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Interval Valued Fuzzy Ideals of Near-rings and its Anti- homomorphism

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Abstract: Aim of this study is to investigate anti-homomorphic images and pre-images of semiprime and primary ideals in interval valued fuzzy Near-rings. Further some results on f-invariant interval valued fuzzy ideal, f-invariant strongly primary interval valued fuzzy ideal and f-invariant semiprime interval valued fuzzy ideals of Near-rings are discussed.

2010 AMS Classification: 03E72, 08A72 and 13A15

Keywords: Semiprime, primary ideal and f-invariant.

1 Introduction

The notion of a fuzzy set was introduced by Zadeh [13] in 1965, utilizing which Rosenfeld [11] has defined fuzzy subgroups. In 1975, Zadeh [16] investigated the notion of interval valued fuzzy subsets (in short i-v fuzzy subsets) where the values of the membership functions are closed intervals of numbers instead of single numbers. Liu introduced the concept of a fuzzy ideal of a near-ring in [8]. The concepts of prime fuzzy ideals, primary fuzzy ideals for ring were introduced in [9]. In 1991, Abou-Zaid [1] also exposed some results in fuzzy subnear-rings and fuzzy ideals in

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near-rings. Jun and Kim [4] and Davvaz [5] applied a few concepts of interval valued fuzzy subsets in near-rings. Sheikabdullah and Jeyaraman has discussed anti-homomorphic images and pre-images of prime fuzzy ideals and anti-homomorphic image of primary fuzzy ideals in a ring in [13].
The aim of this paper is to define and study i-v fuzzy primary ideals of a near ring N and investigate anti-homomorphic images and pre-images of semi-prime, strongly primary i-v fuzzy ideals.

2 Preliminaries

Definition 2.1. *[15] A non-empty set N with two binary operations $+$ and $.$ is called a near-ring if:*
i. $(N,+)$ is a group
ii. $(N,.)$ is a semigroup
iii. $x.(y+z) = x.y + x.z$ for all $x, y, z \in N$.
We will use the word Near-ring to mean left near-ring.

Definition 2.2. *[15] Let X be a non-empty universal set. A fuzzy subset μ of X is a function $\mu : X \to [0,1]$.*

Example 2.3. *Let $N = \{a, b, c, d\}$ be the Klein's four group. Define addition and multiplication in N as follows.*

$+$	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

$\cdot$	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	a
d	a	a	b	b

Here $(N,+,.)$ is a left near-ring. Define an interval valued fuzzy subset $\overline{\mu} : N \to D[0,1]$ by
$\overline{\mu}(a) = [0.7, 0.8]$, $\overline{\mu}(b) = [0.5, 0.6]$, $\overline{\mu}(c) = [0.3, 0.4] = \overline{\mu}(d)$.
It can be verified that $\overline{\mu}$ is an i-v fuzzy ideal of N.

Definition 2.4. *[15] An interval number $\overline{a}$ on [0,1] is a closed subinterval of [0,1], that is, $\overline{a} = [a^-, a^+]$ such that $0 \leq a^- \leq a^+ \leq 1$ where a^- and a^+ are the lower and upper end limits of $\overline{a}$ respectively. The set of all closed subintervals of $[0,1]$ is*

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denoted by $D[0,1]$. *We also identify the interval* $[a,a]$ *by the number* $a \in [0,1]$. *For any interval numbers* $\overline{a}_i = [a_i^-, a_i^+], \overline{b}_i = [b_i^-, b_i^+] \in D[0,1], i \in I$, *we define*
$max^i\{\overline{a}_i, \overline{b}_i\} = [max^i\{a_i^-, b_i^-\}, max^i\{a_i^+, b_i^+\}]$,
$min^i\{\overline{a}_i, \overline{b}_i\} = [min^i\{a_i^-, b_i^-\}, min^i\{a_i^+, b_i^+\}]$,
$inf^i \overline{a}_i = \left[\bigcap_{i\in I} a_i^-, \bigcap_{i\in I} a_i^+\right], sup^i \overline{a}_i = \left[\bigcup_{i\in I} a_i^-, \bigcup_{i\in I} a_i^+\right]$

In this notation $\overline{0} = [0,0]$ **and** $\overline{1} = [1,1]$. **For any interval numbers** $\overline{a} = [a^-, a^+]$ **and** $\overline{b} = [b^-, b^+]$ **on** $[0,1]$, **define**
(1) $\overline{a} \leq \overline{b}$ **if and only if** $a^- \leq b^-$ **and** $a^+ \leq b^+$**.**
(2) $\overline{a} = \overline{b}$ **if and only if** $a^- = b^-$ **and** $a^+ = b^+$**.**
(3) $\overline{a} < \overline{b}$ **if and only if** $\overline{a} \leq \overline{b}$ **and** $\overline{a} \neq \overline{b}$
(4) $k\overline{a} = [ka^-, ka^+]$, **whenever** $0 \leq k \leq 1$.

Definition 2.5. *[15] Let* X *be any set. A mapping* $\overline{A} : X \to D[0,1]$ *is called an interval-valued fuzzy subset (briefly, i-v fuzzy subset) of* X *where* $D[0,1]$ *denotes the family of all closed subintervals of* $[0,1]$ *and* $\overline{A}(x) = [A^-(x),\ A^+(x)]$ *for all* $x \in X$, *where* A^- *and* A^+ *are fuzzy subsets of* X *such that* $A^-(x) \leq A^+(x)$ *for all* $x \in X$.

Note that $\overline{A}(x)$ **is an interval (a closed subset of [0,1]) and not a number from the interval [0,1] as in the case of fuzzy subset.**

Definition 2.6. *[15] A mapping* $min^i : D[0,1] \times D[0,1] \to D[0,1]$ *defined by* $min^i(\overline{a}, \overline{b}) = [min\{a^-, b^-\},\ min\{a^+, b^+\}]$ *for all* $\overline{a}, \overline{b} \in D[0,1]$ *is called an interval min-norm.*

Definition 2.7. *[15] A mapping* $max^i : D[0,1] \times D[0,1] \to D[0,1]$ *defined by* $max^i(\overline{a}, \overline{b}) = [max\{a^-, b^-\},\ max\{a^+, b^+\}]$ *for all* $\overline{a}, \overline{b} \in D[0,1]$ *is called an interval max-norm.*

Definition 2.8. *[15] Let* N *be a near-ring. An i-v fuzzy set* $\overline{\mu}$ *of* N *is called an i-v fuzzy subnear-ring of* N *if for all* $x, y \in N$,
(i) $\overline{\mu}(x - y) \geq min^i\{\overline{\mu}(x),\ \overline{\mu}(y)\}$,
(ii) $\overline{\mu}(xy) \geq min^i\{\overline{\mu}(x),\ \overline{\mu}(y)\}$.

Definition 2.9. *[15] An i-v fuzzy subset* $\overline{\mu}$ *of a Near-ring* N *is called an i-v fuzzy ideal of* N *if* $\overline{\mu}$ *is an i-v fuzzy sub near-ring of* N *and*
(i) $\overline{\mu}(x) = \overline{\mu}(y + x - y)$
(ii) $\overline{\mu}(xy) \geq \overline{\mu}(y)$
(iii) $\overline{\mu}((x + i)y - xy) \geq \overline{\mu}(i)$ *for any* $x, y, i \in N$

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Proposition 2.10. *[15] The anti-homomorphic image of an i-v fuzzy ideal of N is an i-v fuzzy ideal of N'.*

Proposition 2.11. *[15] The homomorphic pre-image of an i-v fuzzy ideal of N' is an i-v fuzzy ideal of N.*

3 Main Results

Definition 3.1. *An i-v fuzzy ideal $\overline{\mu}$ of a near-ring N is called an i-v prime fuzzy ideal if for any two i-v fuzzy ideals $\overline{\sigma}$ and $\overline{\theta}$ of N the condition $\overline{\sigma}\overline{\theta} \subseteq \overline{\mu}$ implies that $\overline{\sigma} \subseteq \overline{\mu}$ or $\overline{\theta} \subseteq \overline{\mu}$.*

Definition 3.2. *For an i-v fuzzy ideal $\overline{\mu}$ of a near-ring, the i-v fuzzy radical of $\overline{\mu}$, denoted by $\sqrt{\overline{\mu}}$, is defined by $\sqrt{\overline{\mu}} = \cap\{\overline{\sigma} : \overline{\sigma}$ is an i-v fuzzy prime ideal of N, $\overline{\sigma} \subseteq \overline{\mu}$, $\overline{\sigma}_* \subseteq \overline{\mu}_*\}$. We denote $\overline{\mu}_* = \{x \in N : \overline{\mu}(x) = \overline{\mu}(0)\}$*

Definition 3.3. *An i-v fuzzy ideal $\overline{\mu}$ of a near-ring N is known as i-v fuzzy primary ideal if $\overline{\sigma}\overline{\theta} \subseteq \overline{\mu}$, then either $\overline{\sigma} \subseteq \overline{\mu}$ or $\overline{\theta} \subseteq \sqrt{\overline{\mu}}$.*

Definition 3.4. *An i-v fuzzy ideal $\overline{\mu}$ if a near-ring N is called i-v strongly primary fuzzy ideal of a near-ring N if $\overline{\mu}$ is an i-v primary fuzzy ideal and $(\sqrt{\overline{\mu}})^n \subseteq \overline{\mu}$ for some $n \in N$.*

Definition 3.5. *An i-v fuzzy ideal $\overline{\mu}$ of a near-ring N is called i-v semi-prime if for any i-v fuzzy ideal $\overline{\sigma}$ of N, $\overline{\sigma}^2 \subseteq \overline{\mu}$, then $\overline{\sigma} \subseteq \overline{\mu}$.*

Definition 3.6. *Let X and Y be two non-empty sets, $f : X \to Y$, $\overline{\mu}$ and $\overline{\sigma}$ be an i-v fuzzy subsets of X and Y respectively then $f(\overline{\mu})$, the image of $\overline{\mu}$ under f is an i-v fuzzy subset of Y denoted by*

$$f(\overline{\mu})(y) = \begin{cases} sup^i(x) : f(x) = y & if\ f^{-1}(y) \neq \phi, \\ 0 & if\ f^{-1}(y) = \phi. \end{cases}$$

And $f^{-1}(\overline{\sigma})$, the pre-image of $\overline{\sigma}$ under f is an i-v fuzzy subset of X defined by $f^{-1}(\overline{\sigma})(x) = \overline{\sigma}(f(x)) \ \forall\ x \in X$.

Definition 3.7. *If $\overline{\lambda}$ is an i-v fuzzy subset of N, then $\overline{\lambda}$ is said to have the sup property if for every subset Y of N, there exists $y_0 \in Y$ such that $\overline{\lambda}(y_0) = \{\overline{\lambda}(y) | y \in Y\}$.*

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Definition 3.8. *Let I be a non-empty i-v fuzzy subset of N. Define a function $\overline{C}_I : N \to D[0,1]$ by*

$$\overline{C}_I(x) = \begin{cases} \overline{1} & \text{if } x \in I, \\ \overline{0} & \text{otherwise} \end{cases}$$

for all $x \in N$. Clearly $\overline{C}_I$ is an i-v fuzzy subset of N. $\overline{C}_I$ is called the i-v characteristic function of I. If the replace I by N, $\overline{C}_N$ is the i-v characteristic function of N.

Definition 3.9. *Let N and N' be two near-rings, a mapping $f : N \to N'$ is called an i-v fuzzy homomorphism if $f(\overline{\mu}+\overline{\sigma}) = f(\overline{\mu}) + f(\overline{\sigma})$ and $f(\overline{\mu}\ \overline{\sigma}) = f(\overline{\mu})f(\overline{\sigma})$ where $\overline{\mu}$ and $\overline{\sigma}$ are i-v fuzzy ideals of N.*

Definition 3.10. *Let N and N' be two near-rings, a mapping $f : N \to N'$ is called an i-v fuzzy anti-homomorphism if $f(\overline{\mu}+\overline{\sigma}) = f(\overline{\mu}) + f(\overline{\sigma})$ and $f(\overline{\mu}\ \overline{\sigma}) = f(\overline{\sigma})\ f(\overline{\mu})$ where $\overline{\mu}$ and $\overline{\sigma}$ are i-v fuzzy ideals of N.*

Definition 3.11. *Let $f : N \to N'$. An i-v fuzzy subset $\overline{\mu}$ of a near-ring is called f-invariant if*
$f(x) = f(y)$ implies $\overline{\mu}(x) = \overline{\mu}(y)$, $x, y \in N$.

Definition 3.12. *N is called a fuzzy multiplication near-ring if for any two i-v fuzzy ideals $\overline{g}$ and $\overline{h}$ of N such that $\overline{g} \subseteq \overline{h}$, there exists a fuzzy ideal $\overline{f}$ of N such that $\overline{g} = \overline{h} \circ \overline{f}$.*

Theorem 3.13. *If $\overline{h}$ is a prime i-v fuzzy ideal of a fuzzy multiplication near ring N and $\overline{g}$ is any i-v fuzzy ideal of N such that $\overline{h} \subseteq \overline{g}$, then $\overline{h} = \overline{h} \circ \overline{g}$ and $\overline{h} = \overline{g}^{\ \omega}$ or $\overline{h} = \overline{h} \circ \overline{g}^{\ \omega}$, where $\overline{g}^{\ \omega} = \cap\{\overline{g}^{\ i} | i \in N \backslash \{0\}\}$.*

Proof. Since $\overline{h} \subseteq \overline{g}$ and N is an i-v fuzzy multiplication near-ring, there exists an i-v fuzzy ideal $\overline{k}$ of N such that $\overline{h} = \overline{g} \circ \overline{k}$. Then since $\overline{h}$ is prime, $\overline{h} \supseteq \overline{k}$. Now $\overline{h} = \overline{g} \circ \overline{k} \subseteq \overline{k}$. Thus $\overline{h} = \overline{k}$ and hence $\overline{h} = \overline{g} \circ \overline{h}$. It now follows that $\overline{h} = \overline{g}^{\ \omega}$ or $\overline{h} = \overline{h} \circ \overline{g}^{\ \omega}$.

Theorem 3.14. *If $\sqrt{\overline{f}}$ is an i-v prime fuzzy ideal, then $\overline{f}$ is an i-v primary.*

Proof. Let $\overline{g} \equiv \sqrt{\overline{f}}$. If $\overline{g} = \overline{C}_N$, then clearly $\overline{f}$ is an i-v primary. Assume $\overline{g} \neq \overline{C}_N$. Suppose that f is not i-v primary. Then there exist i-v fuzzy points $\overline{x}_r$, $\overline{y}_t$ such that $\overline{x}_r \circ \overline{y}_t \subseteq \overline{f}$, $\overline{x}_r \subseteq \overline{g}$, but $\overline{x}_r \nsubseteq \overline{f}$ and $\overline{y}_t^{\ n} \nsubseteq \overline{f}$ for all $n > 0$. Let $\overline{k} = \overline{f} \cup \overline{g} \circ (\overline{x}_r \circ \overline{C}_N)$. Clearly, $\underline{k}$ is an i-v fuzzy ideal of N. Suppose $\overline{x}_r \subseteq \overline{k}$. Then since $\overline{x}_r \nsubseteq \overline{f}$, $\overline{x}_r \subseteq \overline{g} \circ (\overline{x}_r \circ C_N)$. Thus $(\overline{g \circ (x_r \circ C_N)})(x) \geq r$, or $\vee\{\overline{g(a)} \wedge \overline{(x_r \circ C_N)(b)} | x = ab\} \geq r$.

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Since $\overline{f}$ has the sup property, $\overline{g}$ also possesses the sup property. Hence there exists $z \in S$ such that $\overline{g}(z) \geq \overline{r}$ and $x = zxs = xzs$. Thus $\overline{f}(z^n) \geq \overline{r}$, for some $n > 0$. Now $x = xz^n s^n$ and since $\overline{f}$ is an i-v fuzzy ideal, $\overline{f}(x) = \overline{f}(xz^n s^n) \geq \overline{f}(z^n) \geq \overline{r}$, i.e $\overline{x}_r \subseteq \overline{f}$, a contradiction. Hence $\overline{x}_r \not\subseteq \overline{k}$. Now, $\overline{k} \cup \overline{x}_r \circ \overline{C}_N \subseteq \overline{g}$. Thus there exists an i-v fuzzy ideal $\overline{h}$ of N such that $\overline{k} \cup \overline{x}_r \circ \overline{C}_N = \overline{g} \circ \overline{h}$. Again since $\overline{y}_t \not\subseteq \overline{g}$, $\overline{g} \subseteq \overline{g} \cup \overline{y}_t \circ \overline{C}_N$. Then by Theorem 3.13 $\overline{g} = \overline{g} \circ (\overline{g} \cup \overline{y}_t \circ \overline{C}_N)$. Now $\overline{k} \cup \overline{x}_r \circ \overline{C}_N = \overline{g} \circ \overline{h} = \overline{g} \circ (\overline{g} \cup \overline{y}_t \circ \overline{C}_N) \circ \overline{h} = \overline{g} \circ \overline{h} \circ (\overline{g} \cup \overline{y}_t \circ \overline{C}_N)$ (since N is commutative) $= (\overline{k} \cup \overline{x}_r \circ \overline{C}_N) \circ (\overline{g} \cup \overline{y}_t \circ \overline{C}_N) \subseteq \overline{k}$. Hence $\overline{x}_r \subseteq \overline{k}$, a contradiction. Therefore, $\overline{f}$ is an i-v primary.

Corollary 3.15. *Let $\overline{f}$ be an i-v prime. Then for all positive integers n, $\overline{f}^{\,n}$ is an i-v primary and its i-v fuzzy radical is $\overline{f}$.*

Proof. We first prove that $\sqrt{\overline{f}^{\,n}} = \overline{f}$ for all $n > 0$. If $n = 1$, the result is obvious. Let $n > 1$. Then $\sqrt{\overline{f}^{\,n}(x)} = \vee\{\overline{f}^{\,n}(x^m) | m > 0\} \geq \overline{f}^{\,n}(x^n) \geq \overline{f}(x)$ for all $x \in N$. Since $\overline{f}$ is an i-v prime, $\overline{f}(x) = \sqrt{\overline{f}(x)} = \vee\{\overline{f}(x^m) | m > 0\} \geq \{\overline{f}^{\,n}(x^m) | m > 0\} = \sqrt{\overline{f}^{\,n}(x)}$ for all $x \in N$. Hence $\sqrt{\overline{f}^{\,n}} = \overline{f}$. The desired result follows from Theorem 3.14

Theorem 3.16. *Let $\overline{f}$ be an i-v prime fuzzy ideal and $\overline{f}^{n} \neq \overline{f}^{n+1}$ for all $n > 0$. Then $\overline{f}^{\omega}$ is an i-v prime fuzzy ideal.*

Proof. Let $\overline{x}_l, \overline{y}_m$ an i-v fuzzy points such that $\overline{x}_l \not\subseteq \overline{f}^{\omega}$ and $\overline{y}_m \not\subseteq \overline{f}^{\omega}$. We show that $\overline{x}_l \circ \overline{y}_m \not\subseteq \overline{f}^{\omega}$. If $\overline{x}_l \not\subseteq \overline{f}, \overline{y}_m \not\subseteq \overline{f}$, then since $\overline{f}$ is an i-v prime, $\overline{x}_l \circ \overline{y}_m \not\subseteq \overline{f}$ and so $\overline{x}_l \circ \overline{y}_m \not\subseteq \overline{f}^{\omega}$. Suppose $\overline{x}_l \subseteq \overline{f}, \overline{y}_m \not\subseteq \overline{f}$. Since $\overline{x}_l \not\subseteq \overline{f}^{\omega}$, there exists a positive integer p such that $\overline{x}_l \subseteq \overline{f}^{p}, \overline{x}_l \not\subseteq \overline{f}^{p+1}$. Since by Corollary 3.15 $\overline{f}^{P+1}$ is an i-v primary fuzzy ideal with i-v fuzzy radical $\overline{f}, \overline{x}_l \circ \overline{y}_m \not\subseteq \overline{f}^{p+1}$ and so $\overline{x}_l \circ \overline{y}_m \not\subseteq \overline{f}^{\omega}$. The case when $\overline{x}_l \not\subseteq \overline{f}, \overline{y}_m \subseteq \overline{f}$ is similar.
Finally, let $\overline{x}_l, \overline{y}_m \subseteq \overline{f}$. Then there exist positive integers q, r such that $\overline{x}_l \subseteq \overline{f}^{q}, \overline{x}_l \not\subseteq \overline{f}^{q+1}$ and $\overline{y}_m \subseteq \overline{f}^{r}, \overline{y}_m \not\subseteq \overline{f}^{r+1}$. Then $\overline{x}_l \circ \overline{C}_N \subseteq \overline{f}^{q}, \overline{y}_m \circ \overline{C}_N \subseteq \overline{f}^{r}$. Since N is an i-v fuzzy multiplication near ring, there exist i-v fuzzy ideals $\overline{g}, \overline{h}$ of N such that $\overline{x}_l \circ \overline{C}_N = \overline{f}^{q} \circ \overline{g}, \overline{y}_m \circ \overline{C}_N = \overline{f}^{r} \circ \overline{h}, \overline{h}, \overline{g} \not\subseteq \overline{f}$. Now, if $\overline{x}_l \circ \overline{y}_m \subseteq \overline{f}^{q+r+1}$, then $\overline{f}^{q+r} \circ \overline{h} \circ \overline{g} = (\overline{f}^{q} \circ \overline{g})(\overline{f}^{r} \circ h) = \overline{x}_l \circ \overline{y}_m \circ \overline{C}_N \subseteq \overline{f}^{q+r+1}$. Since $\overline{f}^{q+r+1}$ is an i-v primary fuzzy ideal with i-v fuzzy radical $\overline{f}$ and $\overline{h} \circ \overline{g} \not\subseteq \overline{f}$ (since $\overline{f}$ is an i-v prime), $\overline{f}^{q+r} \subseteq \overline{f}^{q+r+1}$. Also $\overline{f}^{q+r} \supseteq \overline{f}^{q+r+1}$. Thus $\overline{f}^{q+r} = \overline{f}^{q+r+1}$, a contradiction. Hence $\overline{x}_l \circ \overline{y}_m \not\subseteq \overline{f}^{q+r+1}$, i.e., $\overline{x}_l \circ \overline{y}_m \not\subseteq \overline{f}^{\omega}$. Thus $\overline{f}^{\omega}$ is an i-v prime fuzzy ideal.

Theorem 3.17. *If $\overline{f}$ is an i-v primary fuzzy ideal, then $\overline{f} = \overline{g}^n$ for some positive integer n, where $\overline{g} = \sqrt{\overline{f}}$.*

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Proof. If $\overline{g} = \overline{C}_N$, then $\overline{f} = \overline{C}_N$. Assume $\overline{g} \neq \overline{C}_N$. Suppose $\overline{f} \subseteq \overline{g}^{\omega}$. Now $\overline{g}$ is an i-v prime fuzzy ideal of N having the sup property. If $\overline{g}^n \neq \overline{g}^{n+1}$ for all $n > 0$, then by Theorem 3.16, $\overline{g}^{\omega}$ is an i-v prime. Thus $\overline{g} = \sqrt{\overline{f}} \subseteq \sqrt{\overline{g}^{\omega}} = \overline{g}^{\omega}$, a contradiction. Thus either $\overline{f} \subseteq \overline{g}^n = \overline{g}^{n+1}$ for some $n > 0$, or $\overline{f} \subseteq \overline{g}^n$, but $\overline{g} \nsubseteq \overline{g}^{n+1}$ for some $n > 0$. In the first case, let $\overline{x}_r \subseteq \overline{g}^n$. Then $\overline{x}_r \circ \overline{C}_N \subseteq \overline{g}^n$. Also, there exists an i-v fuzzy ideal $\overline{h}$ of N such that $\overline{x}_r \circ \overline{C}_N = \overline{g}^n \circ \overline{h}$. Thus $\overline{x}_r \subseteq \overline{x}_r \circ \overline{C}_N = \overline{g}^n \circ \overline{h} = \overline{g}^{n+1} \circ \overline{h} = \overline{g} \circ (\overline{x}_r \circ \overline{C}_N)$. Then as in Theorem 3.14, it can be shown that $\overline{x}_r \subseteq \overline{f}$. Hence $\overline{f} = \overline{g}^n$.
In second case, there exists an i-v fuzzy ideal $\overline{k}$ of N such that $\overline{f} = \overline{g}^n \circ \overline{k}, \overline{k} \nsubseteq \overline{g}$. Since $\overline{f}$ is an i-v primary and $\overline{k} \nsubseteq \overline{g}, \overline{g}^n \nsubseteq \overline{f}$. Hence $\overline{f} = \overline{g}^n$.
Let $\overline{f}, \overline{g}$ be two i-v fuzzy ideals of N. Define the fuzzy subset $\overline{f} : \overline{g}$ of N as follows:
$\overline{f} : \overline{g} = \cup\{\overline{h} | \overline{h}$ is an i-v fuzzy ideal of N such that $\overline{h} \circ \overline{g} \subseteq \overline{f}\}$.
It follows easily that $\overline{f} : \overline{g}$ is an i-v fuzzy ideal of N.

Theorem 3.18. *If $\overline{f}$ is a proper prime i-v fuzzy ideal and $\overline{g}$ is an i-v fuzzy ideal of N such that $\overline{g} \subseteq \overline{f}^n$ and $\overline{g} \nsubseteq \overline{f}^{n+1}$ for some $n > 0$, then $\overline{f}^n = \overline{g} : (\overline{y}_t \circ \overline{C}_N)$, where $\overline{y}_t \nsubseteq \overline{f}$.*

Proof. Since $\overline{g} \subseteq \overline{f}^n$, there exists an i-v fuzzy ideal $\overline{h}$ of N such that $\overline{g} = \overline{f}^n \circ \overline{h}, \overline{h} \nsubseteq \overline{f}$. Let $\overline{y}_t \subseteq \overline{h}, \overline{y}_t \nsubseteq f$. Then $\overline{y}_t \circ \overline{C}_N \subseteq \overline{h}$ and $\overline{f}^n \circ (\overline{y}_t \circ C_N) \subseteq \overline{f}^n \circ \overline{h} = \overline{g}$.
Thus $\overline{f}^n \subseteq g : (\overline{y}_t \circ \overline{C}_N)$. Now let $\overline{k}$ be any i-v fuzzy ideal of N such that $\overline{k} \circ (\overline{y}_t \circ \overline{C}_N) \subseteq g$. Then $\underline{k} \circ (\overline{y}_t) \circ \overline{C}_N \subseteq \overline{f}^n$. Now since by Corollary 3.15, $\overline{f}^n$ is i-v primary with fuzzy radical f and $\overline{y}_t \circ \overline{C}_N \nsubseteq f, k \subseteq \overline{f}^n$. Therefore, $\overline{g} : (\overline{y}_t \circ \overline{C}_N) \subseteq \overline{f}^n$ and hence $\overline{f}^n = \overline{g} : (\overline{y}_t \circ \overline{C}_N)$.

Proposition 3.19. *Let $f : N \to N'$ be a surjective near-ring anti-homomorphism and $\overline{\mu}'$ is an i-v fuzzy prime ideal of N', then $f^{-1}(\overline{\mu}')$ is an i-v fuzzy prime ideal of N.*

Proof. Let $\overline{\mu}$ and $\overline{\sigma}$ be two i-v fuzzy ideals of N such that
$\overline{\mu}\ \overline{\sigma} \subseteq f^{-1}(\overline{\mu}')$
$\Rightarrow f(\overline{\mu}\ \overline{\sigma} \subseteq \overline{\mu}')$
$\Rightarrow f(\overline{\sigma}) f(\overline{\mu}) \subseteq \overline{\mu}'$
Since $\overline{\mu}'$ is an i-v fuzzy prime ideal of N'
$\Rightarrow f(\overline{\sigma}) \subseteq \overline{\mu}'$ or $f(\overline{\mu}) \subseteq \overline{\mu}'$
$\Rightarrow \overline{\sigma} \subseteq f^{-1}(\overline{\mu}')$ or $\overline{\mu} \subseteq f^{-1}(\overline{\mu}')$
Therefore $f^{-1}(\overline{\mu}')$ is an i-v fuzzy prime ideal of N.

Proposition 3.20. *Let $f : N \to N'$ be a surjective near ring anti- homomorphism and $\overline{\mu}'$ is an i-v primary fuzzy ideal of N', then $f^{-1}(\overline{\mu}')$ is an i-v primary fuzzy ideal of N.*

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Proof. Let $\overline{\mu}$ and $\overline{\sigma}$ be two i-v fuzzy ideals of N. Such that
$\overline{\mu\sigma} \subseteq \overline{f}^{-1}(\overline{\mu}')$ and $\overline{\sigma} \nsubseteq f^{-1}(\overline{\mu}')$
$\Rightarrow f(\overline{\mu\sigma}) \subseteq \overline{\mu}'$ and $f(\overline{\sigma}) \nsubseteq \overline{\mu}'$
$\Rightarrow f(\overline{\sigma})f(\overline{\mu}) \subseteq \overline{\mu}'$ and $f(\overline{\sigma}) \nsubseteq \overline{\mu}'$
$\Rightarrow f(\overline{\mu}) \subseteq \sqrt{\overline{\mu}'}$ (Since $\overline{\mu}'$ is an i-v primary fuzzy ideal)
$\Rightarrow \overline{\mu} \subseteq f^{-1}\sqrt{(\overline{\mu}')}$
$\Rightarrow \overline{\mu} \subseteq \sqrt{f^{-1}(\overline{\mu}')}$
Therefore $f^{-1}(\overline{\mu}')$ is an i-v primary fuzzy ideal of N.

Lemma 3.21. *If $\overline{\mu}$ is an i-v primary fuzzy ideal of a near-ring N, then $\sqrt{\overline{\mu}}$ is an i-v prime fuzzy ideal of N.*

Proof. Let $\overline{\sigma}$ and $\overline{\theta}$ be two i-v fuzzy ideals of N such that $\overline{\sigma}\,\overline{\theta} \subseteq \sqrt{\overline{\mu}}$ and $\overline{\sigma} \nsubseteq \sqrt{\overline{\mu}}$
$\Rightarrow \overline{\sigma}\,\overline{\theta} \subseteq \overline{\mu}$ and $\overline{\sigma} \nsubseteq \overline{\mu}$.
Since $\overline{\mu}$ is an i-v primary fuzzy ideal, $\overline{\theta} \subseteq \sqrt{\overline{\mu}}$.
Therefore $\sqrt{\overline{\mu}}$ is an i-v prime fuzzy ideal of N.

Proposition 3.22. *Let $f : N \to N'$ be a surjective near-ring anti-homomorphism. If $\overline{\mu}$ is an f-invariant i-v fuzzy ideal of N and $\overline{\mu}$, an i-v fuzzy primary ideal of N, then $f(\overline{\mu})$ is an i-v fuzzy primary ideal of N'.*

Proof. Let $\overline{\sigma}'$ and $\overline{\theta}'$ be two i-v fuzzy ideals of N' such that $\overline{\sigma}'\overline{\theta}' \subseteq f(\overline{\mu})$ and $\overline{\sigma}' \nsubseteq f(\overline{\mu})$
$\Rightarrow f^{-1}(\overline{\sigma}'\overline{\theta}') \subseteq f^{-1}f(\overline{\mu})$
$\Rightarrow f^{-1}(\overline{\sigma}'\overline{\theta}') \subseteq \overline{\mu}$ and $f^{-1}(\overline{\sigma}' \subseteq \overline{\mu})$
$f^{-1}(\overline{\sigma}')f^{-1}(\overline{\theta}') \subseteq \overline{\mu}$ and $f^{-1}(\overline{\sigma}') \nsubseteq \overline{\mu}$
$\Rightarrow f^{-1}(\overline{\theta}') \subseteq \sqrt{\overline{\mu}}$ (Since $\overline{\mu}$ is an i-v primary fuzzy ideal)
$\Rightarrow \overline{\theta}' \subseteq \sqrt{f(\overline{\mu})}$.
Therefore $f(\overline{\mu})$ is an i-v fuzzy primary ideal of N'.

Proposition 3.23. *For a surjective near-ring anti-homomorphism $f : N \to N'$, if $\overline{\mu}$ is an f-invariant i-v strongly primary fuzzy ideal of N then $f(\overline{\mu})$ is an i-v strongly primary fuzzy ideal of N'.*

Proof. Let $\overline{\mu}$ be an f - invariant i-v strongly primary fuzzy ideal of N.
$\Rightarrow \overline{\mu}$ is an i-v primary fuzzy ideal and $(\sqrt{\overline{\mu}})^n \subset \overline{\mu}$ for some $n \in N$
$\Rightarrow f(\overline{\mu})$ is an i-v primary fuzzy ideal of N'.
Since $f(\overline{\mu})$ is an i-v primary fuzzy ideal of N', $\sqrt{f(\overline{\mu})}$ is an i-v prime fuzzy ideal of N'. (By Lemma 3.21)
Since $\sqrt{f(\overline{\mu})} = \wedge\{f(\overline{\sigma}), f(\overline{\sigma})$ is an i-v fuzzy prime ideal of N', $f(\overline{\sigma}) \subseteq f(\overline{\mu})\}$.

Therefore $(\sqrt{f(\overline{\mu})}^n \subset f(\overline{\mu})$ for some $N \in N$.
Hence $f(\overline{\mu})$ is an i-v strongly primary fuzzy ideal of N'.

Proposition 3.24. *For a surjective near-ring homomorphism $f : N \to N'$, if $\overline{\mu}$ is an i-v semi prime fuzzy ideal of N', then $f^{-1}(\overline{\mu}')$ is an i-v semi prime fuzzy ideal of N.*

Proof. Given $\overline{\mu}'$ is an i-v semi prime fuzzy ideal of N'.
$\Rightarrow \overline{\mu}'$ is an i-v fuzzy ideal of N' and $\overline{\mu}^2(x) = \overline{\mu}(x)$ for all $x \in N$.
$\Rightarrow f^{-1}(\overline{\mu}')$ is an i-v fuzzy ideal of N.
Let $f^{-1}(\overline{\mu}') = \overline{\mu} \Rightarrow (\overline{\mu}') = f(\overline{\mu})$
Now $\overline{\mu}' = \overline{\mu}'\overline{\mu}' = f(\overline{\mu})\ f(\overline{\mu}) = f(\overline{\mu}\ \overline{\mu}) = f(\overline{\mu}^2)$
$\Rightarrow \overline{\mu}^2 = f^{-1}(\overline{\mu}') = \overline{\mu} \Rightarrow [f^{-1}(\overline{\mu}')]^2(x) = f^{-1}(\overline{\mu}')(x)$ for all $x \in N$.

Conclusion

In this article it is shown that for an i-v primary fuzzy ideal of a near ring $N, \sqrt{\overline{\mu}}$ is an i-v prime fuzzy ideal. Further it has been proved for a f-invariant and i-v fuzzy primary ideal $\overline{\mu}$ of $N, f(\overline{\mu})$ is also an i-v fuzzy primary ideal.

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