Numerical solution to Eighth order Linear Differential Equation Using the Octic B-spline collocation Method

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Abstract *:-* In this paper, Collocation method using recursive form of Octic degree B-spline functions as basis is developed and employed to find the numerical solution for eighth order differential equation with boundary condition problems. Numerical examples are considered to test the performance, stability and accuracy of the present developed method.

Keywords *: B-Spline, Collocation, Recursive, Linear differential equation, Octic*

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1. Introduction

The general eighth order linear differential equation with the boundary conditions is given as

$$P0(x)\frac{d^8U}{dx^8}+P_1(x)\frac{d^7U}{dx^7}+P2(x)\frac{d^6U}{dx^6}+P3(x)\frac{d^5U}{dx^5}+P_4(x)\frac{d^4U}{dx^4}+P5(x)\frac{d^3U}{dx^3}+P_6(x)\frac{d^2U}{dx^2}+P_7(x)\frac{dU}{dx}+P_8(x)U=Q(x)$$

$$x\in(a,b) \quad , \qquad (1)$$

with the boundary conditions

$$U(a)=d1, U(b)=d2 \;\; U^{'}(a)=d3, U^{'}(b)=d4, U^{''}(a)=d5, U^{''}(b)=d6, U^{'''}(a)=d7, U^{'''}(b)=d8 \qquad (2)$$

where $a,b,d_1,d_2,d_3,d_4,d_5,d_6,d_7,d8$ **are constants.** $P_0(x), P_1(x), P_2(x), P_3(x), P_4(x)$, $P_5(x), P_6(x)$, $P_7(x), P_8(x), Q(x)$ **are function of** x**.**

Solving such higher order linear differential equations and getting exact solutions is sometimes difficult. Authors developed methods to obtain numerical methods. Some of the selected numerical methods are mentioned. Finite difference method is used to solve such equations by Boutayeb and Twizell [1] .Vishwanadam and Ballem [2] used Galerkin method with quintic B- spline, Siddiqi and iftikhar [3] solved by using homotopy analysis method.

In this paper, A Octic degree B-spline based collocation method has been elaborated for the solution of linear eighth order differential equation with boundary conditions defined in Eq.(1) with Eq.(2).

2. The numerical scheme

Let $[a,b]$ be the domain of the governing differential equation and is partitioned as $X=\{a=x_0,x_1,x_2 \ldots\ldots x_{n-1},x_n=b\}$ without any restriction on length of n sub domains. Let $N_i(x)$ be Octic B- splines with the knots at the points x_i, i=0,1,…n. The set $\{N_{-8}, N_{-7}, N_{-6}, \ldots, N_6, N_7, N8\}$ forms a basis for functions defined over [a,b].

Let $U^h(x)=\sum_{i=-8}^{n+8} C_i N_{i,p}(x)$ (3) , where C_i's are constants to be determined and $N_{i,p}(x)$ are the Octic B-spline functions , be the approximate global solution to the exact solution $U(x)$ of the considered eighth order linear differential equation (1). A zero degree and other than zero degree B-spline basis functions [5, 6] are defined at x_i recursively over the knot vector space $X=\{x_1,x_2,x_3 \ldots\ldots x_{n-1},x_n\}$ as

$$i) \quad if \quad p=0$$

$N_{i,p}(x)=1 \quad if \quad x\in(x_i,x_{i+i})$ $\quad N_{i,p}(x)=0$
$if \quad x\notin(x_i,x_{i+i})$

$$ii) \quad if \quad p\geq 1 \qquad N_{i,p}(x)=\frac{x-x_i}{x_{i+p}-x_i}N_{i,p-1}(x)+\frac{x_{i+p+1}-x}{x_{i+p+1}-x_{i+1}}N_{i+1,p-1}(x) \qquad \ldots\ldots\ldots\ (4)$$

where p is the degree of the B-spline basis function and x is the parameter belongs to X .When evaluating these functions, ratios of the form 0/0 are defined as zero

Derivatives of B-splines

If p=8, we have

$$N'_{i,p}(x)=\frac{x-x_i}{x_{i+p}-x_i}N'_{i,p-1}(x)+\frac{N_{i,p-1}(x)}{x_{i+p}-x_i}+\frac{x_{i+p+1}-x}{x_{i+p+1}-x_{i+1}}N'_{i+1,p-1}(x)-\frac{N_{i+1,p-1}(x)}{x_{i+p+1}-x_{i+1}}$$

$$N^{viii}{}_{i,p}(x)=8\frac{N^{vii}{}_{i,p-1}(x)}{x_{i+p}-x_i}-8\frac{N^{vii}{}_{i+1,p-1}(x)}{x_{i+p+1}-x_{i+1}} \qquad \ldots\ldots\ldots \qquad (5)$$

$$(U^h)^{viii}(x)=\sum_{i=-8}^{n+8}C_i N^{viii}{}_{i,p}(x) \qquad \ldots\ldots\ldots\ldots \qquad (6)$$

The x_i's are known as nodes, the nodes are treated as knots in collocation B-spline method where the B-spline basis functions are defined and these nodes are used to make the residue equal to zero to determine unknowns C_i's in (2).Eight extra knots are taken into consideration besides the domain of problem to maintain the partition of unity[7] when evaluating the Octic B-spline basis functions at the nodes which are within the considered domain.

Substituting the equations (3) and (6) in equation (1) for U (x) and derivatives of U (x).Then system of (n+1) linear equations are obtained in (n+8) constants. Applying the boundary conditions to equation (2), eight more equations are generated in constants. Finally, we have (n+9) equations in (n+9) constants.

Solving the system of equations for constants and substituting these constants in equation (3) then assumed solution becomes the known approximation solution for equation (1) at corresponding the collocation points. This is implemented using the Matlab programming.

3. Numerical Experiments

Some examples [4] are considered to measure the accuracy of the present method. Numerical results obtained by the present method show the betterment of the method.

Example1

$$\frac{d^8 y}{dx^8}+xy=-(48+15x+x^3)\ e^x$$

with the boundary conditions $y(0)=0 \;\; y(1)=0 \;\; y'(0)=1 \;\; y'(1)=-e \;\; y''(0)=0 \;\; y''(1)=-4e$
$y'''(0)=-3 \;\; y'''(1)=-9e$

The Exact solution of example1 is given as $y=x(1-x)\,e^x$

 ISSN: 2321-8169
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Table1 Comparison of numerical results with the exact values

x	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
OBCS*	0.0000	0.0995	0.1954	0.2835	0.3581	0.4123	0.4375	0.4230	0.3562	0.2214
Exact Solution	0.0000	0.0995	0.1954	0.2835	0.3581	0.4123	0.4375	0.4230	0.3562	0.2214

- Octic B-spline based collocation solution

The proposed method is implemented to problem1 for 10 collocation points and compared the values with exact values which are tabulated in table1

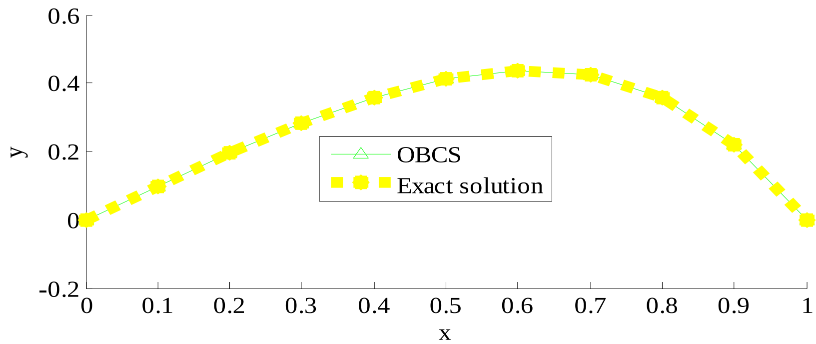

Figure1: Comparison of OBCS and Exact solution for11 Collocation points

Example2

$$\frac{d^8y}{dx^8}+\frac{d^7y}{dx^7}+2\frac{d^6y}{dx^6}+2\frac{d^5y}{dx^5}+2\frac{d^4y}{dx^4}+2\frac{d^3y}{dx^3}+2\frac{d^2y}{dx^2}+\frac{dy}{dx}+y=(14\cos x-16\sin x-4x\sin x)$$ with

the boundary conditions $y(0)=0$ $y(1)=0$ $y'(0)=1$ $y'(1)=2\sin(1)$ $y''(0)=0$

$y''(1)=4\cos(1)+2\sin(1)$ $y'''(0)=7$ $y'''(1)=6\cos(1)-6\sin(1)$

The exact solution for the example2 is $y=(x^2-1)\sin x$

Table2 Maximum Relative Error for various number of collocation points

Number of collocation points	11	41	51
Maximum Relative Error	2.0294e-004	2.0182e-004	1.9958e-004

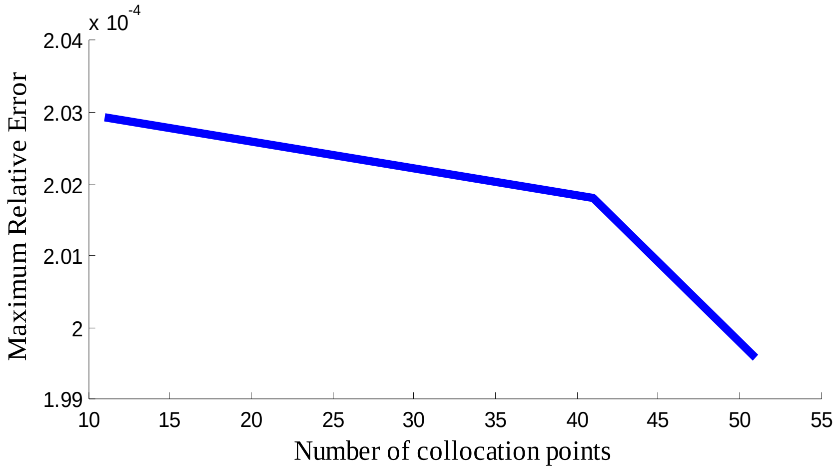

Figure2

Maximum relative error is calculated by using the present method for different size of collocation points. The behavior of maximum relative error is presented in figure2 as the number of collocation points is increased

Conclusion

In this paper, collocation method using the B-splines as basis function has been developed to approximate the linear eighth-order boundary value problems. In this method, the assumed approximate solution is directly substituted in given differential equation and evaluated unknown values in that approximate solution with the help of collocation points and boundary conditions. It is observed that the approximate values is almost equal to exact values and consequently approximate values converging to the exact values by increasing the number of collocation points.Also implementing of proposed method is very easy.

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