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Penta Partitioned Pythagorean Fuzzy Weak Bi-Ideal of γ – Near- Ring

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Abstract:

We introduce the concept of Penta partitioned Pythagorean fuzzy Weak Bi-Ideals of γ-near-ring and examine some of its characteristics. Additionally defined is the homomorphism of Penta partitioned Pythagorean fuzzy weak bi-ideals of γ-near-rings.

Keywords: Penta partitioned, Pythagorean, Bi-Ideal, Weak Bi-Ideal, Near ring, Gamma near -ring.

1. Introduction:x

Fuzzy sets were defined by Zadeh [8] to handle uncertainty. Chinnadurai and Kadalarasi [2] investigated the near-ring qualities of Fuzzy Weak Bi-Ideals, whereas Yager [6][7] defined and examined the properties of Pythagorean Fuzzy Set (PFS) and Booth [1] presented the properties of γ-Near Ring. Fuzzy ideals were introduced by Jun et al. [3] and their characteristics in the γ – Near-ring were examined. Near ring features of fuzzy bi-ideals were introduced by Manikandan and Selvam [4]. The Penta partitioned Neutrosophic set of Rama Malick and Surapati Pramanik [5] and some of its attributes.

Penta partitioned Pythagorean Fuzzy Weak Bi-Ideal of γ – Near-ring is the concept we provide. A few characteristics of the Penta partitioned Pythagorean Fuzzy Weak Bi-Ideal of γ – Near-ring are discussed. Additionally defined is the homomorphism of Penta partitioned Pythagorean fuzzy weak bi-ideals of γ-near-ring.

2. Preliminaries:

Definition 2.1.

A fuzzy subset τ of U is called a fuzzy bi-ideal of Near-ring U, if (i) $\tau(\zeta-\eta) \geq \min\{\tau(\zeta), \tau(\eta)\}$, (ii) $\tau(\zeta z) \geq \min\{\tau(\zeta), \tau(z)\}$

Definition 2.2.

A fuzzy set $\boldsymbol{\sigma}$ of a γ – Near Ring ꝑ is known as a fuzzy left (resp. right) ideal of ꝑ. If

i. $\sigma(\zeta-\eta) \geq \min[\sigma(x), \sigma(\eta)]$, $\forall\ \zeta, \eta \in$ ꝑ

ii. $\sigma(\eta+\zeta-\eta) \geq \sigma(\zeta)\ \forall\ \zeta, \eta \in$ ꝑ

iii. $\sigma[ua(\zeta+v)-uav] \geq \sigma(\zeta) \rightarrow \sigma(\zeta au) \geq \sigma(\zeta)$

Definition 2.3.

A fuzzy set $'\boldsymbol{\sigma}'$ of a γ – Near Ring ꝑ is said a fuzzy bi-ideal of ꝑ, If

i. $\sigma(\zeta-\eta) \geq \min[\sigma(\zeta), \sigma(\eta)]$, $\forall\ x, \eta \in$ ꝑ

ii. $\sigma(\zeta\alpha\eta\beta z) \geq \min[\sigma(\zeta), \sigma(z)]$, $\forall\ \zeta, \eta, z \in$ ꝑ, $\alpha, \beta \in \gamma$.

Definition 2.4.

A Pythagorean fuzzy subsets P is a non-empty set ℵ is a target having the form

$P = \{(\aleph, \pi_p(\zeta), \gamma_p(\zeta)/\ \zeta \in \aleph\}$, where the function $\pi_p: \aleph \rightarrow [0,1]$

And $\gamma_p: \aleph \rightarrow [0,1]$ denote the degree of membership and non- membership of each element

$\zeta \in \aleph$ to the set P individually in order and $0 \leq [\pi_p(\zeta)]^2 + [\gamma_p(\zeta)]^2 \leq 1$, $\forall\ \zeta \in \aleph$

Definition 2.5.

Consider ꝑ be a γ – Near Ring and Let Ç be a Penta partitioned sets defined a mapping from ꝑ to 5-parts of fuzzy sets $Ç(\zeta) = \{\pi(\zeta), \gamma(\zeta), \boldsymbol{\vartheta}(\zeta), \mu(\zeta), \theta(\zeta)\}$,

Where $\pi(\zeta)$- strongly true; $\gamma(\zeta)$- weakly true; $\boldsymbol{\vartheta}(\zeta)$- ignorance; $\mu(\zeta)$ - weakly falsity; $\theta(x)$ –

strongly falsity. With all values [0,1] and bounded condition

$0 \leq \{\pi(\zeta)+\gamma(\zeta)+\vartheta(\zeta)+\mu(\zeta)+\theta(\zeta)\} \leq 5$

Definition 2.6.

A subgroup V of (ƥ, +) is said to be a weak bi-ideal of ƥ. If VγVγVCV.

3. Penta partitioned Pythagorean Fuzzy Weak Bi-Ideal of γ – Near Ring Definition 3.1.

The Penta partition Pythagorean fuzzy subsets of ϱ a non-empty set ℵ is a having the form $\varrho = \{\zeta, \pi(\zeta), \gamma(\zeta), \vartheta(\zeta), \mu(\zeta), \theta(\zeta) / \zeta \in \aleph\}$,

where $\pi(\zeta)$- strongly true; $\gamma(\zeta)$- weakly true; $\vartheta(\zeta)$- ignorance; $\mu(\zeta)$ - weakly falsity; $\theta(\zeta)$ – strongly falsity with all values [0,1] and bounded condition is

$0 \leq \pi_\varrho(\zeta)^2 + \gamma_\varrho(\zeta)^2 + \vartheta_\varrho(\zeta)^2 + \mu_\varrho(\zeta)^2 + \theta_\varrho(\zeta)^2 \leq 3$

Definition 3.2.

A Penta Partition Pythagorean fuzzy set $\varrho = \{\pi(\zeta), \gamma(\zeta), \vartheta(\zeta), \mu(\zeta), \theta(\zeta)\}$

of ƥ is known as a Penta Partitioned Pythagorean fuzzy weak bi-ideal (PPFWBI) of ƥ, If

- $\pi_\varrho(\zeta - \eta) \geq \min\{\pi_\varrho(\zeta), \pi_\varrho(\eta)\}$
- $\gamma_\varrho(\zeta - \eta) \geq \min\{\gamma_\varrho(\zeta), \gamma_\varrho(\eta)\}$
- $\vartheta_\varrho(\zeta - \eta) \geq \min\{\vartheta_\varrho(\zeta), \vartheta_\varrho(\eta)\}$
- $\mu_\varrho(\zeta - \eta) \leq \max\{\mu_\varrho(\zeta), \mu_\varrho(\eta)\}$
- $\theta_\varrho(\zeta - \eta) \leq \max\{\theta_\varrho(\zeta), \theta_\varrho(\eta)\}$
- $\pi_\varrho(\zeta\alpha\eta\beta z) \geq \min\{\pi_\varrho(\zeta), \pi_\varrho(\eta), \pi_\varrho(z)\}$
- $\gamma_\varrho(\zeta\alpha\eta\beta z) \geq \min\{\gamma_\varrho(\zeta), \gamma_\varrho(\eta), \gamma_\varrho(z)\}$

Example:

- $\vartheta_\varrho(\zeta\alpha\eta\beta z) \geq \min\{\vartheta_\varrho(\zeta), \vartheta_\varrho(\eta), \vartheta_\varrho(z)\}$
- $\mu_\varrho(\zeta\alpha\eta\beta z) \leq \max\{\mu_\varrho(\zeta), \mu_\varrho(\eta), \mu_\varrho(z)\}$
- $\theta_\varrho(\zeta\alpha\eta\beta z) \leq \max\{\theta_\varrho(\zeta), \theta_\varrho(\eta), \theta_\varrho(z)\}$ $\forall$ ζ ,η, z ∈ ƥ , α, β ∈ γ

Let ƥ = {á, ò, í, é}be non-empty set with binary operation '+' and 'γ' be non-empty set with binary operation as the following table.

+	Á	ò	í	É
Á	Á	ò	í	É
Ò	Ò	á	é	Í
Í	Í	é	á	Ò
É	É	í	ò	Á

γ	á	ò	Í	é
á	á	ò	Í	é
ò	ò	á	É	í
í	í	é	Á	ò
é	é	í	Ò	á

Let π_p: ƥ → [0,1] be a Penta partitioned fuzzy subsets defined by

π_ϱ(á) =0.7: π_ϱ(ò)= 0.6; π_ϱ(í)= 0. 5; π_ϱ(é)=0.5

γ_ϱ(á) =0.5: γ_ϱ(ò)= 0.3; γ_ϱ(í)= 0.8; γ_ϱ(é)=0.6

ϑ_ϱ(á) =0.2: ϑ_ϱ(ò)= 0.3; ϑ_ϱ(í)= 0.2; ϑ_ϱ(é)=0.3

μ_v(á) =0.1: μ_ϱ(ò)= 0.2; μ_ϱ(í)= 0.1; μ_ϱ(é)=0.8

θ_ϱ(á) =0.6: θ_v(ò)= 0.1; θ_ϱ(í)= 0.1; θ_v(é)=0.5

Theorem 3.1.

Let $\varrho = \{\pi_\varrho, \gamma_\varrho, \vartheta_\varrho, \mu_\varrho, \theta_\varrho\}$ be a Penta partitioned Pythagorean fuzzy subset of ƥ. Then

$\varrho = \{\pi_\varrho, \gamma_\varrho, \vartheta_\varrho, \mu_\varrho, \theta_\varrho\}$ is a Penta partitioned PFWBI of ƥ. If and only if $\pi_\varrho * \pi_\varrho * \pi_\varrho \subseteq \pi_\varrho$

$\mu_\varrho * \mu_\varrho * \mu_\varrho \supseteq \mu_\varrho$ and $\theta_\varrho * \theta_\varrho * \theta_\varrho \supseteq \theta_\varrho$.

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Proof:

Assume that, $\varrho = \{\pi_\varrho, \gamma_\varrho, \boldsymbol{\vartheta}_\varrho, \mu_p, \varrho_p\}$ is a Penta partitioned Pythagorean fuzzy weak Bi-ideal of ${\rm p}$. Let $\zeta, \eta, z, q, r \in {\rm p}$, $\alpha,\beta \in \gamma$, such that $\zeta = y\alpha z$ and $y = q\beta r$.

After that,

$(\pi_\varrho * \pi_\varrho * \pi_\varrho)(\zeta) = \sup_{\zeta = y\alpha z}\{\min\{(\pi_\varrho * \pi_\varrho)(\eta), \pi_\varrho(z)\}\}$

$= \sup_{\zeta=\eta\alpha z}\{\min\{\sup_{\eta=q\beta r}\{\min(\pi_\varrho(q), \pi_\varrho(q)), \pi_\varrho(z)\}\}\}$

$= \sup_{\zeta=\eta\alpha z}\sup_{\eta=q\beta r}\{\min\{\pi_\varrho(q), \pi_\varrho(q), \pi_\varrho(z)\}\}$

$= \sup_{\zeta=q\beta r\alpha z}\{\min\{\pi_\varrho(q), \pi_\varrho(q), \pi_\varrho(z)\}\}$ ---(1),

since π_ϱ is a fuzzy weak bi – ideal of ${\rm p}$.

Therefore $\pi_p(\zeta) \geq \min\{\pi_\varrho(q), \pi_\varrho(q), \pi_\varrho(z)\}$ -------------------------------(2),

(2) sub in (1),

$\leq \sup_{\zeta=q\beta r\alpha z}\pi_\varrho(q\beta r\alpha z)$

$\leq \pi_\varrho(\zeta)$ {since $\zeta = q\beta r\alpha z$}

$(\pi_\varrho * \pi_\varrho * \pi_\varrho)(v) \leq \pi_\varrho(\zeta)$

$(\pi_\varrho * \pi_\varrho * \pi_\varrho)(\zeta) \subseteq \pi_\varrho(\zeta)$

$(\gamma_\varrho * \gamma_\varrho * \gamma_\varrho)(\zeta) = \sup_{\zeta=\eta\alpha z}\{\min\{(\gamma_\varrho * \gamma_\varrho)(\eta), \gamma_\varrho(z)\}\}$

$= \sup_{\zeta=\eta\alpha z}\{\min\{\sup_{\eta=q\beta r}\{\min(\gamma_\varrho(q), \gamma_\varrho(q)), \gamma_\varrho(z)\}\}\}$

$= \sup_{\zeta=\eta\alpha z}\sup_{\eta=q\beta r}\{\min\{\gamma_\varrho(q), \gamma_\varrho(q), \gamma_\varrho(z)\}\}$

$= \sup_{\zeta=q\beta r\alpha z}\{\min\{\gamma_\varrho(q), \gamma_\varrho(q), \gamma_\varrho(z)\}\}(3)$,

since γ_ϱ is a fuzzy weak bi – ideal of ${\rm p}$.

Therefore $\gamma_\varrho(\zeta) \geq \min\{\gamma_\varrho(q), \gamma_\varrho(q), \gamma_\varrho(z)\}(4)$

(3) Substitute in (4),

$\leq \sup_{\zeta)=q\beta r\alpha z}\{\gamma_\varrho(q\beta r\alpha z)\}$

$\leq \gamma_\varrho(\zeta)$ {since $\zeta = q\beta r\alpha z$}

$(\gamma_\varrho * \gamma_\varrho * \gamma_\varrho)(\zeta) \leq \gamma_\varrho(\zeta)$

$(\gamma_\varrho * \gamma_\varrho * \gamma_\varrho)(\zeta) \subseteq \gamma_p(\zeta)$

$(\boldsymbol{\vartheta}_\varrho * v_\varrho * \boldsymbol{\vartheta}_\varrho)(\zeta) = \sup_{\zeta=\eta\alpha z}\{\min\{(\boldsymbol{\vartheta}_\varrho * \boldsymbol{\vartheta}_\varrho)(y), \boldsymbol{\vartheta}_\varrho(z)\}\}$

$= \sup_{\zeta=\eta\alpha z}\{\min\{\sup_{\eta=q\beta r}\{\min(\boldsymbol{\vartheta}_\varrho(q), v_\varrho(q)), \boldsymbol{\vartheta}_\varrho(z)\}\}\}$

$= \sup_{\zeta=\eta\alpha z}\sup_{\eta=q\beta r}\{\min\{\boldsymbol{\vartheta}_\varrho(q), v_\varrho(q), v_\varrho(z)\}\}$

$= \sup_{\zeta=q\beta r\alpha z}\{\min\{v_\varrho(q), \boldsymbol{\vartheta}_\varrho(q), \boldsymbol{\vartheta}_\varrho(z)\}\}(5)$,

since π_ϱ is a fuzzy weak bi – ideal of ${\rm p}$.

Therefore $\pi_\varrho(\zeta) \geq \min\{\boldsymbol{\vartheta}_\varrho(q), \boldsymbol{\vartheta}_\varrho(q), \boldsymbol{\vartheta}_\varrho(z)\}(6)$

(6) Substitute in (5),

$\leq \sup_{\zeta=q\beta r\alpha z}\boldsymbol{\vartheta}_\varrho(q\beta r\alpha z)$

$\leq \boldsymbol{\vartheta}_\varrho(\zeta)$ {since $\zeta = q\beta r\alpha z$}

$(\boldsymbol{\vartheta}_\varrho * \boldsymbol{\vartheta}_v * \boldsymbol{\vartheta}_\varrho)(\zeta) \leq \boldsymbol{\vartheta}_\varrho(\zeta)$

$(\boldsymbol{\vartheta}_\varrho * \boldsymbol{\vartheta}_\varrho * \boldsymbol{\vartheta}_\varrho)(\zeta) \subseteq \boldsymbol{\vartheta}_\varrho(\zeta)$

$(\mu_\varrho * \mu_\varrho * \mu_\varrho)(\zeta) = \inf_{\zeta=y\alpha z}\{\max\{(\mu_\varrho * \mu_\varrho)(\eta), \mu_\varrho(z)\}\}$

$= \inf_{\zeta=y\alpha z}\{\max\{\inf_{\eta=q\beta r}\{\inf(\mu_\varrho(q), \mu_\varrho(q)), \mu_\varrho(z)\}\}\}$

$= \inf_{\zeta=\eta\alpha z}\inf_{\eta=q\beta r}\{\max\{\mu_\varrho(q), \mu_\varrho(q), \mu_\varrho(z)\}\}$

$= \inf_{\zeta=q\beta r\alpha z}\{\max\{\mu_v(q), \mu_\varrho(q), \mu_\varrho(z)\}\}(7)$,

since μ_ϱ is a fuzzy weak bi – ideal of ${\rm p}$.

Therefore $\mu_\varrho(\zeta) \geq \min\{\mu_\varrho(q), \mu_\varrho(q), \mu_\varrho(z)\}(8)$

(8) Substitute in (7),

$\geq \inf_{\zeta=q\beta r\alpha z}\mu_\varrho(q\beta r\alpha z)$

$\geq \mu_\varrho(\zeta)$ {since $\zeta = q\beta r\alpha z$}

$(\theta_\varrho * \theta_\varrho * \theta_\varrho)(\zeta) = \inf_{\zeta=\eta\alpha z}\{\max\{(\theta_\varrho * \theta_\varrho)(y), \theta_\varrho(z)\}\}$

$= \inf_{\zeta=\eta\alpha z}\{\max\{\inf_{\eta=q\beta r}\{\inf(\theta_v(q), \theta_\varrho(q)), \theta_\varrho(z)\}\}\}$

$= \inf_{\zeta=\eta\alpha z}\inf_{\eta=q\beta r}\{\max\{\theta_\varrho(q), \theta_\varrho(q), \theta_\varrho(z)\}\}$

$= \inf_{\zeta=q\beta r\alpha z}\{\max\{\theta_\varrho(q), \theta_\varrho(q), \theta_\varrho(z)\}\}(9)$,

since θ_ϱ is a fuzzy weak bi – ideal of ${\rm p}$.

Therefore $\theta_\varrho(\zeta) \geq \max\{\theta_\varrho(q), \theta_\varrho(q), \theta_\varrho(z)\}(10)$

(10) sub in (9),

$\geq \theta_\varrho(\zeta)$ {since $\zeta = q\beta r\alpha z$}

If ζ can't be express as $\zeta = q\beta r\alpha z$, then $(\pi_\varrho * \pi_\varrho * \pi_\varrho)(\zeta) = 0 \leq \pi_\varrho(\zeta)$; $(\gamma_\varrho * \gamma_\varrho * \gamma_\varrho)(\zeta) = 0 \leq \gamma_\varrho(\zeta)$; $(\boldsymbol{\vartheta}_\varrho * \boldsymbol{\vartheta}_\varrho * \boldsymbol{\vartheta}_\varrho)(\zeta) = 0 \leq \boldsymbol{\vartheta}_\varrho(\zeta)$;

$(\mu_\varrho * \mu_\varrho * \mu_\varrho)(\zeta) = 0 \geq \mu_\varrho(\zeta)$; $(\theta_\varrho * \theta_\varrho * \theta_\varrho)(\zeta) = 0 \geq \theta_\varrho(\zeta)$. In case all, $(\pi_\varrho * \pi_v * \pi_\varrho)(\zeta) = 0 \subseteq \pi_\varrho(\zeta)$;

$(\gamma_\varrho * \gamma_\varrho * \gamma_\varrho)(\zeta) = 0 \subseteq \gamma_\varrho(\zeta)$; $(\boldsymbol{\vartheta}_\varrho * \boldsymbol{\vartheta}_\varrho * \boldsymbol{\vartheta}_\varrho)(\zeta) = 0 \subseteq \boldsymbol{\vartheta}_\varrho(\zeta)$; $(\mu_\varrho * \mu_v * \mu_\varrho)(\zeta) = 0 \geq \mu_\varrho(\zeta)$; $(\theta_\varrho * \theta_\varrho *$

$\theta_\varrho)(\zeta) = 0 \geq \theta_\varrho(\zeta)$.

Conversely,

assume that $(\pi_\varrho * \pi_\varrho * \pi_v)(\zeta) = 0 \subseteq \pi_\varrho(\zeta)$

For $x', \zeta, \eta, z \in \varrho$ and $\alpha, \beta, \alpha_1, \beta_1 \in \gamma$.

Let $x' = x\ \alpha\ y\ \beta\ z$ then

$\pi_\varrho(\zeta\alpha y\beta z) = \pi_\varrho(x')$

$\geq (\pi_\varrho * \pi_\varrho * \pi_\varrho)(x')$

$= \sup_{x'=p\alpha_1 q}\{\min\{\pi_\varrho * \pi_\varrho\}(p), \pi_\varrho(q)\}$

$= \sup_{x'=p\alpha_1 q}\{\min\{\sup_{p=p_1\beta_1 p_2}\{\min(\pi_\varrho(p_1), \pi_\varrho(p_2)), \pi_\varrho(q)\}\}\}$

$= \sup_{x'=p\alpha_1 q}\sup_{p=p_1\beta_1 p_2}\{\min\{\min\{\pi_\varrho(p_1), \pi_\varrho(p_2), \pi_\varrho(q)\}\}\}$

$= \sup_{x'=p_1\beta_1 p_2\alpha_1 q}\{\text{nin}[\pi_\varrho(p_1), \pi_\varrho(p_2), \pi_\varrho(q)]\}$

$\geq \min\{\pi_\varrho(\zeta), \pi_\varrho(y), \pi_\varrho(z)\}$

Hence $\pi_\varrho(\zeta\alpha y\beta z) \geq \min\{\pi_\varrho(\zeta), \pi_\varrho(y), \pi_\varrho(z)\}$

and

$\gamma_\varrho(\zeta\alpha y\beta z) = \gamma_\varrho(x')$

$\geq (\gamma_\varrho * \gamma_\varrho * \varrho_p)(x')$

$= \sup_{x'=p\alpha_1 q}\{\min\{\gamma_\varrho * \gamma_\varrho\}(p), \gamma_\varrho(q)\}$

$= \sup_{x'=p\alpha_1 q}\{\min\{\sup_{p=p_1\beta_1 p_2}\{\min(\gamma_\varrho(p_1), \gamma_\varrho(p_2)), \gamma_v(q)\}\}\}$

$= \sup_{x'=p\alpha_1 q}\sup_{p=p_1\beta_1 p_2}\{\min\{\min\{\gamma_\varrho(p_1), \gamma_\varrho(p_2), \gamma_\varrho(q)\}\}\}$

$= \sup_{x'=p_1\beta_1 p_2\alpha_1 q}\{\text{nin}[\gamma_\varrho(p_1), \gamma_\varrho(p_2), \gamma_\varrho(q)]\}$

$\geq \min\{\gamma_\varrho(\zeta), \gamma_\varrho(y), \gamma_\varrho(z)\}$

Hence $\gamma_\varrho(\zeta\alpha y\beta z) \geq \min\{\gamma_\varrho(\zeta), \gamma_\varrho(y), \gamma_\varrho(z)\}$

And also $\boldsymbol{\vartheta}_\varrho(\zeta\alpha y\beta z) = \boldsymbol{\vartheta}_\varrho(x')$

$\geq (\boldsymbol{\vartheta}_\varrho * \gamma_\varrho * \boldsymbol{\vartheta}_v)(x')$

$= \sup_{x'=p\alpha_1 q}\{\min\{\boldsymbol{\vartheta}_\varrho * \boldsymbol{\vartheta}_\varrho\}(p), \boldsymbol{\vartheta}_\varrho(q)\}$

$= \sup_{x'=p\alpha_1 q}\{\min\{\sup_{p=p_1\beta_1 p_2}\{\min(\boldsymbol{\vartheta}_\varrho(p_1), \boldsymbol{\vartheta}_\varrho(p_2)), \boldsymbol{\vartheta}_\varrho(q)\}\}\}$

$= \sup_{x'=p\alpha_1 q}\sup_{p=p_1\beta_1 p_2}\{\min\{\min\{\boldsymbol{\vartheta}_\varrho(p_1), \boldsymbol{\vartheta}_\varrho(p_2), \boldsymbol{\vartheta}_\varrho(q)\}\}\}$

$= \sup_{x'=p_1\beta_1 p_2\alpha_1 q}\{\text{nin}[\boldsymbol{\vartheta}_\varrho(p_1), \boldsymbol{\vartheta}_\varrho(p_2), \boldsymbol{\vartheta}_\varrho(q)]\}$

$\geq \min\{\boldsymbol{\vartheta}_\varrho(\zeta), \boldsymbol{\vartheta}_\varrho(\eta), \boldsymbol{\vartheta}_\varrho(z)\}$

Hence $\boldsymbol{\vartheta}_\varrho(\zeta\alpha y\beta z) \geq \min\{\boldsymbol{\vartheta}_\varrho(\zeta), \boldsymbol{\vartheta}_\varrho(y), \boldsymbol{\vartheta}_\varrho(z)\}$

$\mu_\varrho(\zeta\alpha y\beta z) = \mu_\varrho(x')$

$\leq (\mu_\varrho * \mu_\varrho * \mu_\varrho)(x')$

$= \inf_{x'=p\alpha_1 q}\{\max\{\mu_\varrho * \mu_\varrho\}(p), \mu_\varrho(q)\}$

$= \inf_{x'=p\alpha_1 q}\{\max\{\inf_{p=p_1\beta_1 p_2}\{\max(\mu_\varrho(p_1), \mu_\varrho(p_2)), \mu_\varrho(q)\}\}\}$

$= \inf_{x'=p\alpha_1 q}\inf_{p=p_1\beta_1 p_2}\{\max\{\max\{\mu_\varrho(p_1), \mu_\varrho(p_2), \mu_\varrho(q)\}\}\}$

$= \inf_{x'=p_1\beta_1 p_2\alpha_1 q}\{\max[\mu_\varrho(p_1), \mu_\varrho(p_2), \mu_\varrho(q)]\}$

$\leq \max\{\mu_\varrho(\zeta), \mu_\varrho(\eta), \mu_\varrho(z)\}$

Henc, $\mu_\varrho(\zeta\alpha\eta z) \leq \max\{\mu_\varrho(\zeta), \mu_\varrho(y), \mu_\varrho(z)\}$

$\theta_\varrho(\zeta\alpha\eta\beta z) = \theta_\varrho(x')$

$\leq (\theta_\varrho * \theta_\varrho * \theta_\varrho)(x')$

$= \inf_{x'=p\alpha_1 q}\{\max\{\theta_\varrho * \theta_\varrho\}(p), \theta_\varrho(q)\}$

$= \inf_{x'=p\alpha_1 q}\{\max\{\inf_{p=p_1\beta_1 p_2}\{\max(\theta_\varrho(p_1), \theta_\varrho(p_2)), \theta_\varrho(q)\}\}\}$

$= \inf_{x'=p\alpha_1 q}\inf_{p=p_1\beta_1 p_2}\{\max\{\max\{\theta_\varrho(p_1), \theta_\varrho(p_2), \theta_\varrho(q)\}\}\}$

$= \inf_{x'=p_1\beta_1 p_2\alpha_1 q}\{\max[\theta_\varrho(p_1), \theta_\varrho(p_2), \theta_\varrho(q)]\}$

$\leq \max\{\theta_\varrho(\zeta), \theta_\varrho(\eta), \theta_\varrho(z)\}$

Hence $\theta_\varrho(\zeta\alpha\eta\beta z) \leq \max\{\theta_\varrho(\zeta), \theta_\varrho(y), \theta_\varrho(z)\}$

Theorem 3.2.

Let $\{(\pi_{\varrho_i}, \gamma_{\varrho_i}, \boldsymbol{\vartheta}_{\varrho_i}, \mu_{\varrho_i}, \theta_{p_i}) / i \in \Delta\}$ be a family of Panta partitioned PFWBI of γ- near ring ꝓ. Then $\cap_{i\in\Delta}\pi_{\varrho_i}$; $\cap_{i\in\Delta}\gamma_{\varrho_i}$; $\cap_{i\in\Delta}\boldsymbol{\vartheta}_{\varrho_i}$; $\cup_{i\in\Delta}\mu_{\varrho_i}$ and $\cup_{i\in\Delta}\theta_{\varrho_i}$ are also a pentapartitioned Pythagorean fuzzy weak bi-ideal of γ- near ring ꝓ, where Δ is any index set.

Proof:

Let $\{\pi_{\varrho_i}\}_{i\in\Delta}$ be a family of Penta partitioned PFWBI of γ- near ring ꝓ. Let $\zeta, \eta, z \in$ ꝓ, $\alpha, \beta \in \gamma$

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and $\pi = \cap_{i\in\Delta} \pi_{\wp_I}$; $\gamma = \cap_{i\in\Delta} \gamma_{\wp_I}$; $\boldsymbol{\vartheta} = \cap_{i\in\Delta} \boldsymbol{\vartheta}_{\wp_i}$; $\mu = \cap_{i\in\Delta} \mu_{\wp_i}$ & $\mu = \cap_{i\in\Delta} \mu_{\wp_i}$ Then

$\cap_{i\in\Delta} \pi_{\wp_i}(\zeta) = [\inf_{i\in\Delta}\pi_{\wp_i}](\zeta)$ [or] $\inf_{i\in\Delta}\pi_{\wp_i}(\zeta)$ and

$\cup_{i\in\Delta} \pi_{\wp_i}(\zeta) = [\sup_{i\in\Delta}\pi_{\wp_i}](\zeta)$ [or] $\sup_{i\in\Delta}\pi_{\wp_i}(\zeta)$

(i) $\cap_{i\in\Delta} \pi_{\wp_i}(\zeta-\eta) = \inf_{i\in\Delta}\pi_{\wp_i}(\zeta-\eta)$

$\geq \inf_{i\in\Delta}\{\min(\pi_{\wp_i}(\zeta), \pi_{\wp_i}(y))\}$

$=\min\{\inf_{i\in\Delta}[\pi_{\wp_i}(\zeta)], \inf_{i\in\Delta}[\pi_{\wp_i}(\eta)]\}$

$=\min\{\cap_{i\in\Delta} \pi_{\wp_i}(\zeta), \cap_{i\in\Delta} \pi_{\wp_i}(\eta)\}$

(ii) $\cap_{i\in\Delta} \gamma_{\wp_i}(\zeta-\eta) = \inf_{i\in\Delta}\gamma_{\wp_i}(\zeta-\eta)$

$\geq \inf_{i\in\Delta}\{\min(\gamma_{\wp_i}(\zeta), \gamma_{\wp_i}(\eta))\}$

$= \min\{\inf_{i\in\Delta}[\gamma_{\wp_i}(\zeta)], \inf_{i\in\Delta}[\gamma_{\wp_i}(\eta)]\}$

$= \min\{\cap_{i\in\Delta} \gamma_{\wp_i}(\zeta), \cap_{i\in\Delta} \gamma_{\wp_i}(\eta)\}$

(iii) $\cap_{i\in\Delta} \boldsymbol{\vartheta}_{\wp_i}(\zeta-\eta) = \inf_{i\in\Delta} \boldsymbol{\vartheta}_{\wp_i}(\zeta-\eta)$

$\geq \inf_{i\in\Delta}\{\min(\boldsymbol{\vartheta}_{\wp_i}(\zeta), \boldsymbol{\vartheta}_{\wp_i}(\eta))\}$

$= \min\{\inf_{i\in\Delta}[\boldsymbol{\vartheta}_{\wp_i}(\zeta)], \inf_{i\in\Delta}[\boldsymbol{\vartheta}_{\wp_i}(\eta)]\}$

$= \min\{\cap_{i\in\Delta} \boldsymbol{\vartheta}_{\wp_i}(\zeta), \cap_{i\in\Delta} \boldsymbol{\vartheta}_{\wp_i}(\eta)\}$

(iv) $\cup_{i\in\Delta} \mu_{\wp_i}(\zeta-\eta) = \sup_{i\in\Delta} \mu_{\wp_i}(\zeta-\eta)$

$\leq \sup_{i\in\Delta}\{\max(\mu_{\wp_i}(\zeta), \mu_{\wp_i}(\eta))\}$

$= \max\{\sup_{i\in\Delta}[\mu_{\wp_i}(\zeta)], \inf_{i\in\Delta}[\mu_{\wp_i}(\eta)]\}$

$= \max\{\cup_{i\in\Delta} \mu_{\wp_i}(\zeta), \cup_{i\in\Delta} \mu_{\wp_i}(y)\}$

(v) $\cup_{i\in\Delta} \theta_{\wp_i}(\zeta-\eta) = \sup_{i\in\Delta} \theta_{\wp_i}(\zeta-\eta)$

$\leq \sup_{i\in\Delta}\{\max(\theta_{\wp_i}(\zeta), \theta_{\wp_i}(\eta))\}$

$= \max\{\sup_{i\in\Delta}[\theta_{\wp_i}(\zeta)], \inf_{i\in\Delta}[\theta_{\wp_i}(\eta)]\}$

$= \max\{\cup_{i\in\Delta} \theta_{\wp_i}(\zeta), \cup_{i\in\Delta} \theta_{\wp_i}(\eta)\}$

(i) $\cap_{i\in\Delta} \pi_{\wp_i}(\zeta\alpha\eta\beta z) = \inf_{i\in\Delta}\pi_{\wp_i}(\zeta\alpha\eta\beta z)$

$\geq \inf_{i\in\Delta}\{\min(\pi_{\wp_i}(\zeta), \pi_{\wp_i}(y), \pi_{\wp_i}(z))\}$

$=\min\{\inf_{i\in\Delta}[\pi_{\wp_i}(\zeta)], \inf_{i\in\Delta}[\pi_{\wp_i}(\eta)], \inf_{i\in\Delta}[\pi_{\wp_i}(z)]\}$

$=\min\{\cap_{i\in\Delta} \pi_{\wp_i}(\zeta), \cap_{i\in\Delta} \pi_{\wp_i}(\eta), \cap_{i\in\Delta} \pi_{\wp_i}(z)\}$

(ii) $\cap_{i\in\Delta} \gamma_{\wp_i}(\zeta\alpha\eta\beta z) = \inf_{i\in\Delta}\gamma_{\wp_i}(\zeta\alpha\eta\beta z)$

$\geq \inf_{i\in\Delta}\{\min(\gamma_{\wp_i}(\zeta), \gamma_{\wp_i}(\eta), \gamma_{\wp_i}(z))\}$

$= \min\{\inf_{i\in\Delta}[\gamma_{\wp_i}(\zeta)], \inf_{i\in\Delta}[\gamma_{\wp_i}(\eta)], \inf_{i\in\Delta}[\gamma_{\wp_i}(z)]\}$

$=\min\{\cap_{i\in\Delta} \gamma_{\wp_i}(\zeta), \cap_{i\in\Delta} \gamma_{\wp_i}(\eta), \cap_{i\in\Delta} \gamma_{\wp_i}(z)\}$

(iii) $\cap_{i\in\Delta} \boldsymbol{\vartheta}_{\wp_i}(\zeta\alpha\eta\beta z) = \inf_{i\in\Delta}\boldsymbol{\vartheta}_{\wp_{i_i}}(\zeta\alpha\eta\beta z)$

$\geq \inf_{i\in\Delta}\{\min(\boldsymbol{\vartheta}_{\wp_i}(\zeta), \boldsymbol{\vartheta}_{\wp_i}(v), \boldsymbol{\vartheta}_{\wp_i}(z))\}$

$= \min\{\inf_{i\in\Delta}[\boldsymbol{\vartheta}_{\wp_i}(\zeta)], \inf_{i\in\Delta}[\boldsymbol{\vartheta}_{\wp_i}(\eta)], \inf_{i\in\Delta}[\boldsymbol{\vartheta}_{\wp_i}(z)]\}$

$= \min\{\cap_{i\in\Delta} \boldsymbol{\vartheta}_{\wp_i}(\zeta), \cap_{i\in\Delta} \boldsymbol{\vartheta}_{\wp_i}(\eta), \cap_{i\in\Delta}, \boldsymbol{\vartheta}_{\wp_i}(z)\}$

(iv) $\cup_{i\in\Delta} \mu_{\wp_i}(\zeta\alpha\eta\beta z) = \sup_{i\in\Delta} \mu_{\wp_i}(\zeta\alpha\eta\beta z)$

$\leq \sup_{i\in\Delta}\{\max(\mu_{\wp_i}(\zeta), \mu_{\wp_i}(\eta), \mu_{\wp_i}(z))\}$

$=\max\{\sup_{i\in\Delta}[\mu_{\wp_i}(\zeta)], \sup_{i\in\Delta}[\mu_{\wp_i}(\eta)], \sup_{i\in\Delta}[\mu_{\wp_i}(z)]\}$

$= \max\{\cap_{i\in\Delta} \mu_{\wp_i}(\zeta), \cap_{i\in\Delta} \mu_{\wp_i}(\eta), \cap_{i\in\Delta}, \mu_{\wp_i}(z)\}$

(iv) $\cup_{i\in\Delta} \theta_{\wp_i}(\zeta\alpha y\beta z) = \sup_{i\in\Delta}\theta_{\wp_i}(\zeta\alpha\eta\beta z)$

$\leq \sup_{i\in\Delta}\{\max(\theta_{\wp_i}(\zeta), \theta_{\wp_i}(\eta), \theta_{\wp_i}(z))\}$

$= \max\{\sup_{i\in\Delta}[\theta_{\wp_i}(\zeta)], \sup_{i\in\Delta}[\theta_{\wp_i}(\eta)], \sup_{i\in\Delta}[\theta_{\wp_i}(z)]\}$

$= \max\{\cap_{i\in\Delta} \theta_{\wp_i}(\zeta), \cap_{i\in\Delta} \theta_{\wp_i}(\eta), \cap_{i\in\Delta}, \theta_{\wp_i}(z)\}$

Hence the set $\cap_{i\in\Delta} \pi_{\wp_i}$; $\cap_{i\in\Delta} \gamma_{\wp_i}$; $\cap_{i\in\Delta} \boldsymbol{\vartheta}_{p_i}$; $\cup_{i\in\Delta} \mu_{\wp_i}$ & $\cup_{i\in\Delta} \theta_{\wp_i}$ are also a family of Penta partitioned PFWBI of ᵽ.

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Theorem 3.3.

$(\pi_\varrho, \gamma_\varrho, \boldsymbol{\vartheta}_\varrho, \mu_\varrho, \theta_\varrho)$ be a Penta partitioned PFWBI of ⲣ. Then the set

$ⲣ_{(\pi,\gamma,\vartheta,\mu,\theta)} = \{\pi_\varrho(\zeta) = \pi_\varrho(0); \gamma_\varrho(\zeta) = \gamma_\varrho(0); \boldsymbol{\vartheta}_\varrho(\zeta) = \boldsymbol{\vartheta}_\varrho(0); \mu_p(\zeta) = \mu_\varrho(0); \theta_\varrho(\zeta) = \theta_\varrho(0) / \zeta \in ⲣ)\}$ is a weak bi-ideals of ⲣ.

Proof:

Let $\zeta, \eta \in ⲣ(\pi_\varrho,\gamma_\varrho,\boldsymbol{\vartheta}_\varrho,\mu_\varrho,\theta_\varrho)$. Then $\pi_\varrho(\zeta) = \pi_\varrho(0)$; $\gamma_\varrho(\zeta) = \gamma_\varrho(0)$; $\boldsymbol{\vartheta}_\varrho(\zeta) = \boldsymbol{\vartheta}_\varrho(0)$; $\mu_\varrho(\zeta) = \mu_\varrho(0)$; $\theta_\varrho(\zeta) = \theta_\varrho(0)$.

Similarly, 'η' element is zero.

(i) $\pi_\varrho(\zeta - \eta) \geq \min\{\pi_\varrho(\zeta), \pi_\varrho(\eta)\} = \min\{\pi_\varrho(0), \pi_\varrho(0)\} = \pi_\varrho(0)$

(ii) $\gamma_\varrho(\zeta - \eta) \geq \min\{\gamma_\varrho(\zeta), \gamma_\varrho(\eta)\} = \min\{\gamma_\varrho(0), \gamma_\varrho(0)\} = \gamma_\varrho(0)$

(iii) $\boldsymbol{\vartheta}_\varrho(\zeta - \eta) \geq \min\{\boldsymbol{\vartheta}_\varrho(\zeta), \boldsymbol{\vartheta}_\varrho(\eta)\} = \min\{\boldsymbol{\vartheta}_\varrho(0), \boldsymbol{\vartheta}_\varrho(0)\} = \boldsymbol{\vartheta}_\varrho(0)$

(iv) $\mu_\varrho(\zeta - \eta) \leq \max\{\mu_\varrho(\zeta), \mu_\varrho(\eta)\} = \{\max\{\mu_\varrho(0), \mu_\varrho(0)\} = \mu_\varrho(0)$

(iv) $\theta_\varrho(\zeta - \eta) \leq \max\{\theta_\varrho(\zeta), \theta_\varrho(\eta)\} = \max\{\theta_\varrho(0), \theta_\varrho(0)\} = \theta_\varrho(0)$

Thus, $\zeta - \eta \in ⲣ(\pi_\varrho,\gamma_\varrho,\boldsymbol{\vartheta}_\varrho,\mu_\varrho,\theta_\varrho)$. For every x, y, z $\in ⲣ(\pi_\varrho,\gamma_\varrho,\boldsymbol{\vartheta}_\varrho,\mu_\varrho,\theta_\varrho)$ and $\alpha,\beta \in \gamma$. We have,

$\pi_\varrho(\zeta\alpha\eta\beta z) \geq \min\{\pi_\varrho(\zeta), \pi_\varrho(\eta), \pi_\varrho(z)\}$

$= \min\{\pi_\varrho(0), \pi_\varrho(0), \pi_\varrho(0)\} = \pi_\varrho(0)$

$\gamma_\varrho(\zeta\alpha\eta\beta z) \geq \min\{\gamma_\varrho(\zeta), \gamma_\varrho(\eta), \gamma_\varrho(z)\}$

$= \min\{\gamma_\varrho(0), \gamma_\varrho(0), \gamma_\varrho(0)\} = \gamma_\varrho(0)$

$\boldsymbol{\vartheta}_\varrho(\zeta\alpha\eta\beta z) \geq \min\{\boldsymbol{\vartheta}_\varrho(\zeta), \boldsymbol{\vartheta}_\varrho(\eta), \boldsymbol{\vartheta}_\varrho(z)\}$

$= \min\{\boldsymbol{\vartheta}_\varrho(0), \boldsymbol{\vartheta}_\varrho(0), \boldsymbol{\vartheta}_\varrho(0)\} = \boldsymbol{\vartheta}_\varrho(0)$

$\mu_\varrho(\zeta\alpha\eta\beta z) \leq \max\{\mu_\varrho(\zeta), \mu_\varrho(\eta), \mu_\varrho(z)\}$

$\leq \max\{\mu_\varrho(0), \mu_\varrho(0), \mu_\varrho(0)\} = \mu_\varrho(0)$

$\theta_\varrho(\zeta\alpha\eta\beta z) \leq \max\{\theta_\varrho(\zeta), \theta_\varrho(\eta), \theta_\varrho(z)\}$

$\leq \max\{\theta_\varrho(0), \theta_\varrho(0), \theta_\varrho(0)\} = \theta_\varrho(0)$

Thus $x\alpha y\beta z \in ⲣ(\pi_\varrho,\gamma_\varrho,\boldsymbol{\vartheta}_\varrho,\mu_\varrho,\theta_\varrho)$. Hence $ⲣ(\pi_\varrho,\gamma_\varrho,\boldsymbol{\vartheta}_\varrho,\mu_\varrho,\theta_\varrho)$. is a weak bi-ideal of ⲣ.

4. Homomorphism of Penta partitioned Pythagorean Fuzzy Weak Bi-Ideal Of γ-Near-Rings

This portion uses Homomorphism to characterise the Penta partitioned PFWBI Of γ-Near-Rings

Definition 4.1.

Let f be a mapping from a set ⲣ to the set T. Let f $= (\pi_\varrho, \gamma_\varrho, \boldsymbol{\vartheta}_\varrho, \mu_\varrho, \theta_\varrho)$ be a

Penta partitioned Pythagorean fuzzy subsets of M and T respectively. Then f is image of

$\pi_\varrho, \gamma_\varrho, \boldsymbol{\vartheta}_\varrho, \mu_\varrho$ & θ_ϱ below f is a fuzzy subset of T is given by

$f((\pi_\varrho)(\eta) = \{$

$\sup_{\zeta \in f'(y)} \pi_\varrho(\zeta)$ if $f'(\eta) \neq \emptyset$

0 if otherwise

$f(\gamma_\varrho)(\eta) = \{$

$\sup_{\zeta \in f'(y)} \gamma_\varrho(\zeta)$ if $f'(\eta) \neq \emptyset$

0 if otherwise

$f(\boldsymbol{\vartheta}_\varrho)(\eta) = \{$

$\sup_{\zeta \in f'(\eta)} \boldsymbol{\vartheta}_\varrho(\zeta)$ if $f'(\eta) \neq \emptyset$

0 if otherwise

$f(\mu_\varrho)(\eta) = \{$

$\inf_{\zeta \in f'} \mu_\varrho(\zeta)$ if $f'(v) \neq \emptyset$

$f(\theta_\varrho)(\eta) = \{$

(v)

0 if otherwise

$\inf_{\zeta \in f'(y)} \theta_\varrho(\zeta)$ if $f'(v) \neq \emptyset$

0

f otherwise

And the pre-image of $\pi_\varrho, \gamma_\varrho, \boldsymbol{\vartheta}_\varrho, \mu_\varrho$ & θ_ϱ below f is a fuzzy subset of M denoted by

$f'(\pi_\varrho(\zeta)) = \pi_\varrho(f(\zeta)), f'(\gamma_\varrho(\zeta)) = \gamma_\varrho(f(\zeta)), f'(\boldsymbol{\vartheta}_\varrho(\zeta)) = \boldsymbol{\vartheta}_\varrho(f(\zeta)), f'(\mu_\varrho(\zeta)) = \mu_\varrho(f(\zeta)), f'(\theta_\varrho(\zeta)) = \theta_\varrho(f(\zeta)). \forall\ \zeta \in M$

and $f'(y) = \{\zeta \in ꝑ \mid f(\zeta) = \eta\}$

Theorem 4.1.

Assume f: ꝑ → T be a homomorphism between γ-near-rings ꝑ and T, if

ꝑ = $\{\pi_\varrho, \gamma_\varrho, \boldsymbol{\vartheta}_\varrho, \mu_\varrho$ & $\theta_\varrho\}$ is a Penta partitioned PFWBI of T, then

$f'(ꝑ) = \{f'(\pi_\varrho, \gamma_\varrho, \boldsymbol{\vartheta}_\varrho, \mu_\varrho, \theta_\varrho)\}$ is Penta partitioned PFWBI of ꝑ.

Proof:

Let f be Penta partitioned Pythagorean fuzzy weak bi-ideal of ꝑ. and Let ζ, η, z ∈ ꝑ and

α, β ϵγ. Then

$f'(\pi_\varrho)(\zeta - \eta) = \pi_\varrho[f(\zeta - \eta)]$

$=\pi_\varrho\{f(\zeta) - f(y)\}$

$\geq \min\{\pi_\varrho(f(\zeta)), \pi_\varrho(f(\eta))\}$

$=\min\{f'(\pi_\varrho(\zeta)), f'(\pi_\varrho(v))\}$

$f'(\gamma_\varrho)(\zeta - \eta) = \gamma_\varrho[f(\zeta - \eta)]$

$=\gamma_\varrho\{f(\zeta) - f(\eta)\}$

$\geq \min\{\gamma_\varrho(f(\zeta)), \gamma_\varrho(f(\eta))\}$

$=\min\{f'(\gamma_\varrho(\zeta)), f'(\gamma_\varrho(y))\}$

$f'(\boldsymbol{\vartheta}_\varrho)(\zeta - y) = \boldsymbol{\vartheta}_\varrho[f(\zeta - \eta)]$

$=\boldsymbol{\vartheta}_\varrho\{f(\zeta) - f(y)\}$

$\geq \min\{\boldsymbol{\vartheta}_\varrho(f(\zeta)), \boldsymbol{\vartheta}_\varrho(f(\eta))\}$

$=\min\{f'(\boldsymbol{\vartheta}_\varrho(\zeta)), f'(\boldsymbol{\vartheta}_\varrho(\eta))\}$

$f'(\mu_\varrho)(\zeta - \eta) = \mu_\varrho[f(\zeta v y)]$

$=\mu_\varrho\{f(\zeta) - f(\eta)\}$

$\leq \max\{\mu_\varrho(f(\zeta)), \mu_\varrho(f(v))\}$

$= \max\{f'(\mu_\varrho(\zeta)), f'(\mu_\varrho(\eta))\}$

$f'(\theta_\varrho)(\zeta - \eta) = \theta_\varrho[f(\zeta - \eta)]$

$=\theta_\varrho\{f(\zeta) - f(y)\}$

$\leq \max\{\theta_\varrho(f(\zeta)), \theta_\varrho(f(v))\}$

$= \max\{f'(\theta_\varrho(\zeta)), f'(\theta_\varrho(v))\}$

$f'(\pi_\varrho)(\zeta\alpha v\beta z) = \pi_\varrho\{f(\zeta\alpha y\beta z)\} = \pi_\varrho[f(\zeta)f(\eta)f(z)]$

$\geq \min\{\pi_\varrho(f(\zeta)), \pi_\varrho(f(v)), \pi_\varrho(f(z))\}$

$= \min\{f'(\pi_\varrho(\zeta)), f'(\pi_\varrho(v)), f'(\pi_\varrho(z))\}$

$f'(\gamma_\varrho)(\zeta\alpha\eta\beta z) = \gamma_\varrho\{f(\zeta\alpha v\beta z)\} = \gamma_\varrho[f(\zeta)f(\eta)f(z)]$

$\geq \min\{\gamma_\varrho(f(\zeta)), \gamma_\varrho(f(\eta)), \gamma_\varrho(f(z))\}$

$= \min\{f'(\gamma_\varrho(\zeta)), f'(\gamma_\varrho(y)), f'(\gamma_\varrho(z))\}$

$f'(\boldsymbol{\vartheta}_\varrho)(\zeta\alpha\eta\beta z) = \boldsymbol{\vartheta}_\varrho\{f(\zeta\alpha\eta\beta z)\} = \boldsymbol{\vartheta}_\varrho[f(\zeta)f(\eta)f(z)]$

$\geq \min\{\boldsymbol{\vartheta}_\varrho(f(\zeta)), \boldsymbol{\vartheta}_\varrho(f(\eta)), \boldsymbol{\vartheta}_\varrho(f(z))\}$

$= \min\{f'(\boldsymbol{\vartheta}_\varrho(\zeta)), f'(\boldsymbol{\vartheta}_\varrho(\eta)), f'(\boldsymbol{\vartheta}_\varrho(z))\}$

$f'(\mu_\varrho)(\zeta\alpha\eta\beta z) = \varrho = \mu_\varrho[f(\zeta)f(\eta)f(z)]$

$\leq \max\{\mu_\varrho(f(\zeta)), \mu_\varrho(f(\eta)), \mu_\varrho(f(z))\}$

$= \max\{f'(\mu_\varrho(\zeta)), f'(\mu_\varrho(\eta)), f'(\mu_\varrho(z))\}$

$f'(\theta_\varrho)(\zeta\alpha\eta\beta z) = \theta_\varrho\{f(\zeta\alpha\eta\beta z)\} = \theta_\varrho[f(\zeta)f(\eta)f(z)]$

$\leq \max\{\theta_\varrho(f(\zeta)), \theta_\varrho(f(\eta)), \theta_\varrho(f(z))\}$

$= \max\{f'(\theta_\varrho(\zeta)), f'(\theta_\varrho(\eta)), f'(\theta_\varrho(z))\}$

Consequently, $f'(ꝑ) = \{f'(\pi_\varrho, \gamma_\varrho, \boldsymbol{\vartheta}_\varrho, \mu_\varrho, \theta_\varrho)\}$ is a Pentapartitioned Pythagorean fuzzy weak bi-ideal of ꝑ

Conclusion:

This work presents a Penta partitioned Pythagorean fuzzy weak ideal. fuzzy weak bi-ideal ꝑ with Penta partitioning. Penta partitioned Pythagorean fuzzy weak bi-ideal Homomorphism of γ-Near-rings. And also, we develop the concept of Penta partitioned fuzzy weak bi- ideals of γ- near-rings to the Fermatean concept.

Reference:

[1] Booth, G. L. (1988). A note on gamma near-rings. Stud. Sci. Math. Hung, 23,471-475.

[2] Chinnadurai, v., & Kadalarasi, S. (2016). Fuzzy weak bi-ideals of near-rings. Annuals of fuzzy mathematics and informatics, 11(5), 799-808.

[3] Jun, Y. B., SAPANCI, M., & OZTURK, M. A. (1999). Fuzzy ideals in gamma near-rings. Turkish journal of mathematics, 22(4), 449-460.

[4] Manikantan, T. (2009). Fuzzy bi-ideals of near-rings. Journal of fuzzy mathematics,17(3), 359-674.

[5] Rama Malick and Surapati Pramanik. Neutrosophic Sets and System, University of New Mexico, Vol. 36, 2020

[6] Yager, R. R. (2013). Pythagorean fuzzy sets. In 2013joint IFSA world congress and NAFIPS annual meeting (IFSA/NAFIPS) (pp. 57-61). IEEE.

[7] Yager, R. R. (2013). Pythagorean membership grades in multicriteria decision making. IEEE transactions on fuzzy systems, 22(4), 958-965.

[8] Zadeth, L. A. (1965). Fuzzy sets. Information and control, 8(3),338-353.