 2.4. Alternate Triangular Belt graph

Let $L_n = P_n \times P_2 (n \geq 2)$ be the ladder graph with vertex sets u_i and v_i for $i = 1,2,\dots,n$. The Alternate Triangular Belt is obtained from the ladder by adding the edges $u_{2i+1}v_{2i+2}$ for all $i = 0,1,2,\dots,n-1$ and $v_{2i}u_{2i+1}$ for all $i = 1,2,\dots,n-1$.

Definition 2.5. Alternate triangular snake graph

The alternate triangular snake graph $A(TS_n)$ is obtained from the path P_n with vertices $v_1, v_2, \dots, v_n$ by joining v_i and v_{i+1} alternatively to two new vertex v_i for all $i = 1,\dots,n$. In other words, every alternate edge of the path P_n is replaced by a cycle C_3.

Definition 2.6. Duplication of top vertex Alternate Triangular snake graph

Consider the alternate triangular snake graph $D(AT_n)$ with the vertex set $u_1, u_2 \dots, u_n \cup w_1, w_2 \dots, w_{n-1}$. Now let us duplicate each of w_i, for $1 \leq i \leq n-1$ with w_i' for $1 \leq i \leq n-1$ and attach edges that are adjacent to the vertices, x_i, w_i, and $1 \leq i \leq n-1$ to form a new graph denoted by $D(AT_n)$.

Definition 2.7. Jelly Fish Graph

The jellyfish graph J_{xy} is obtained from four cycles with vertices x, y, u, and v by joining x and y with a prime edge and appending x pendent edges to u and y pendent edges to v.

Definition 2.8. Pentagonal snake graph

The pentagonal snake graph $P(S_n)$ is obtained from a path $u_1, u_2, \dots, u_n$ by joining u_i and $u_i + 1$ for $1 \leq i \leq n_1$ to two new vertices v_i, w_i, x_i and then joining v_i, x_i and x_i, w_i that is the path P_n by replacing each edge of the path by a cycle.

Definition 2.9. Duplication of top vertex of Pentagonal snake graph

Duplication of top vertex of Pentagonal snake graph $D(PS_n)$ with vertex $u_1, u_2 \dots, u_n \cup v_1, v_2, \dots, v_{n-1}$ Now let us duplicate each of v_i, forall $1 \leq i \leq n_1$ with v_i' for $1 \leq i \leq n_1$ attaching edges that are adjacent to vertex $x_i, w_i 1 \leq i \leq n_1$ to form a new graph denoted by $D(PS_n)$ whose order is 5 .

Definition 2.10. Spider Graph $S_{n,2}$

The Spider graph $S_{n,2}$ is obtained by attaching a pendent edge to each vertex of the star graph.

Definition 2.11. Join Spider graph $JS_{n,2}$

Join Spider Graph $JS_{n,2}$ by joining the centre vertex of two spider graph $S_{n,2}$.

III. MAIN RESULTS

Theorem 3.1. The Spider Graph with n spokes $S_{n,2}$ is sum divisor cordial labeling graph.

Proof. Let $S_{n,2}$ be the Spider Graph, where n is the number of vertices of the star graph, and the vertices linking the star graph to the pendent vertices are indicated by bi where i=1,2,...,m. Assume that the central vertex of the star graph is a_o=0. $S_{n,2}$ is a graph with 2n+1 and 2n edges.

The vertex set $V = \{a_i, b_i, a_o, 1 \leq i, \leq n\}$

The edge set $E = \{(a_i b_i); \cup (a_o a_i); ,1 \leq i \leq n\}$.

Now let us define a function,

$\rho: V \rightarrow \{1,2,3,\dots,p\}$.

Let us label the vertices as follows,

$$\rho(a_o) = 2;\ \rho(a_1) = 1;\ \rho(a_{i+1}) = 2i + 2;\ 1 \leq i \leq n-1$$
$$\rho(b_i) = 2i + 1;\ 1 \leq i \leq n$$

The following is the induced edge labeling:

$$\rho^*(a_o a_1) = 0;\ \rho^*(a_o a_{i+1}) = 1;\ 1 \leq i \leq n-1$$
$$\rho^*(a_1 b_1) = 1;\ \rho^*(a_{i+1} b_{i+1}) = 0;\ 1 \leq i \leq n-1$$

We find that the induced edge labeling satisfies the condition $|e_\rho(1) - e_\rho(0)| \leq 1$ (Table 1).

TABLE I. EDGE LABELING OF $S_{n,2}$

Number of edges	
Labeled with 0	Labeled with 1
N	n

Example 3.1. Spider Graph with n spokes $S_{n,2}$ shown in Fig. 1.

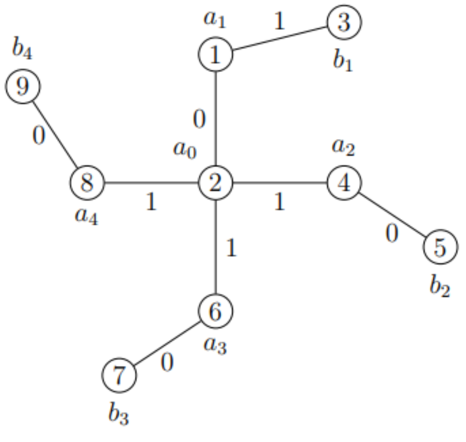

Figure 1. Spider Graph

Theorem 3.2. The Join Spider Graph with n spokes $JS_{n,2}$ is sum divisor cordial labeling graph.

Proof. Let $JS_{n,2}$ be the Join Spider Graph where n is the number of vertices of the star graph, The vertices of the star graph are denoted by a_i where i=1,2,...,n and the vertices attaching the star graph to the pendent vertices are denoted by b_i where i=1,2,...,m. Fix the central vertex of the star graph to a_0=0. S_(n,2) consists of 2n+1 and 2n edges. By connecting the central vertex of two Spider graphs. The vertex set $V = \{a_i, b_i, a_b, a_i', b_i'; 1 \leq i, \leq n\}$ The edge set $E = \{(a_i b_i); \cup (a_o a_i); \cup a_i' b_i'); ' \cup (a_o' a_i'); 1 \leq i \leq n\}$.

Now let us define a function,

$\rho: V \rightarrow \{1,2,3, \dots, p\}$

Let us label the vertices as follows,

$$\rho(a_o) = 1\ \rho(a_o') = 1;\ \rho(a_i) = 4i - 1;\ 1 \leq i \leq n$$
$$\rho(b_1) = 2;\ \rho(b_{i+1}) = 4i + 4;\ 1 \leq i \leq n - 1$$
$$\rho(a_1') = 4;\ \rho(a_{i+1}') = 4i + 5;\ 1 \leq i \leq n - 1$$
$$\rho(b_i) = 4i + 2;\ 1 \leq i \leq n$$

The following is the induced edge labeling:

$$\rho^*(a_o a_o') = 1;\ \rho^*(a_o a_i) = 1; \quad 1 \leq i \leq n$$
$$\rho^*(a_i b_i) = 0; \quad 1 \leq i \leq n$$
$$\rho^*(a_o' a_1') = 0;\ \rho^*(a_o' a_{i+1}') = 1; \quad 1 \leq i \leq n - 1$$
$$\rho^*(a_1' b_1') = 1;\ \rho^*(a_{i+1}' b_{i+1}') = 0; \quad 1 \leq i \leq n - 1$$

We find that the induced edge labeling satisfies the condition $|e_\rho(1) - e_\rho(0)| \leq 1$ (Table 2).

TABLE II. EDGE LABELING OF $JS_{n,2}$

Number of edges	
Labeled with 0	Labeled with 1
2n	2n + 1

Example 3.2. Join Spider Graph with n spokes $JS_{n,2}$ displayed in Fig 2.

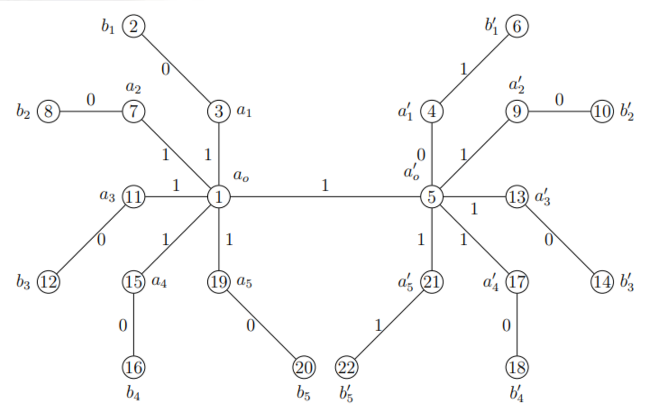

Figure 2. Join Spider Graph

Theorem 3.3. Twig graph $T(n)$ is sum divisor cordial labeling graph.

Proof. Consider the graph $G: T(n)$ with the vertex set $V = \{a_i \cdot b_j, c_j, 1 \leq i \leq n, 1 \leq j \leq n - 1\}$ and the edge set $E = \{(a_i a_{i+1}), 1 \leq i \leq n - 1\} \cup \{(a_i b_j), 2 \leq i \leq n, 1 \leq j \leq n - 1\} \cup \{(a_i c_j), 2 \leq i \leq n, 1 \leq j \leq n - 1\}$.

Now let us define a function,

$\rho: V \rightarrow \{1,2,3, \dots, p\}$.

Let us label the vertices as follows,

$$\rho(b_1) = 2;\ \rho(b_{2j+1}) = 6j - 4;\ \rho(b_j) = 6j;\ \text{for } 1 \leq j \leq \frac{n}{2}$$
$$\rho(a_1) = 1\ \rho(a_{2i+1}) = 6i - 1;\ \rho(a_{2i}) = 6i - 3;\ \text{for } 1 \leq i \leq n$$
$$\rho(c_{2j-1}) = 6j - 2;\ \rho(c_{2j}) = 6j + 1;\ \text{for } 1 \leq j \leq \frac{n}{2}$$

The following is the induced edge labeling:

$$\rho^*(a_i a_{i+1}) = 1;\quad \rho^*(a_{2i+1} c_{2i}) = 1;\quad i \equiv 1(\text{mod}2)$$
$$\rho^*(b_i a_{i+1}) = 0;\quad \rho^*(a_{2i} c_{2i-1}) = 0;\quad i \equiv 0(\text{mod}2)$$

We find that the induced edge labeling satisfies the condition $|e_\rho(1) - e_\rho(0)| \leq 1$ (Table 3).

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TABLE III.EDGE LABELING OF $T(n)$

Nature	Number of edges	
	Labeled with 0	Labeled with 1
Case 1: n is Odd	3n-1	3n-2
Case 2: n is Even	3n	3n

Example 3.3. Fig. 3. depicts the Twig graph $T(5)$

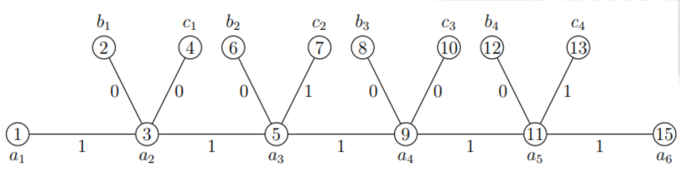

Figure 3. Twig graph $T(5)$

Theorem 3.4. Alternate Triangular belt graph $ATB(n)$ is sum divisor cordial labeling graph.

Proof. Consider the graph $G: ATB(n)$ with the vertex set $V = \{a_i, b_j; 1 \le i \le n, 1 \le j \le n\}$ and the edge set $E = \{(a_i a_{i+1}), 1 \le i \le n-1\} \cup \{(a_i b_i), 1 \le i \le n\} \cup \{(b_i b_{i+1}), 1 \le i \le n-1\} \cup \{(a_{i+1} b_{2i-1}, 1 \le i \le n\} \cup \{a_{2i} b_{2i+1}, 1 \le i \le n\}$.

Now let us define a function,

$$\rho: V \to \{1,2,3,\dots,p\}.$$

Let us label the vertices as follows,

$\rho(b_1) = 1; \rho(b_{2j}) = 4j;\ \rho(b_{2j+1}) = 4j+2;\ \text{ for } 1 \le j \le n$

$\rho(a_i) = 3; i \equiv 1(\text{mod}2)$

$= 2; i \equiv 0(\text{mod}2) \text{ for } 1 \le i \le 2$

$\rho(a_{i+2}) = 2i+3;\ \text{for } 1 \le i \le n$

The following is the induced edge labeling:

$\rho^*(a_i b_i) = 1;\ \text{for } 1 \le i \le 2\ \rho^*(a_i b_i) = 0;\ \text{for } 3 \le i \le n$

$\rho^*(a_i a_{i+1}) = 0;\ \text{ for } 1 \le i \le 2\ \rho^*(a_i a_{i+1}) = 1; \text{for } 3 \le i \le n-1$

$\rho^*(a_2 b_3) = 1\ \rho^*(a_i b_{i+1}) = 0;\ \text{ for } 4 \le i \le n\ \rho^*(a_{2i} b_{2i-1}) = 0;\ i \equiv 0(\text{mod}2)$

We find that the induced edge labeling satisfies the condition $|e_\rho(1) - e_\rho(0)| \le 1$ (Table 4).

TABLE IV. EDGE LABELING OF $ATB(n)$

Number of edges	
Labeled with 0	Labeled with 1
2n + 1	2n

Example 3.4. Alternate Triangular belt graph $ATB(4)$ is shown in Fig. 4.

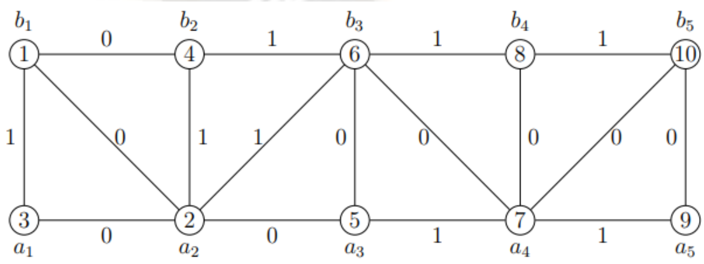

Figure 4. Alternate Triangular belt graph

Theorem 3.5. Duplication of top vertex Alternate Triangular snake graph DAT(n) is sum divisor cordial labeling graph.

Proof. Consider the graph $G: AT(n)$ with the vertex set $V = \{a_i, b_j, c_j; 1 \le i \le n, 1 \le j \le \frac{n}{2}\}$ and the edge set $E = \{(a_i a_{i+1}), 1 \le i \le n-1\} \cup \{(a_{2i-1} c_i), 1 \le i \le n\} \cup \{(a_{2i} c_i), 1 \le i \le n\} \cup \{(a_{2i-1} b_i), 1 \le i \le n\} \cup \{(a_{2i} b_i), 1 \le i \le n\}$.

Now let us define a function,

$\rho: V \to \{1,2,3,\dots,p\}$.

Let us label the vertices as follows,

$$\rho(b_1) = 1\ \rho(b_{2j}) = 8j-2;\ \rho(b_{2j+1}) = 8j+1;\ \text{ for } 1 \le j \le \frac{n}{2} - 1$$

$$\rho(c_1) = 4;\ \rho(c_{2j}) = 8j-1;\ \rho(c_{2j+1}) = 8j+4;\ \text{for } 1 \le j \le \frac{n}{2} - 1$$

$$\rho(a_{4i-3}) = 8i-6;\ \rho(a_{4i-2}) = 8i-5;\ \rho(a_{4i-1}) = 8i-3;\ \rho(a_{4i}) = 8i;\ \text{ for } 1 \le i \le n$$

The following is the induced edge labeling:

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$$\rho^*(a_{2i-1}b_i) = 0;\ \rho^*(a_{2i}b_i) = 1;\ \rho^*(a_{2i-1}c_i) = 1;\ \rho^*(a_{2i}c_i) = 0;\ 1 \le i \le n-1$$

$$\rho^*(a_{2i-1}a_{2i}) = 0;\ i \equiv 0(\mathrm{mod}2)\ \rho^*(a_{2i}a_{2i+1}) = 1;\ i \equiv 1(\mathrm{mod}2)$$

We find that the induced edge labeling satisfies the condition $|e_\rho(1) - e_\rho(0)| \le 1$ (Table 5).

TABLE V. EDGE LABELING OF $DAT(n)$

Number of edges	
Labeled with 0	Labeled with 1
3n	3n-1

Example 3.5. Duplication of top vertex Alternate Triangular snake graph $AT(4)$ shown in Fig. 5.

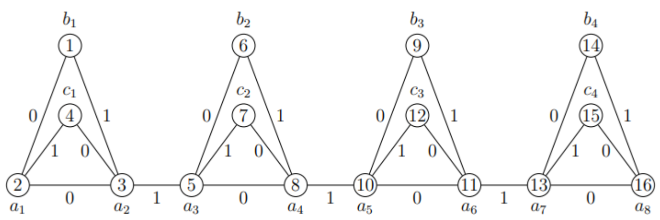

Figure 5. Duplication of top vertex Alternate Triangular snake graph

Theorem 3.6. Jelly fish $J(x, y)$ is sum division cordial labeling graph.

Proof. Consider the graph $G = J(x, y)$ with the vertex set $V = \{a_i, c_j, 1 \le i, j \le n\} \cup \{b_i, 1 \le i \le 4\}$. and the edge set $E = \left\{(a_i b_2), (b_2 a_i'), 1 \le i \le \frac{n}{2}\right\} \cup \left\{(b_4 c_i), (b_4 c_i'), 1 \le i \le \frac{n}{2}\right\} \cup \{(b_1 b_4), (b_4 b_2), (b_2 b_3), (b_3 b_4)$ $\{(b_3 b_4)$.

Now let us define a function,

$\rho: V \to \{1,2,3, \dots, p\}$.

Let us label the vertices as follows,

$$\rho(b_3) = 4\ \rho(b_4) = 5\ \rho(b_1 c_j) = 4j + 3;\ \rho(b_1 c'j) = 4j + 4;\ \text{ for } 1 \le j \le n$$

$$\rho(b_2 a_1) = 1\ \rho(b_2 a_1') = 2\ \rho(b_2 a_{i+1}) = 4i + 5;\ \rho(b_2 a_{i+1}) = 4i + 6;\ \text{ for } 1 \le i \le n$$

The following is the induced edge labeling:

$$\rho^*(b_4 b_1) = 1;\ \rho^*(b_1 b_3) = 0;\ \rho^*(b_2 b_3) = 1;\ \rho^*(b_4 b_3) = 1;\ \rho^*(b_1 b_2) = 0$$

$$\rho^*(b_1 b_i') = 0;\ \rho^*(b_2 a_i') = 0;\ i \equiv 0(\mathrm{mod}2)$$

$$\rho^*(b_i c_i') = 1;\ \rho^*(b_2 a_i) = 1;\ i \equiv 1(\mathrm{mod}2)$$

We find that the induced edge labeling satisfies the condition $|e_\rho(1) - e_\rho(0)| \le 1$ (Table 6).

TABLE VI. EDGE LABELING OF $J(x, y)$

Number of edges	
Labeled with 0	Labeled with 1
2n + 2	2n + 3

Example 3.6. Jelly fish $J(4,4)$ displayed in Fig. 6.

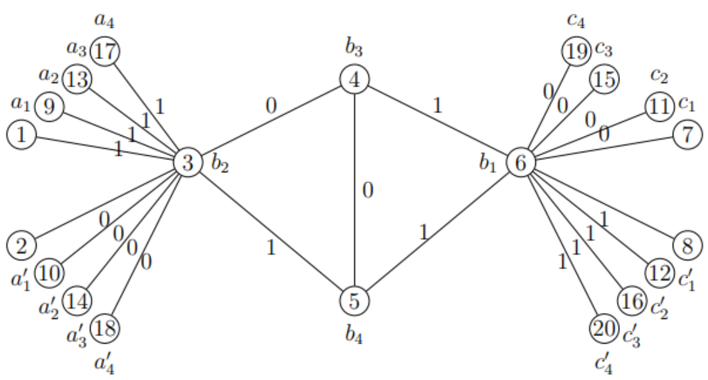

Figure 6. Jelly fish $J(4,4)$

Theorem 3.7. Duplication of Top vertex Pentagonal Snake $D(P(S_n))$ is sum divisor Cordial labeling graph.

Proof. Consider the graph $G = DP(S_n)$ with vertex set $V = \{a_i, b_i, a_i' b_i'; 1 \le i \le n\}$ and edge set $E = \{a_i' a_{2i-1}; 1 \le i \le n-1\} \cup \{a_{i+1}' a_{2i}; 1 \le i \le n-1\} \cup \{b_i a_{2i-1}; 1 \le i \le n-1\} \cup \{b_i a_{2i}; 1 \le i \le n-1\} \cup \{b_i' a_{2i-1}'; 1 \le i \le n-1\} \cup \{b_i' a_{2i}'; 1 \le i \le n-1\}$

Now let us define a function

$\rho: v \to \{1,2,3, \cdots p\}$

Let us label the vertices as follows.

$\rho(b'_i) = 6;\ \rho(b'_{2i}) = 10i + 1;\ \rho(b'_{2i+1}) = 10i + 5;\ 1 \le i \le \frac{n}{2}$

$\rho(b_1) = 1;\ \rho(b_{2i}) = 10i;\ \rho(b_{2i+1}) = 10i + 6;\ 1 \le i \le \frac{n}{2}$

$\rho(a_1) = 2;\ \rho(a_2) = 3;\ \rho(a_{4i-1}) = 10i - 3;\ 1 \le i \le n$

$\rho(a_{4i}) = 10i - 1;\ \rho(a_{4i+1}) = 10i + 3;\ \rho(a_{4i+2}) = 10i + 4;\ 1 \le i \le n$

$\rho(a'_1) = 4;\ \rho(a'_2) = 5;\ \rho(a'_{2i+1}) = 10i - 2;\ \rho(a'_{2i+2}) = 10i + 2;\ 1 \le i \le n$

The following is the induced edge labeling:

$\rho^*(a'_i a'_{i+1}) = 0;\ 1 \le i \le 2\ \rho^*(a'_i a'_{i+1}) = 1;\ 3 \le i \le n$

$\rho^*(a'_i a_i) = 1;\ 1 \le i \le 2\ \rho^*(a'_2 a_3) = 1;\ \rho^*(a'_3 a_4) = 0$

$\rho^*(a_i b_i) = 0\ 1 \le i \le \frac{n}{2}$

$\rho^*(a_{4i-2} b_{2i-1}) = 1\ 1 \le i \le n$

$\rho^*(a'_{2i+1} a_{2i+3}) = 0\ \rho^*(a_{4i} b_{2i}) = 0\ \rho^*(a_{4i-2} b'_{2i-1}) = 0\ i \equiv 0 (\mathrm{mod} 2)$

$\rho^*(a'_{2i+2} a_{4i+2}) = 1\ \rho^*(a_{4i} b'_{2i}) = 1\ \rho^*(a_i b'_i) = 1\ i \equiv 1 (\mathrm{mod} 2)$

We find that the induced edge labeling satisfies the condition $|e_\rho(1) - e_\rho(0)| \le 1$ (Table 7).

TABLE VII. EDGE LABELING OF $D(P(S_n))$

Nature	Number of edges	
	Labeled with 0	Labeled with 1
Case 1: n is Odd	7n − 4	7n − 3
Case 2: n is Even	7n	7n

Example 3.7. Duplication of Top vertex Pentagonal snake graph $DP(S_6)$ shown in Fig. 7.

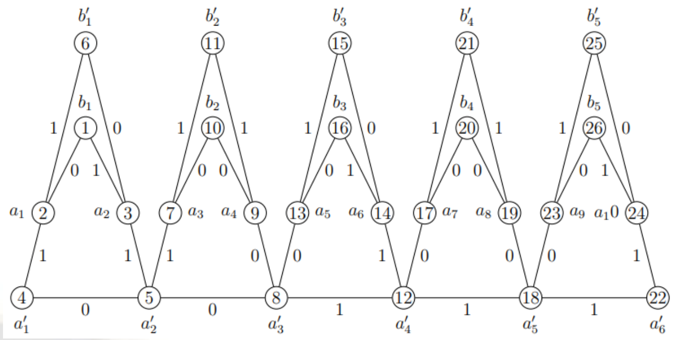

Figure 7. Duplication of Top vertex Pentagonal snake graph

IV. Application of Jelly Fish Graph for Affline Cipher Security through Cryptography

For any cipher text y and plain text x, a basic form of affline cipher text for encryption is $U = e(x) = ax + b(\mathrm{modm})$, and decryption is $x \equiv a^{-1}(Z - b)\mathrm{modm}$, where a, b, and m are natural numbers. We have used this affline cipher text procedure by applying it to the pendant vertex of Jelly Fish Graph $J(4,4)$.

Consider the Jelly Fish Graph $J(4,4)$ given in Example.3.4.

Algorithm for Encoding

- The vowels a,e,i, o, and u are assigned to multiples of four, i.e., 4, 8, 12, 16, 20, and other numbers 1,2,3,5,6,7,9,10,11,13,14,15,17,18,19,21,22,23,25 , and 26 are assigned to consonants in the order.
- The affline cipher text for encryption is $y = e(x) = ax + b(\mathrm{modm})$, and let a = 7 , where 7 is coprime,where a, b, and m are natural numbers.
- Assigned for a_i, a'_i and c_i, c'_i for the the Jelly Fish Graph $J(4,4)$
- Sending the cipher text as the jellyfish graph along with the new label is the key to decoding the message.

Algorithm for Decoding

- The affline cipher text for decryption is $x \equiv a^{-1}(z - b)\mathrm{modm}$, where a, b, and m are natural numbers, and let $a^1 = 4$, which is the inverse of 7.
- Assigned for c_i, c'_i and a_i, a'_i for the jellyfish graph $J(4,4)$.

A. Numbering Alphabets: Multiples of four are vowels; others are consonants

Step -1

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A	B	C	D	E	F	G	H	I
4	1	2	3	8	5	6	7	12
J	K	L	M	N	O	P	Q	R
9	10	11	13	14	16	15	17	18
S	T	U	V	W	X	Y	Z	space
19	21	20	22	23	24	25	26	0

- Consider the vertex label for the pendant vertices of the jellyfish graph $J(4,4)$ representing the a_i, a_i'

1	9	13	17	2	10	14	18

- In our example, let us choose the word labeled, which is to be encrypted, as proposed in our system.

L	A	B	E	L	I	N	G
11	4	1	8	11	12	14	6

- Now let us use the affline encryption and compute $U_i = (ax + g_i) \bmod \mathrm{m}$ Let us take in this example $a = 7$, where 7 is coprime.
- consider g_i as the Vertex label, $m = 27$ and $a = 7$
- Now computing $U_1 = 7 \times 11 + 1 = 78(\mathrm{mod}27) = 24 = X$.

$U_2 = 7 \times 4 + 9 = 37(\mathrm{mod}27) = 10 = K$.
$U_3 = 7 \times 1 + 13 = 20(\mathrm{mod}27) = 20 = U$.
$U_4 = 7 \times 8 + 17 = 73(\mathrm{mod}27) = 19 = S$.
$U_5 = 7 \times 11 + 2 = 79(\mathrm{mod}27) = 25 = Y$.
$U_6 = 7 \times 12 + 10 = 94(\mathrm{mod}27) = 13 = M$.
$U_7 = 7 \times 14 + 14 = 112(\mathrm{mod}27) = 4 = A$.
$U_8 = 7 \times 6 + 18 = 60(\mathrm{mod}27) = 6 = G$.

- We identify the corresponding new word for plain text, "LABELING," which is encrypted as "XKUSYMAG.".
- Now let us proceed with the second step of encrypting the new word "XKUSYMAG" by using the vertex labels 7,11,15,19,8,12,16, and 20 assigned for the vertex label c_i, c_i' as follows:

Step-2

Hence, the new word on encryption is "XKUSYMAG" which is assigned as

X	K	U	S	Y	M	A	G
24	11	15	19	8	12	16	20

- Consider the vertex label for the pendant vertices of the Jelly Fish Graph $J(4,4)$ representing the c_i, c_i'.

7	11	15	19	8	12	16	20

- Now compute V_i as follows

$V_1 = 7 \times 24 + 7 = 175(\mathrm{mod}27) = 13 = M$
$V_2 = 7 \times 10 + 11 = 81(\mathrm{mod}27) = 0 =$ Space
$V_3 = 7 \times 20 + 15 = 155(\mathrm{mod}27) = 20 = U$
$V_4 = 7 \times 19 + 19 = 152(\mathrm{mod}27) = 17 = Q$
$V_5 = 7 \times 25 + 8 = 183(\mathrm{mod}27) = 21 = T$
$V_6 = 7 \times 13 + 12 = 103(\mathrm{mod}27) = 22 = V$
$V_7 = 7 \times 4 + 16 = 44(\mathrm{mod}27) = 17 = Q$
$V_8 = 7 \times 6 + 20 = 62(\mathrm{mod}27) = 8 = E$

With the introduction of the word "XKUSYMAG," we have obtained the new word "MSpaceUQTVQE."

Step-3

- Now let us decrypt by using the function definition $d(X_i) = a^{-1}(V - g_i)\mathrm{mod}m$, by considering $a^{-1} = 4$, which is the inverse of 7.
- Proceeding for the word "MSpaceUQTVQE"

M	Space	U	Q	T	V	Q	E
13	0	20	17	21	22	17	8

- Consider the vertex label for the pendant vertices of the Jelly Fish Graph $J(4,4)$ representing the c_i, c_i' We

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compute $X_1 = 4 \times (13 - 7) = 24(\text{mod}27) = 24 = X$,

7	11	15	19	8	12	16	20

$X_2 = 4 \times (0 - 11) = -154(\text{mod}27) = 10 = K$,
$X_3 = 4 \times (20 - 15) = -20(\text{mod}27) = 20 = U$
$X_4 = 4 \times (17 - 19) = -28(\text{mod}27) = 19 = S$
$X_5 = 4 \times (21 - 8) = 52(\text{mod}27) = 25 = Y$
$X_6 = 4 \times (22 - 12) = 40(\text{mod}27) = 13 = M$
$X_7 = 4 \times (17 - 16) = 4(\text{mod}27) = 4 = A$
$X_8 = 4 \times (8 - 20) = -48(\text{mod}27) = 6 = G$

Through the decryption of the word "MZSpaceNZS," we have obtained the new word "XKUSYMAG."

Step-4

- Now let us decrypt by using the function definition $d(y_i) = a^{-1}(U - g_i)\text{mod}m$, by considering $a^{-1} = 4$, which is the inverse of 7.
- Proceeding for the word "XKUSYMAG"

X	K	U	S	Y	M	A	G
24	10	20	19	25	13	4	6

- Consider the vertex label for the pendant vertices of the Jelly Fish Graph $J(4,4)$ representing the a_i, a_i' We compute

1	9	13	17	2	10	14	18

$Y_1 = 4 \times (24 - 1) = 92(\text{mod}27) = 11 = L$,
$Y_2 = 4 \times (10 - 9) = 4(\text{mod}$*This* is the process of encrypting and decrypting plain text into cipher text $4 = A$,

$Y_3 = 4 \times (20 - 13) = 28(\text{mod}27) = 1 = B$
$Y_4 = 4 \times (19 - 17) = 8(\text{mod}27) = 8 = E$
$Y_5 = 4 \times (25 - 2) = 92(\text{mod}27) = 11 = L$
$Y_6 = 4 \times (13 - 10) = 12(\text{mod}27) = 12 = I$
$Y_7 = 4 \times (4 - 14) = -40(\text{mod}27) = 14 = N$
$Y_8 = 4 \times (6 - 18) = -48(\text{mod}27) = 6 = G$

We find that the original plain text is obtained as "LABELING".

This is the process of encrypting and decrypting plain text into cipher text. We find that safety and security can be implemented when data is processed through computer networks. We have consumed the idea and have executed it by identifying the Jelly Spider graph $J(4,4)$, which can be labeled and proved to be sum divisor cordial labeling, and we have also used the pendent vertex label for the execution of the process of cipher text.

V. Conclusion

In this paper, we have considered sum divisor cordial labeling and have proved alternate triangular belt graphs, twig graphs, and duplication of the top vertex. Alternate triangular snake graph, duplication of top vertex Pentagon snake graph, jellyfish $J(x, y)$ when x and y are even, Spider graph n spokes $S_{n,2}$ are sum divisor cordial graphs identified graphs that can be proved to be sum divisor cordial graphs. In the process of labeling the graphs according to the condition required for a sum-divisor cordial graph We have added a duplicate vertex for the alternate triangular snake graph and a duplicate vertex for the pentagonal snake graph. In our process, we will further study some more graphs for which a duplicate vertex can be constructed and prove that they are sum-divisor cordial labeling graphs.

References

[1] J.A. Gallian, A Dynamic Survey of Graph labeling, The Electronic J. Combin, 2015, vol. 18, pp. D56.
[2] F. Harary, Graph Theory, Addison -wesley, Reading, Mass, (1972).
[3] P. Lawrence Rozario Raj and R. Lawence Joseph Manoharan, Some Result on Divisor Cordial Labeling of Graphs, Int. J. Innocative Sci., 2014, vol. 1(10), pp. 226 - 231.
[4] A. Lourdusay and F. Patrick, Sum Divisor cordial graphs, Proyecciones Journal Mathematics, 2016, vol. 35(1), pp. 115-132.
[5] A. Nellai Murugan, G. Devakiruba and S. Navanaeethakrishan, Cycle Related Divisor Cordial Graphs, Int. J. Math. Trends and tech., 2014, vol. 12(1), pp. 34 - 43.
[6] A. Nellai Murugan, G. Devakiruba and S. Navanaeethakrishan, Star Attached Divisor Cordial Graphs, Int. J. Sci. Engineering and Tech., 2014, vol. 1(5), pp.165 – 171.
[7] A. Sugumaran and K. Rajesh, Further Results on sum divisor cordial labeling. Journal of Computer and Mathematical Science, 2017, vol. 8(12), pp. 802-807.
[8] A. Sugumaran and K. Rajesh, Some new results on sum divisor cordial graphs, Annals of Pure and Applied Mathematics, 2017, vol. 14(1), pp. 45 - 52.
[9] S. K. Vaidya and N. H. Shah, Some Star and Bistar Related Cordial Graphs, Annals Pure Appl. Math., 2013, vol. 3(1), pp. 67-77.
[10] R. Varatharajan, S. Navaneethakrishnan and K. Nagarajan, Divisor cordial graphs, International J. Math. combin, 2011, vol.4, pp. 15-25.