"Integration through Pre-Differentiation" in Theory of PDE Application: Equations of Navier-Stockes

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Abstract. This article applies one method of ordinary differential equations which can be applied for partial differential equations. Let us have Lagrange's equation

$$y = x\varphi(y') + f(y'), \quad y = y(x)$$

This equation is linear on x and y. We denote y' with p and differentiate:

$$p = \varphi(p) + x\varphi'(p)\frac{dp}{dx} + f'(p)\frac{dp}{dx} \Leftrightarrow$$
$$\Leftrightarrow (p - \varphi(p))\frac{dx}{dp} - x\varphi'(p) = f'(p)$$

Then the solution of the Lagrange's equation is determined by the system

$$\left| \begin{array}{l} y = x\varphi(p) + f(p) \\ (p - \varphi(p))\frac{dx}{dp} - x\varphi'(p) = f'(p) \end{array} \right.$$

The latter equation is linear ODE with unknown function ***x*** *and independent*

*Variable**p**. This is a familiar task.Then the solution in parametric form is:*

$$x = C\varphi_1(p) + f_1(p) \quad , \quad y = C\varphi_2(p) + f_2(p)$$

We will apply this scheme and in ***PDE****at first to the* ***Burger'sequation*** *and*

After that for the equationsof ***Navier-Stockes***

Keywords *ODE, PDE, determinant, derivative of the determinant.*

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I. Burger's equation.

$$\frac{\partial u_1}{\partial t} + u_1\frac{\partial u_1}{\partial x} = 0; \; u_1(x,0) = x$$

The classic solution is**:**

$$\frac{dt}{1} = \frac{dx}{u_1} = \frac{du_1}{0}$$

$$\left| \begin{array}{l} \frac{dt}{1} = \frac{dx}{u_1} \\ \frac{\mathrm{dt}}{1} = \frac{du_1}{0} \end{array} \right. \Rightarrow$$

$$\begin{array}{c} x - u_1 t = C_1 \\ \Rightarrow u_1 = C_2 \\ u_1(x,0) = x \end{array} \Rightarrow$$

Hence $C_1 = F(C_2) \Rightarrow \begin{array}{l} x - u_1 t = F(u_1) \\ t = 0 \to x = F(x) \\ x - u_1 t = F(x - u_1 t) \end{array}$ **I.e.** $u_1 = \frac{x}{1+t}$

Now we will solve the same equation by pre-differentiation. Namely, if we denote

$$p_1 = \frac{\partial u_1}{\partial t}, \quad q_1 = \frac{\partial u_1}{\partial x}, \quad \mathrm{q}_1 \neq 0$$

Then $\frac{\partial u_1}{\partial t} + u_1 \frac{\partial u_1}{\partial x} = 0; \quad u_1(x,0) = x;$ denote $u_1 = -\frac{p_1}{q_1}; q_1 \neq 0$ and hence

$$\frac{\partial u_1}{\partial t} = \frac{\frac{\partial p_1}{\partial t} q_1 - p_1 \frac{\partial q_1}{\partial t}}{q_1^2}$$

$$\frac{\partial u_1}{\partial x} = -\frac{\frac{\partial p_1}{\partial x} q_1 - p_1 \frac{\partial q_1}{\partial x}}{q_1^2}$$

But $\frac{\partial p_1}{\partial x} = \frac{\partial q_1}{\partial t}$

Or we have the system

$$\left| \begin{array}{l} \mathrm{p}_1 q_1^2 = p_1 \frac{\partial p_1}{\partial x_1} - q_1 \frac{\partial p_1}{\partial t} \\ q_1^3 = p_1 \frac{\partial q_1}{\partial x_1} - q_1 \frac{\partial q_1}{\partial t} \end{array} \right.$$

Hence we have

$$\left| \begin{array}{l} \frac{dx_1}{p_1} = \frac{dt}{-q_1} = \frac{dp_1}{p_1 q_1^2} \\ \frac{dx_1}{p_1} = \frac{dt}{-q_1} = \frac{dq_1}{q_1^3} \end{array} \right. \text{And}$$

$$\left| \begin{array}{l} \frac{dx_1}{p_1} = \frac{dt}{-q_1} \\ \frac{dx_1}{p_1} = \frac{dp_1}{p_1 q_1^2} \\ \frac{dx_1}{p_1} = \frac{dt}{-q_1} \\ \frac{dx_1}{p_1} = \frac{dq_1}{q_1^3} \end{array} \right.$$

Hence

$$\left| \begin{array}{l} p_1 t + q_1 x_1 = C_1 \\ p_1 t - q_1^2 x_1 = C_2 \\ p_1 t + q_1 x_1 = C_1 \\ \mathrm{p}_1 q_1 - q_1^3 x_1 = C_3 \end{array} \right. \Rightarrow \left| \begin{array}{l} \mathrm{C}_1 = F(C_2) \\ C_1 = G(C_3) \end{array} \right. \Rightarrow F = G \Rightarrow \left| \begin{array}{l} \mathrm{C}_1 = F(C_2) \\ C_1 = F(C_3) \end{array} \right. \text{and}$$

$$p_1 - q_1^2 x_1 = p_1 q_1 - q_1^3 x_1 = 0 \Leftrightarrow$$
$$\Leftrightarrow (1 - q_1)(p_1 - q_1^2 x_1) = 0$$

For $q_1 = 1$ we have $\frac{\partial u_1}{\partial t} + u_1 = 0, \quad u_1(x_1, 0) = 0$

Consistently, we find

$$\frac{dt}{1} = \frac{du_1}{-u_1}, \quad u_1 + u_1 t = C, \Rightarrow t = 0 \quad C = x \Rightarrow u_1 = \frac{x}{1+t}$$

The second equation $p_1 - q_1^2 x_1 = 0$, considered in a system with the given equation, gets the same result:

$$\left| \begin{array}{l} \frac{\partial u_1}{\partial t} + u_1 \frac{\partial u_1}{\partial x} = 0; \; u_1(x, 0) = x \\ \frac{\partial u_1}{\partial t} - \left(\frac{\partial u_1}{\partial x} \right)^2 x_1 = 0 \end{array} \right. \Rightarrow \frac{\partial u_1}{\partial x} x_1 - u_1 = 0 \Rightarrow \frac{x}{1+t} - u_1 = 0$$

We see that the two equations have equivalent solutions thatcoincide with a solution obtained earlier

II. The two-dimensional case of the Navies-Stocks equations

The easiest is when the algebraic part in this system is linear. We look at the system

$$\left|\begin{array}{l} \frac{\partial u_1}{\partial t}+u_1\frac{\partial u_1}{\partial x_1}+u_2\frac{\partial u_1}{\partial x_2}=L(u_1)+f_1(x_1,x_2,t)-\frac{\partial \rho}{\partial x_1} \\ \frac{\partial u_2}{\partial t}+u_1\frac{\partial u_2}{\partial x_1}+u_2\frac{\partial u_2}{\partial x_2}=L(u_2)+f_2(x_1,x_2,t)-\frac{\partial \rho}{\partial x_2} \\ \frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2}=0 \end{array}\right.$$

$$x\in R^2, t\geq 0, u(x,0)=u^0(x)$$

This system is linear about$\boldsymbol{u}$.For that reason, we divide its linear part

$$\left|\begin{array}{l} u_1\frac{\partial u_1}{\partial x_1}+u_2\frac{\partial u_1}{\partial x_2}=-\frac{\partial \rho}{\partial x_1}+L(u_1)-\frac{\partial u_1}{\partial t}+f_1(x_1,x_2,t) \\ u_1\frac{\partial u_2}{\partial x_1}+u_2\frac{\partial u_2}{\partial x_2}=-\frac{\partial \rho}{\partial x_2}+L(u_2)-\frac{\partial u_2}{\partial t}+f_2(x_1,x_2,t) \\ \frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2}=0 \end{array}\right.$$

If denote $L_1=L(u_1)-\frac{\partial u_1}{\partial t}$ $L_2=L(u_2)-\frac{\partial u_2}{\partial t}$ Then our system is

$$\left|\begin{array}{l} u_1\frac{\partial u_1}{\partial x_1}+u_2\frac{\partial u_1}{\partial x_2}=-\frac{\partial \rho}{\partial x_1}+L_1+f_1(x_1,x_2,t) \\ u_1\frac{\partial u_2}{\partial x_1}+u_2\frac{\partial u_2}{\partial x_2}=-\frac{\partial \rho}{\partial x_2}+L_2+f_2(x_1,x_2,t) \\ \frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2}=0 \end{array}\right.$$

Where

$$\Delta=\begin{vmatrix} \frac{\partial u_1}{\partial x_1} & \frac{\partial u_1}{\partial x_2} \\ \frac{\partial u_1}{\partial x_2} & \frac{\partial u_2}{\partial x_2} \end{vmatrix}\neq 0;\ \text{and}\ \Delta_1=\begin{vmatrix} -\frac{\partial \rho}{\partial x_1}+L_1+f_1 & \frac{\partial u_1}{\partial x_2} \\ -\frac{\partial \rho}{\partial x_2}+L_2+f_2 & \frac{\partial u_2}{\partial x_2} \end{vmatrix}=$$

$$=\begin{vmatrix} -\frac{\partial \rho}{\partial x_1} & \frac{\partial u_1}{\partial x_2} \\ -\frac{\partial \rho}{\partial x_2} & \frac{\partial u_2}{\partial x_2} \end{vmatrix}+\begin{vmatrix} L_1 & \frac{\partial u_1}{\partial x_2} \\ L_2 & \frac{\partial u_2}{\partial x_2} \end{vmatrix}+\begin{vmatrix} f_1 & \frac{\partial u_1}{\partial x_2} \\ f_2 & \frac{\partial u_2}{\partial x_2} \end{vmatrix}\rightarrow$$

$$\rightarrow \Delta_1=\Delta_1(\rho)+\Delta_1(L)+\Delta_1(f)$$

$$\Delta_2 = \begin{vmatrix} \dfrac{\partial u_1}{\partial x_1} & -\dfrac{\partial \rho}{\partial x_1} + L_1 + f_1 \\ \dfrac{\partial u_2}{\partial x_1} & -\dfrac{\partial \rho}{\partial x_2} + L_2 + f_2 \end{vmatrix} = \Delta_2(\rho) + \Delta_2(L) + \Delta_2(f)$$

Then we have for $\Delta \neq 0$

$$u_1 = \frac{\Delta_1}{\Delta}, \quad u_2 = \frac{\Delta_2}{\Delta}$$

We now have to prepare to use the third equation of the Naviep-Stokes system. This means from the last expressions for $u_1, \ u_2$ to find $\frac{\partial u_1}{\partial x_1}$ and $\frac{\partial u_2}{\partial x_2}$

$$\frac{\partial u_1}{\partial x_1} = \frac{\dfrac{\partial \Delta_1}{\partial x_1}\Delta - \Delta_1 \dfrac{\partial \Delta}{\partial x_1}}{\Delta^2} \ \Delta \neq 0, \Leftrightarrow \left(\Delta^2 \neq 0\right)$$

$$\frac{\partial u_2}{\partial x_2} = \frac{\dfrac{\partial \Delta_2}{\partial x_2}\Delta - \Delta_2 \dfrac{\partial \Delta}{\partial x_2}}{\Delta^2} \ \Delta \neq 0, \Leftrightarrow \left(\Delta^2 \neq 0\right)$$

$$\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} = 0$$

$$\frac{\partial \Delta_1}{\partial x_1}\Delta - \Delta_1 \frac{\partial \Delta}{\partial x_1} + \frac{\partial \Delta_2}{\partial x_2}\Delta - \Delta_2 \frac{\partial \Delta}{\partial x} = 0 \Leftrightarrow$$

$$\Leftrightarrow \frac{\partial\left(\Delta_1(\rho) + \Delta_1(L_1) + \Delta_1(f_1)\right)}{\partial x_1}\Delta - \left(\Delta_1(\rho) + \Delta_1(L_1) + \Delta_1(f_1)\right)\frac{\partial \Delta}{\partial x_1} +$$
$$+ \frac{\partial\left(\Delta_2(\rho) + \Delta_2(L_2) + \Delta_2(f_2)\right)}{\partial x_2}\Delta - \left(\Delta_2(\rho) + \Delta_2(L_2) + \Delta_2(f_2)\right)\frac{\partial \Delta}{\partial x_2} = 0$$

$$\left(\frac{\partial \Delta_1(\rho)}{\partial x_1}\Delta - \Delta_1(\rho)\frac{\partial \Delta}{\partial x_1} + \frac{\partial \Delta_2(\rho)}{\partial x_2}\Delta - \Delta_2(\rho)\frac{\partial \Delta}{\partial x}\right) +$$
$$+\left(\frac{\partial \Delta_1(L_1)}{\partial x_1}\Delta - \Delta_1(L_1)\frac{\partial \Delta}{\partial x_1} + \frac{\partial \Delta_2(L_2)}{\partial x_2}\Delta - \Delta_2(L_2)\frac{\partial \Delta}{\partial x}\right)$$
$$+\left(\frac{\partial \Delta_1(f_1)}{\partial x_1}\Delta - \Delta_1(f_1)\frac{\partial \Delta}{\partial x_1} + \frac{\partial \Delta_2(f_2)}{\partial x_2}\Delta - \Delta_2(f_2)\frac{\partial \Delta}{\partial x}\right) = 0$$

This is a partial differential equation for ρ apparently from second order, written with determinants.

If we denote

$$K_0 = \frac{\partial \Delta_1(f_1)}{\partial x_1}\Delta - \Delta_1(f_1)\frac{\partial \Delta}{\partial x_1} + \frac{\partial \Delta_2(f_2)}{\partial x_2}\Delta - \Delta_2(f_2)\frac{\partial \Delta}{\partial x}$$

$$K_1 = \frac{\partial \Delta_1(L_1)}{\partial x_1}\Delta - \Delta_1(L_1)\frac{\partial \Delta}{\partial x_1} + \frac{\partial \Delta_2(L_2)}{\partial x_2}\Delta - \Delta_2(L_2)\frac{\partial \Delta}{\partial x}$$

Then the equation is

$$\frac{\partial \Delta_1(\rho)}{\partial x_1}\Delta - \Delta_1(\rho)\frac{\partial \Delta}{\partial x_1} + \frac{\partial \Delta_2(\rho)}{\partial x_2}\Delta - \Delta_2(\rho)\frac{\partial \Delta}{\partial x} + K_0 + K_1 = 0$$

Therefore, $\Delta, \frac{\partial \Delta}{\partial x_1}, \frac{\partial \Delta}{\partial x_2}$ we can consider as a parameters too.

Using

$$\frac{\partial \Delta_1(\rho)}{\partial x_1} = \begin{vmatrix} -\frac{\partial^2 \rho}{\partial x_1^2} & \frac{\partial^2 u_1}{\partial x_1 \partial x_2} \\ -\frac{\partial \rho}{\partial x_2} & \frac{\partial u_2}{\partial x_2} \end{vmatrix} + \begin{vmatrix} -\frac{\partial \rho}{\partial x_1} & \frac{\partial u_1}{\partial x_2} \\ -\frac{\partial^2 \rho}{\partial x_1 \partial x_2} & \frac{\partial^2 u_2}{\partial x_1 \partial x_2} \end{vmatrix}$$

$$\frac{\partial \Delta_2(\rho)}{\partial x_2} = \begin{vmatrix} \frac{\partial^2 u_1}{\partial x_1 \partial x_2} & -\frac{\partial\ \rho}{\partial x_1 \partial x} \\ \frac{\partial u_2}{\partial x_1} & -\frac{\partial \rho}{\partial x_2} \end{vmatrix} + \begin{vmatrix} \frac{\partial u_1}{\partial x_1} & -\frac{\partial \rho}{\partial x_1} \\ \frac{\partial^2 u_2}{\partial x_1 \partial x_2} & -\frac{\partial^2 \rho}{\partial x_2^2} \end{vmatrix}$$

$$\Delta_1(\rho) = \begin{vmatrix} -\frac{\partial \rho}{\partial x_1} & \frac{\partial u_1}{\partial x_2} \\ -\frac{\partial \rho}{\partial x_2} & \frac{\partial u_2}{\partial x_2} \end{vmatrix}, \quad \Delta_2(\rho) = \begin{vmatrix} \frac{\partial u_1}{\partial x_1} & -\frac{\partial \rho}{\partial x_1} \\ \frac{\partial u_2}{\partial x_1} & -\frac{\partial \rho}{\partial x_2} \end{vmatrix}$$

Then our PDE about ρ in the classical form is:

$$\left(\frac{\partial u_2}{\partial x_2}\Delta\right)\frac{\partial^2 \rho}{\partial x_1^2} - \left(\left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_1}\right)\Delta\right)\frac{\partial^2 \rho}{\partial x_1 \partial x_2} + \left(\frac{\partial u_1}{\partial x_1}\Delta\right)\frac{\partial^2 \rho}{\partial x_2^2} -$$

$$-\left(\frac{\partial u_2}{\partial x_2}\frac{\partial \Delta}{\partial x_1} - \frac{\partial u_2}{\partial x_1}\frac{\partial \Delta}{\partial x_2}\right)\frac{\partial \rho}{\partial x_1} - \left(\frac{\partial u_1}{\partial x_1}\frac{\partial \Delta}{\partial x_2} - \frac{\partial u_1}{\partial x_2}\frac{\partial \Delta}{\partial x_1}\right)\frac{\partial \rho}{\partial x_2} + K_0 + K_1 = \mathbf{0}$$

Where

$$K_0=\frac{\partial f_1(x_1,t)}{\partial x_1}\frac{\partial u_2}{\partial x_2}\Delta+\frac{\partial f_2(x_1,t)}{\partial x_1}\frac{\partial u_1}{\partial x_1}\Delta-f_1(x_1,t)\frac{\partial u_2}{\partial x_2}\frac{\partial \Delta}{\partial x_1}+f_2(x_2,t)\frac{\partial u_1}{\partial x_2}\frac{\partial \Delta}{\partial x_1}--\frac{\partial f_1(x_1,t)}{\partial x_2}\frac{\partial u_2}{\partial x_1}\Delta$$
$$+\frac{\partial f_2(x_1,t)}{\partial x_2}\frac{\partial u_1}{\partial x_1}\Delta-f_2(x_2,t)\frac{\partial u_1}{\partial x_1}\frac{\partial \Delta}{\partial x_2}+f_1(x_1,t)\frac{\partial u_2}{\partial x_1}\frac{\partial \Delta}{\partial x_2}$$

$$K_1=\frac{\partial L_1}{\partial x_1}\frac{\partial u_2}{\partial x_2}\Delta+\frac{\partial L_2}{\partial x_1}\frac{\partial u_1}{\partial x_1}\Delta-L_1\frac{\partial u_2}{\partial x_2}\frac{\partial \Delta}{\partial x_1}+L_2\frac{\partial u_1}{\partial x_2}\frac{\partial \Delta}{\partial x_1}-\frac{\partial L_1}{\partial x_2}\frac{\partial u_2}{\partial x_1}\Delta+\frac{\partial L_2}{\partial x_2}\frac{\partial u_1}{\partial x_1}\Delta-L_2\frac{\partial u_1}{\partial x_1}\frac{\partial \Delta}{\partial x_2}+L_1\frac{\partial u_2}{\partial x_1}\frac{\partial \Delta}{\partial x_2}$$

The first expression K_o contains besides the parametrs also the free members of the equations fro

the given system and the second K_1- only parameters and their introddused derivatives

Finally, using the pre-differentiation method, which we have already explained, the last equation is

transformed into an another equation

$$\left|\begin{array}{l}
\frac{\partial u_1}{\partial t}+u_1\frac{\partial u_1}{\partial x_1}+u_2\frac{\partial u_1}{\partial x_2}=L(u_1)+f_1(x_1,x_2,t)-\frac{\partial \rho}{\partial x_1}\\
\frac{\partial u_2}{\partial t}+u_1\frac{\partial u_2}{\partial x_1}+u_2\frac{\partial u_2}{\partial x_2}=L(u_2)+f_2(x_1,x_2,t)-\frac{\partial \rho}{\partial x_2}\\
\left(\frac{\partial u_2}{\partial x_2}\Delta\right)\frac{\partial^2\rho}{\partial x_1^2}-\left(\left(\frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_1}\right)\Delta\right)\frac{\partial^2\rho}{\partial x_1\partial x_2}+\left(\frac{\partial u_1}{\partial x_1}\Delta\right)\frac{\partial^2\rho}{\partial x_2^2}-\\
-\left(\frac{\partial u_2}{\partial x_2}\frac{\partial \Delta}{\partial x_1}-\frac{\partial u_2}{\partial x_1}\frac{\partial \Delta}{\partial x_2}\right)\frac{\partial \rho}{\partial x_1}-\left(\frac{\partial u_1}{\partial x_1}\frac{\partial \Delta}{\partial x_2}-\frac{\partial u_1}{\partial x_2}\frac{\partial \Delta}{\partial x_1}\right)\frac{\partial \rho}{\partial x_2}+K_0+K_1=0
\end{array}\right.$$

$$x\in R^2,\quad t\ge 0,\ \mathrm{u}(\mathrm{x},0)=u^0(x)$$

The third equation shows that the given system is compatible

Lemma. Let be given the two next determinants

$$\Delta=\left|\begin{array}{l} a_{11}\quad a_{12}\ \dots\ a_{1,k-1}\quad a_1\quad a_{1,k+1}\dots\ a_{1n}\\ \dots\\ a_{n1}\quad a_{n2}\ \dots\ a_{n,k-1}\quad a_n\quad a_{n,k+1}\dots a_{n,n}\ ..\end{array}\right|\quad \Delta_k=\left|\begin{array}{l} a_{11}\quad a_{12}\ \dots\ a_{1,k-1}\quad b_1\quad a_{1,k+1}\dots\ a_{1n}\\ \dots\\ a_{n1}\quad a_{n2}\ \dots\ a_{n,k-1}\quad b_n\quad a_{n,k+1}\dots\ a_{n,n}\end{array}\right|$$

Then the respective addeed amounts of the two k-pillars are equal.The same applies to their derivatives.

Proof. The proof is elementary and we omit it

Examples:

$$\Delta = \begin{vmatrix} \frac{\partial u_1}{\partial x_1} & \frac{\partial u_1}{\partial x_2} & \frac{\partial u_1}{\partial x_3} \\ \frac{\partial u_2}{\partial x_1} & \frac{\partial u_2}{\partial x_2} & \frac{\partial u_2}{\partial x_3} \\ \frac{\partial u_3}{\partial x_1} & \frac{\partial u_3}{\partial x_2} & \frac{\partial u_3}{\partial x_3} \end{vmatrix} \quad \Delta_1 = \begin{vmatrix} W_1 & \frac{\partial u_1}{\partial x_2} & \frac{\partial u_1}{\partial x_3} \\ W_2 & \frac{\partial u_2}{\partial x_2} & \frac{\partial u_2}{\partial x_3} \\ W_3 & \frac{\partial u_3}{\partial x_2} & \frac{\partial u_3}{\partial x_3} \end{vmatrix} \text{ Hence}$$

$$\Delta = \frac{\partial u_1}{\partial x_1} A_{11}(\Delta) - \frac{\partial u_2}{\partial x_1} A_{21}(\Delta) + \frac{\partial u_3}{\partial x_1} A_{31}(\Delta)$$

$$\Delta_1 = W_1 A_{11}(\Delta_1) - W_2 A_{21}(\Delta_1) + W_3 A_{31}(\Delta_1) = W_1 A_{11}(\Delta) - W_2 A_{21}(\Delta) + W_3 A_{31}(\Delta)$$

$$\frac{\partial \Delta_1}{\partial x_1} = \begin{vmatrix} \frac{\partial W_1}{\partial x_1} & \frac{\partial^2 u_1}{\partial x_1 \partial x_2} & \frac{\partial^2 u_1}{\partial x_1 \partial x_2} \\ W_2 & \frac{\partial u_2}{\partial x_2} & \frac{\partial u_2}{\partial x_3} \\ W_3 & \frac{\partial u_3}{\partial x_2} & \frac{\partial u_3}{\partial x_3} \end{vmatrix} + \begin{vmatrix} W_1 & \frac{\partial u_1}{\partial x_2} & \frac{\partial u_1}{\partial x_3} \\ \frac{\partial W_2}{\partial x_1^2} & \frac{\partial^2 u_2}{\partial x_1 \partial x_2} & \frac{\partial^2 u_2}{\partial x_1 \partial x_2} \\ W_3 & \frac{\partial u_3}{\partial x_2} & \frac{\partial u_3}{\partial x_3} \end{vmatrix} + \begin{vmatrix} W_1 & \frac{\partial u_1}{\partial x_2} & \frac{\partial u_1}{\partial x_3} \\ W_2 & \frac{\partial u_2}{\partial x_2} & \frac{\partial u_2}{\partial x_3} \\ \frac{\partial W_3}{\partial x_1} & \frac{\partial^2 u_3}{\partial x_1 \partial x_2} & \frac{\partial^2 u_3}{\partial x_1 \partial x_2} \end{vmatrix}$$

And using **Lemma** we have

$$\frac{\partial \Delta_1}{\partial x_1} = \frac{\partial W_1}{\partial x_1} A_{11}^{(1)}\left(\frac{\partial \Delta}{\partial x_1}\right) - W_2 A_{21}^{(1)}\left(\frac{\partial \Delta}{\partial x_1}\right) + W_3 A_{31}^{(1)}\left(\frac{\partial \Delta}{\partial x_1}\right) +$$

$$+ W_1 A_{11}^{(2)}\left(\frac{\partial \Delta}{\partial x_2}\right) - \frac{\partial W_2}{\partial x_1} A_{21}^{(2)}\left(\frac{\partial \Delta}{\partial x_2}\right) + W_3 A_{31}^{(2)}\left(\frac{\partial \Delta}{\partial x_2}\right) +$$

$$+ W_1 A_{11}^{(2)}\left(\frac{\partial \Delta}{\partial x_2}\right) - \frac{\partial W_2}{\partial x_1} A_{21}^{(2)}\left(\frac{\partial \Delta}{\partial x_2}\right) + W_3 A_{31}^{(2)}\left(\frac{\partial \Delta}{\partial x_2}\right)$$

The above index (k), k=1, 2, 3 means the line number with the corresponding derivatives. We

Directly see that $A_{ij}^{(k)}\left(\frac{\partial \Delta_1}{\partial x_1}\right) = A_{ij}^{(k)}\left(\frac{\partial \Delta}{\partial x_1}\right)$.This example ends **thelemma** analysis

III. The three-dimensional case of Navier-Stockes equations

We know that in this case the system is

$$\left|\begin{array}{l}\frac{\partial u_1}{\partial t}+u_1\frac{\partial u_1}{\partial x_1}+u_2\frac{\partial u_1}{\partial x_2}+u_3\frac{\partial u_1}{\partial x_3}=L(u_1)+f_1(x_1,x_2,x_3t)-\frac{\partial\rho}{\partial x_1}\\ \frac{\partial u_2}{\partial t}+u_1\frac{\partial u_2}{\partial x_1}+u_2\frac{\partial u_2}{\partial x_2}+u_3\frac{\partial u_2}{\partial x_3}=L(u_2)+f_2(x_1,x_2,x_3.t)-\frac{\partial\rho}{\partial x_2}\\ \frac{\partial u_3}{\partial t}+u_1\frac{\partial u_3}{\partial x_1}+u_2\frac{\partial u_3}{\partial x_2}+u_3\frac{\partial u_3}{\partial x_3}=L(u_3)+f_3(x_1,x_2,x_3.t)-\frac{\partial\rho}{\partial x_3}\\ \frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2}+\frac{\partial u_3}{\partial x_3}=0\end{array}\right.$$

$$x\in R^3,\quad t\geq 0,\ \mathrm{u}(\mathrm{x},0)=u^0(x)$$

Or

$$\left|\begin{array}{l}u_1\frac{\partial u_1}{\partial x_1}+u_2\frac{\partial u_1}{\partial x_2}+u_3\frac{\partial u_1}{\partial x_3}=-\frac{\partial\rho}{\partial x_1}+L(u_1)-\frac{\partial u_1}{\partial t}+f_1(x_1,x_2,x_3t)\\ u_1\frac{\partial u_2}{\partial x_1}+u_2\frac{\partial u_2}{\partial x_2}+u_3\frac{\partial u_2}{\partial x_3}=-\frac{\partial\rho}{\partial x_2}+L(u_2)-\frac{\partial u_2}{\partial t}+f_2(x_1,x_2,x_3.t)\\ u_1\frac{\partial u_3}{\partial x_1}+u_2\frac{\partial u_3}{\partial x_2}+u_3\frac{\partial u_3}{\partial x_3}=-\frac{\partial\rho}{\partial x_3}+L(u_3)-\frac{\partial u_3}{\partial t}+f_3(x_1,x_2,x_3.t)\\ \frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2}+\frac{\partial u_3}{\partial x_3}=0\end{array}\right.$$

We denote

$$\mathrm{L}_1=L(u_1)-\frac{\partial u_1}{\partial t},\quad \mathrm{L}_2=L(u_2)-\frac{\partial u_2}{\partial t},\quad \mathrm{L}_3=L(u_3)-\frac{\partial u_3}{\partial t}$$

Then our system is

$$\left|\begin{array}{l}u_1\frac{\partial u_1}{\partial x_1}+u_2\frac{\partial u_1}{\partial x_2}+u_3\frac{\partial u_1}{\partial x_3}=-\frac{\partial\rho}{\partial x_1}+L_1+f_1(x_1,x_2,x_3t)\\ u_1\frac{\partial u_2}{\partial x_1}+u_2\frac{\partial u_2}{\partial x_2}+u_3\frac{\partial u_2}{\partial x_3}=-\frac{\partial\rho}{\partial x_2}+L_2+f_2(x_1,x_2,x_3.t)\\ u_1\frac{\partial u_3}{\partial x_1}+u_2\frac{\partial u_3}{\partial x_2}+u_3\frac{\partial u_3}{\partial x_3}=-\frac{\partial\rho}{\partial x_3}+L_3+f_3(x_1,x_2,x_3.t)\\ \frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2}+\frac{\partial u_3}{\partial x_3}=0\end{array}\right.$$

Where

$$\Delta = \begin{vmatrix} \frac{\partial u_1}{\partial x_1} & \frac{\partial u_1}{\partial x_2} & \frac{\partial u_1}{\partial x_3} \\ \frac{\partial u_1}{\partial x_2} & \frac{\partial u_2}{\partial x_2} & \frac{\partial u_2}{\partial x_3} \\ \frac{\partial u_1}{\partial x_3} & \frac{\partial u_3}{\partial x_2} & \frac{\partial u_3}{\partial x_3} \end{vmatrix} \neq 0; \text{ and } \Delta_1 = \begin{vmatrix} -\frac{\partial \rho}{\partial x_1} + L_1 + f_1 & \frac{\partial u_1}{\partial x_2} & \frac{\partial u_1}{\partial x_3} \\ -\frac{\partial \rho}{\partial x_2} + L_2 + f_2 & \frac{\partial u_2}{\partial x_2} & \frac{\partial u_2}{\partial x_3} \\ -\frac{\partial \rho}{\partial x_3} + L_3 + f_3 & \frac{\partial u_3}{\partial x_2} & \frac{\partial u_3}{\partial x_3} \end{vmatrix} =$$

$$= \begin{vmatrix} -\frac{\partial \rho}{\partial x_1} & \frac{\partial u_1}{\partial x_2} & \frac{\partial u_1}{\partial x_3} \\ -\frac{\partial \rho}{\partial x_2} & \frac{\partial u_2}{\partial x_2} & \frac{\partial u_2}{\partial x_3} \\ -\frac{\partial \rho}{\partial x_3} & \frac{\partial u_3}{\partial x_2} & \frac{\partial u_3}{\partial x_3} \end{vmatrix} + \begin{vmatrix} L_1 & \frac{\partial u_1}{\partial x_2} & \frac{\partial u_1}{\partial x_3} \\ L_2 & \frac{\partial u_2}{\partial x_2} & \frac{\partial u_2}{\partial x_3} \\ L_3 & \frac{\partial u_3}{\partial x_2} & \frac{\partial u_3}{\partial x_3} \end{vmatrix} + \begin{vmatrix} f_1 & \frac{\partial u_1}{\partial x_2} & \frac{\partial u_1}{\partial x_3} \\ f_2 & \frac{\partial u_2}{\partial x_2} & \frac{\partial u_2}{\partial x_3} \\ f_3 & \frac{\partial u_3}{\partial x_2} & \frac{\partial u_3}{\partial x_3} \end{vmatrix} \rightarrow$$

$$\rightarrow \Delta_1 = \Delta_1(\rho) + \Delta_1(L) + \Delta_1(f)$$

$$\Delta_2 = \begin{vmatrix} \frac{\partial u_1}{\partial x_1} & -\frac{\partial \rho}{\partial x_1} + L_1 + f_1 & \frac{\partial u_1}{\partial x_3} \\ \frac{\partial u_2}{\partial x_1} & -\frac{\partial \rho}{\partial x_2} + L_2 + f_2 & \frac{\partial u_2}{\partial x_3} \\ \frac{\partial u_3}{\partial x_1} & -\frac{\partial \rho}{\partial x_3} + L_3 + f_3 & \frac{\partial u_3}{\partial x_3} \end{vmatrix} = \Delta_2(\rho) + \Delta_2(L) + \Delta_2(f)$$

$$\Delta_3 = \begin{vmatrix} \frac{\partial u_1}{\partial x_1} & \frac{\partial u_1}{\partial x_2} & -\frac{\partial \rho}{\partial x_1} + L_1 + f_1 \\ \frac{\partial u_2}{\partial x_1} & \frac{\partial u_2}{\partial x_2} & -\frac{\partial \rho}{\partial x_2} + L_2 + f_2 \\ \frac{\partial u_3}{\partial x_1} & \frac{\partial u_3}{\partial x_2} & -\frac{\partial \rho}{\partial x_3} + L_3 + f_3 \end{vmatrix} = \Delta_3(\rho) + \Delta_3(L) + \Delta_3(f)$$

We only look at the first three equations of the given system.From this subsystem of the given

$$u_1 = \frac{\Delta_1}{\Delta}, u_2 = \frac{\Delta_2}{\Delta}, u_3 = \frac{\Delta_3}{\Delta}$$

From the last condition of continuity of the given system:

$$\frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3} = 0 \text{ we have}$$

$$\frac{\partial u_1}{\partial x_1} = \frac{\frac{\partial \Delta_1}{\partial x_1}\Delta - \Delta_1 \frac{\partial \Delta}{\partial x_1}}{\Delta^2}, \frac{\partial u_2}{\partial x_2} = \frac{\frac{\partial \Delta_2}{\partial x_2}\Delta - \Delta_2 \frac{\partial \Delta}{\partial x_2}}{\Delta^2}, \frac{\partial u_3}{\partial x_3} = \frac{\frac{\partial \Delta_3}{\partial x_3}\Delta - \Delta_3 \frac{\partial \Delta}{\partial x_3}}{\Delta^2}, (\Delta \neq 0) \text{ or}$$

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$$\sum_{i=1}^{3}\frac{\frac{\partial\Delta_i}{\partial x_i}\Delta-\Delta_i\frac{\partial\Delta}{\partial x_i}}{\Delta^2}=0;\Delta\neq0\rightarrow$$

$$\sum_{i-1}^{3}\left(\frac{\partial\Delta_i}{\partial x_i}\Delta-\Delta_i\frac{\partial\Delta}{\partial x_i}\right)=0\leftrightarrow$$

$$\rightarrow\sum_{i=1}^{3}\left(\frac{\Delta_i(\rho)}{\partial x_i}\Delta-\Delta_i(\rho)\frac{\partial\Delta}{\partial x_i}\right)+\sum_{i=1}^{3}\left(\frac{\Delta_i(L)}{\partial x_i}\Delta-\Delta_i(L)\frac{\partial\Delta}{\partial x_i}\right)+\sum_{i=1}^{3}\left(\frac{\Delta_i(f)}{\partial x_i}\Delta-\Delta_i(f)\frac{\partial\Delta}{\partial x_i}\right)=0$$

$$K_0=\sum_{i=1}^{3}\left(\frac{\partial\Delta_i(f)}{\partial x_i}\Delta-\Delta_i(f)\frac{\partial\Delta}{\partial x_i}\right),K_1=\sum_{i=1}^{3}\left(\frac{\partial\Delta_i(L)}{\partial x_i}\Delta-\Delta_i(L)\frac{\partial\Delta}{\partial x_i}\right);$$

$$\sum_{i=1}^{3}\left(\frac{\partial\Delta_i(\rho)}{\partial x_i}\Delta-\Delta_i(\rho)\frac{\partial\Delta}{\partial x_i}\right)+K_0+K_1=0$$

$$\Delta=\begin{vmatrix}\frac{\partial u_1}{\partial x_1} & \frac{\partial u_1}{\partial x_2} & \frac{\partial u_1}{\partial x_3}\\ \frac{\partial u_2}{\partial x_1} & \frac{\partial u_2}{\partial x_2} & \frac{\partial u_2}{\partial x_3}\\ \frac{\partial u_3}{\partial x_1} & \frac{\partial u_3}{\partial x_2} & \frac{\partial u_3}{\partial x_3}\end{vmatrix}$$

$$\Delta_1(\rho)=\begin{vmatrix}-\frac{\partial\rho}{\partial x_1} & \frac{\partial u_1}{\partial x_2} & \frac{\partial u_1}{\partial x_3}\\ -\frac{\partial\rho}{\partial x_2} & \frac{\partial u_2}{\partial x_2} & \frac{\partial u_2}{\partial x_3}\\ -\frac{\partial\rho}{\partial x_3} & \frac{\partial u_3}{\partial x_2} & \frac{\partial u_3}{\partial x_3}\end{vmatrix}=-\frac{\partial\rho}{\partial x_1}A_{11}(\Delta)+\frac{\partial\rho}{\partial x_2}A_{21}(\Delta)-\frac{\partial\rho}{\partial x_3}A_{31}(\Delta)$$

$$\frac{\partial\Delta_1(\rho)}{\partial x_1}\Delta-\Delta_1(\rho)\frac{\partial\Delta}{\partial x_1}=$$

$$=\left(\begin{vmatrix}-\frac{\partial^2\rho}{\partial x_1^2} & \frac{\partial^2u_1}{\partial x_2\partial x_1} & \frac{\partial^2u_1}{\partial x_3\partial x_1}\\ -\frac{\partial\rho}{\partial x_2} & \frac{\partial u_2}{\partial x_2} & \frac{\partial u_2}{\partial x_3}\\ -\frac{\partial\rho}{\partial x_3} & \frac{\partial u_3}{\partial x_2} & \frac{\partial u_3}{\partial x_3}\end{vmatrix}+\begin{vmatrix}-\frac{\partial\rho}{\partial x_1} & \frac{\partial u_1}{\partial x_2} & \frac{\partial u_1}{\partial x_3}\\ -\frac{\partial^2\rho}{\partial x_1\partial x_2} & \frac{\partial^2u_2}{\partial x_2\partial x_1} & \frac{\partial^2u_2}{\partial x_3\partial x_1}\\ -\frac{\partial\rho}{\partial x_3} & \frac{\partial u_3}{\partial x_2} & \frac{\partial u_3}{\partial x_3}\end{vmatrix}+\begin{vmatrix}-\frac{\partial\rho}{\partial x_1} & \frac{\partial u_1}{\partial x_2} & \frac{\partial u_1}{\partial x_3}\\ -\frac{\partial\rho}{\partial x_2} & \frac{\partial u_2}{\partial x_2} & \frac{\partial u_2}{\partial x_3}\\ \frac{\partial^2\rho}{\partial x_1\partial x_3} & \frac{\partial^2u_3}{\partial x_2\partial x_1} & \frac{\partial^2u_3}{\partial x_3\partial x_1}\end{vmatrix}\right)\Delta-$$

$$-\left(-\frac{\partial\rho}{\partial x_1}A_{11}(\Delta)+\frac{\partial\rho}{\partial x_2}A_{21}(\Delta)-\frac{\partial\rho}{\partial x_3}A_{31}(\Delta)\right)\frac{\partial\Delta}{\partial x_1}.$$

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I.e.using our **lemma we receive in** canonical **form**

$$\frac{\partial\Delta_1(\rho)}{\partial x_1}\Delta-\Delta_1(\rho)\frac{\partial\Delta}{\partial x_1}=-A_{11}^1\left(\frac{\partial\Delta}{\partial x_1}\right)\Delta\frac{\partial^2\rho}{\partial x_1^2}+A_{21}^2\left(\frac{\partial\Delta}{\partial x_1}\right)\Delta\frac{\partial^2\rho}{\partial x_1\partial x_2}+A_{31}^3\left(\frac{\partial\Delta}{\partial x_1}\right)\Delta\frac{\partial^2\rho}{\partial x_1\partial x_3}+$$

$$\left(-A_{11}^2\left(\frac{\partial\Delta}{\partial x_1}\right)\Delta-A_{11}^3\left(\frac{\partial\Delta}{\partial x_1}\right)\Delta+A_{11}(\Delta)\frac{\partial\Delta}{\partial x_1}\right)\frac{\partial\rho}{\partial x_1}+$$

$$+\left(A_{21}^1\left(\frac{\partial\Delta}{\partial x_1}\right)\Delta+A_{21}^3\left(\frac{\partial\Delta}{\partial x_1}\right)\Delta+A_{21}(\Delta)\frac{\partial\Delta}{\partial x_1}\right)\frac{\partial\rho}{\partial x_2}+$$

$$+\left(A_{31}^3\left(\frac{\partial\Delta}{\partial x_1}\right)\Delta-A_{31}^2\left(\frac{\partial\Delta}{\partial x_1}\right)\Delta+A_{31}(\Delta)\frac{\partial\Delta}{\partial x_1}\right)\frac{\partial\rho}{\partial x_3}\ \text{Analogy}$$

$$\frac{\partial\Delta_2(\rho)}{\partial x_2}\Delta-\Delta_2(\rho)\frac{\partial\Delta}{\partial x_2}=-A_{22}^2\left(\frac{\partial\Delta}{\partial x_2}\right)\Delta\frac{\partial^2\rho}{\partial x_2^2}+A_{12}^1\left(\frac{\partial\Delta}{\partial x_2}\right)\Delta\frac{\partial^2\rho}{\partial x_1\partial x_2}+A_{32}^3\left(\frac{\partial\Delta}{\partial x_2}\right)\Delta\frac{\partial^2\rho}{\partial x_2\partial x_3}+$$

$$\left(A_{12}^2\left(\frac{\partial\Delta}{\partial x_2}\right)\Delta-A_{12}^3\left(\frac{\partial\Delta}{\partial x_2}\right)\Delta-A_{12}(\Delta)\frac{\partial\Delta}{\partial x_2}\right)\frac{\partial\rho}{\partial x_1}+$$

$$+\left(-A_{22}^1\left(\frac{\partial\Delta}{\partial x_2}\right)\Delta-A_{22}^3\left(\frac{\partial\Delta}{\partial x_2}\right)\Delta+A_{22}(\Delta)\frac{\partial\Delta}{\partial x_2}\right)\frac{\partial\rho}{\partial x_2}+$$

$$+\left(A_{32}^1\left(\frac{\partial\Delta}{\partial x_2}\right)\Delta+A_{32}^2\left(\frac{\partial\Delta}{\partial x_2}\right)\Delta-A_{32}(\Delta)\frac{\partial\Delta}{\partial x_2}\right)\frac{\partial\rho}{\partial x_3}$$

$$\frac{\partial\Delta_3(\rho)}{\partial x_3}\Delta-\Delta_3(\rho)\frac{\partial\Delta}{\partial x_3}=-A_{33}^3\left(\frac{\partial\Delta}{\partial x_3}\right)\Delta\frac{\partial^2\rho}{\partial x_3^2}-A_{13}^1\left(\frac{\partial\Delta}{\partial x_3}\right)\Delta\frac{\partial^2\rho}{\partial x_1\partial x_3}+A_{23}^2\left(\frac{\partial\Delta}{\partial x_3}\right)\Delta\frac{\partial^2\rho}{\partial x_2\partial x_3}-$$

$$\left(-A_{13}^2\left(\frac{\partial\Delta}{\partial x_3}\right)\Delta-A_{13}^3\left(\frac{\partial\Delta}{\partial x_3}\right)\Delta+A_{13}(\Delta)\frac{\partial\Delta}{\partial x_3}\right)\frac{\partial\rho}{\partial x_1}+$$

$$+\left(A_{23}^1\left(\frac{\partial\Delta}{\partial x_3}\right)\Delta+A_{23}^3\left(\frac{\partial\Delta}{\partial x_3}\right)\Delta-A_{23}(\Delta)\frac{\partial\Delta}{\partial x_3}\right)\frac{\partial\rho}{\partial x_2}+$$

$$+\left(-A_{33}^1\left(\frac{\partial\Delta}{\partial x_3}\right)\Delta-A_{33}^2\left(\frac{\partial\Delta}{\partial x_3}\right)\Delta-A_{33}^3\left(\frac{\partial\Delta}{\partial x_3}\right)\Delta\right)\frac{\partial\rho}{\partial x_3}$$

All determinants will be develop on the suitableelements. Therefore, the coefficients in these

expressions are the corresponding added quantities. All these suitable quantities are expressions of parameters. All system is an example of a parametrical system. Consequently, physicists will be able to solve the task to the end in a practical relationship between speeds.

Or finallywe received

$$\left|\begin{array}{l}\frac{\partial u_1}{\partial x_1}u_1+\frac{\partial u_1}{\partial x_2}u_2+\frac{\partial u_1}{\partial x_3}u_3=-\frac{\partial \rho}{\partial x_1}+L(u_1)-\frac{\partial u_1}{\partial t}+f_1(x_1,x_2,x_3,t)\\ \frac{\partial u_2}{\partial x_1}u_1+\frac{\partial u_2}{\partial x_2}u_2+\frac{\partial u_2}{\partial x_3}u_3=-\frac{\partial \rho}{\partial x_2}+L(u_2)-\frac{\partial u_2}{\partial t}+f_2(x_1,x_2,x_3,t)\\ \frac{\partial u_3}{\partial x_1}u_1+\frac{\partial u_2}{\partial x_2}u_2+\frac{\partial u_3}{\partial x_3}u_3=-\frac{\partial \rho}{\partial x_3}+L(u_3)-\frac{\partial u_3}{\partial t}+f_3(x_1,x_2,x_3,t)\\ A_{11}^1\left(\frac{\partial\Delta}{\partial x_1}\right)\Delta\frac{\partial^2\rho}{\partial x_1^2}-A_{22}^2\left(\frac{\partial\Delta}{\partial x_2}\right)\Delta\frac{\partial^2\rho}{\partial x_2^2}-A_{33}^3\left(\frac{\partial\Delta}{\partial x_3}\right)\Delta\frac{\partial^2\rho}{\partial x_3^2}+\\ +\left(A_{21}^2\left(\frac{\partial\Delta}{\partial x_1}\right)\Delta+A_{12}^1\left(\frac{\partial\Delta}{\partial x_2}\right)\Delta\right)\frac{\partial^2\rho}{\partial x_1\partial x_2}+\left(-A_{31}^3\left(\frac{\partial\Delta}{\partial x_1}\right)\Delta-A_{13}^1\left(\frac{\partial\Delta}{\partial x_3}\right)\Delta\right)\frac{\partial^2\rho}{\partial x_1\partial x_3}+\\ +\left(A_{32}^3\left(\frac{\partial\Delta}{\partial x_2}\right)+A_{23}^2\left(\frac{\partial\Delta}{\partial x_3}\right)\right)\Delta\frac{\partial^2\rho}{\partial x_2\partial x_3}-\\ -\left(\begin{array}{l}-A_{11}^2\left(\frac{\partial\Delta}{\partial x_1}\right)\Delta-A_{11}^3\left(\frac{\partial\Delta}{\partial x_1}\right)\Delta+A_{11}(\Delta)\frac{\partial\Delta}{\partial x_1}+A_{12}^2\left(\frac{\partial\Delta}{\partial x_2}\right)\Delta+A_{12}^3\left(\frac{\partial\Delta}{\partial x_2}\right)\Delta+A_{12}(\Delta)\frac{\partial\Delta}{\partial x_2}\\ +A_{13}^2\left(\frac{\partial\Delta}{\partial x_3}\right)\Delta-A_{13}^3\left(\frac{\partial\Delta}{\partial x_3}\right)\Delta+A_{13}(\Delta)\frac{\partial\Delta}{\partial x_3}\end{array}\right)\frac{\partial\rho}{\partial x_1}+\\ +\left(\begin{array}{l}A_{21}^1\left(\frac{\partial\Delta}{\partial x_1}\right)\Delta+A_{21}^3\left(\frac{\partial\Delta}{\partial x_1}\right)\Delta+A_{21}(\Delta)\frac{\partial\Delta}{\partial x_1}-A_{22}^1\left(\frac{\partial\Delta}{\partial x_2}\right)\Delta-A_{22}^3\left(\frac{\partial\Delta}{\partial x_2}\right)\Delta-\\ -A_{22}(\Delta)\frac{\partial\Delta}{\partial x_2}+A_{23}^1\left(\frac{\partial\Delta}{\partial x_3}\right)\Delta+A_{23}^3\left(\frac{\partial\Delta}{\partial x_3}\right)\Delta+A_{23}(\Delta)\frac{\partial\Delta}{\partial x_3}\end{array}\right)\frac{\partial\rho}{\partial x_2}+\\ +\left(\begin{array}{l}-A_{31}^1\left(\frac{\partial\Delta}{\partial x_1}\right)\Delta-A_{31}^2\left(\frac{\partial\Delta}{\partial x_1}\right)\Delta+A_{31}(\Delta)\frac{\partial\Delta}{\partial x_1}+A_{32}^1\left(\frac{\partial\Delta}{\partial x_2}\right)\Delta+\\ +A_{32}^2\left(\frac{\partial\Delta}{\partial x_2}\right)\Delta+A_{32}(\Delta)\frac{\partial\Delta}{\partial x_2}-A_{33}^1\left(\frac{\partial\Delta}{\partial x_3}\right)\Delta+A_{33}^2\left(\frac{\partial\Delta}{\partial x_3}\right)\Delta-A_{33}(\Delta)\frac{\partial\Delta}{\partial x_3}\end{array}\right)\frac{\partial\rho}{\partial x_3}+K_0+K_1=0\end{array}\right.$$

Where $K_0=\sum_{i=1}^{3}\left(\frac{\partial\Delta_i(f)}{\partial x_i}\Delta-\Delta_i(f)\frac{\partial\Delta}{\partial x_i}\right)$ and $K_1=\sum_{i=1}^{3}\left(\frac{\partial\Delta_i(L)}{\partial x_i}\Delta-\Delta_i(L)\frac{\partial\Delta}{\partial x_i}\right)$

$$x\in R^3,\quad t\geq 0,\quad \mathrm{u}(\mathrm{x},0)=u^0(x)$$

.

The system thus obtained means that the equations of Navier Stokes form a compatible system.

The last equation is a studied task for $\Delta\neq 0$. There is a case $\Delta=0$ that is easier.

Finally, let'sagain look at Lagrange's equation by theoryof ODE.

$$y=x\varphi(y')+f(y'),\quad \mathrm{y}=\mathrm{y}(\mathrm{x})$$

The final comparison with the concept of continuity from the Navier Stokes system shows that and in the Lagrange equation there must be a condition of continuity, and then the Lagrange task is recorded in fullness as follows

$$\left| \begin{array}{l} y = x\varphi(y') + f(y'), \quad \mathrm{y} = \mathrm{y}(\mathrm{x}) \\ \dfrac{dy}{dx} \cdot \dfrac{dx}{dy} = 1 \end{array} \right.$$

The second equation is trivial:“Of infinitely small amendment of **x** corresponds infinitely small

Changes of **y** and vice versa of infinitely small amendment of **y** corresponds infinitely small

Changes of **x.”** What else besides the continuity condition is the second equation?

Namely it completes the correct comparison of ideas from ODE in PDE.

We have to make a meaningful interpretation of the two obtained partial differential equations and to

Prove the equations of Navier-Stokes for the general case. But this is a follow-up development.

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