International Journal on Recent and Innovation Trends in Computing and Communication
ISSN: 2321-8169 Volume: 14 Issue: 2
***Article Received:** 25 April 2026 **Revised:** 12 May 2026 **Accepted:** 30 June 2026*

Fermatean Fuzzy Prime Bi-ideal of Γ-BCK Algebras

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Abstract:

The idea of a Fermatean fuzzy prime bi-ideal of Γ-BCK algebras was proposed in this study, and some of its basic characteristics were examined. The cubic membership and non-membership conditions for bi-ideals are presented in relation to Γ-structures. Additionally, we study the behaviour of these prime bi-ideal under homomorphism and characterise them using cubic level sets. Fermatean logic is used in the results to create a generalised foundation for fuzzy algebraic structures.

Keywords: Γ-BCK algebra, Fermatean fuzzy set, Bi-ideals, Fermatean fuzzy prime bi-ideal cubic level.

1. Introduction:

Zadeh [8] first introduced fuzzy sets (FS) in 1965 as a way to deal with ambiguity, and it was subsequently used in the theory of BCK\BCI algebra. By incorporating a non-membership function, Atanassov [1]expanded FS into intuitionistic fuzzy set (IFS). Senapati and Yager [4] proposed the Fermatean fuzzy set (FFS), which satisfies the condition 0≤ $\mu_A(x) + \vartheta_A(x) \leq 1$

, to provide more flexibility when the sum of membership and non-membership degrees

exceeds 1. While fuzzy ideals in BCK-algebras have been thoroughly studied [7,3], the concept of Γ-BCK algebras was introduced as a more general algebraic structure [5]. Numerous authors have studied bi-ideals in different fuzzy contexts [7,3]. In the context of Fermatean fuzzy BCK algebras, however, the research of prime Bi-ideals is still in its infancy. Inspired by the latest advances in Fermatean fuzzy logic, this study presents and investigates the characteristics of Fermatean fuzzy prime bi-ideal Γ-BCK algebras.

2. Preliminaries:

Definition 2.1[5]: Let X be a non-empty set and Γ-be a non-empty set of binary operations. Then (X, Γ,0) is called a Γ-BCK algebra if for all x, y, z ∈ X and γ, δ ∈ Γ, it satisfies the following axioms:

i) ((x γ y) δ (x γ z)) δ (z γ y) = 0

ii) (x γ (x γ y)) γ y) = 0

iii) x γ x= 0

iv) 0 γ x= 0

v) x γ y = 0 and y γ x = 0 ⇒ $x = y$

Definition 2.2[5]: In a Γ-BCK algebra X, the partial order " ≤ " is defined as x ≤ y if and only if x γ y = 0 for all γ ∈ Γ.

Definition 2.3[2]: A non-empty subset A of a Γ-BCK algebra X is called a Γ-subalgebra of X, if for any x, y ∈ A and γ ∈ Γ, we have x γ y∈ A .

Definition 2.4[7]: Let X be a non-empty set. A Fermatean fuzzy set (FFS) A in X is an object having the form: A= {< x, $\mu_A(x), \vartheta_A(x)$>: x∈ X } were μ_A: X → [0,1] and ϑ_A: X → [0,1] denote the degree of membership and non-membership respectively, satisfying the condition.

0≤ $(\mu_A(x))^3 + (\vartheta_A(x))^3 \leq 1$ for all x∈ X

Definition 2.5[1]: An Intuitionistic fuzzy set (IFS). A in x is defined by A={< x, $\mu_A(x), \vartheta_A(x)$>: x∈ X }, where 0≤ $\mu_A(x) + \vartheta_A(x) \leq 1$

Definition 2.6[2]: A Fermatean fuzzy set A= (μ_A, ϑ_A) of a Γ-BCK algebra X is called a Fermatean fuzzy Γ- subalgebra of X if for all x, y ∈ X and γ ∈ Γ

i) μ_A(x γ y)≥ $min\{\mu_A(x), \mu_A(y)\}$

ii) ϑ_A(x γ y)≤ $max\{\vartheta_A(x), \vartheta_A(y)\}$

Definition 2.7[4]: A Fermatean fuzzy set A= (μ_A, ϑ_A) of a Γ-BCK algebra X is called a Fermatean fuzzy bi-ideal of x if it satisfies:

i) $\mu_A(0) \geq \mu_A(x)$ and $\vartheta_A(0) \leq \vartheta_A(x)$

ii) $\mu_A(x\ \gamma\ y\ \gamma\ z) \geq min\{\mu_A(x), \mu_A(z)\}$

iii) $\vartheta_A(x\ \gamma\ y\ \gamma\ z) \leq max\{\vartheta_A(x), \vartheta_A(z)\}$ for all x, y, z ∈X and γ, δ ∈ Γ

Definition 2.8[3]: A Fermatean fuzzy bi-ideal A= (μ_A, ϑ_A) of a Γ-BCK algebra X is called a Fermatean fuzzy prime bi-ideal if it satisfies the following conditions for all x, y ∈ X and γ ∈ Γ

i) $\mu_A(x\ \gamma\ y) \leq max\{\mu_A(x), \mu_A(z)\}$

ii) $\vartheta_A(x\ \gamma\ y) \geq min\{\vartheta_A(x), \vartheta_A(y)\}$

Alternatively, this can be expressed in terms of the Fermatean boundary condition as:

i) $(\mu_A(x\ \gamma\ y))^3 \leq max\{(\mu_A(x))^3, (\mu_A(y))^3\}$and

ii) $(\vartheta_A(x\ \gamma\ y))^3 \geq min\{(\vartheta_A(x))^3, (\vartheta_A(y))^3\}$

3. Fermatean Fuzzy prime Bi-ideals of Γ-BCK algebra

Definition 3.1: Assume A = (μ_A, ϑ_A) be a Fermatean fuzzy set in a Γ-BCK algebra X. Then A is called a Fermatean fuzzy prime bi-ideal of X if it satisfied the following conditions:

(FFBI - 1) $\mu_A(0) \geq \mu_A(x)$ and $\boldsymbol{\vartheta}_A(0) \leq \boldsymbol{\vartheta}_A(x)$

(FFBI - 2) $\mu_A(x\ \gamma\ y) \geq min\{\mu_A(x), \mu_A(z)\}$

(FFBI - 3) $\vartheta_A(x\ \gamma\ y) \leq max\{\vartheta_A(x), \vartheta_A(z)\}$ for all x, y, z ∈X and γ, δ ∈ Γ (FFBI - 4)$\mu_A(x\ \gamma\ y) \leq max\{\mu_A(x), \mu_A(y)\}$

(FFBI - 5) $\vartheta_A(x\ \gamma\ y) \geq min\{\vartheta_A(x), \vartheta_A(y)\}$ for all x, y ∈X and γ ∈ Γ

Example 3.2: Let a Γ-BCK algebra X = {0,1,2,3} with Γ={γ} and the operation γ defined by the following Cayley table

Γ	0	1	2	3
0	0	0	0	0
1	1	0	0	1
2	2	1	0	2
3	3	3	3	3

Explain a Fermatean fuzzy set A = (μ_A, ϑ_A) in X as shown in Figure 3.2, which is a Fermatean fuzzy prime bi-ideal of X and satisfies the condition $0 \leq (\mu_A(x))^3 + (\vartheta_A(x))^3 \leq 1\ \forall\ x \in X$

X	$\mu_A(x)$	$\vartheta_A(x)$
0	0.9	0.1
1	0.7	0.3
2	0.5	0.5
3	0.4	0.6

1. Bi -ideal property:

i) $\mu_A(x\ \gamma\ y\ \gamma\ z) \geq min\{\mu_A(x), \mu_A(z)\}$

For x = 2, y = 1, z = 1in X and γ ∈ Γ, since (2 γ 1) γ 1 =1 γ 1 = 0, we have:

$\mu_A(2\ \gamma\ 1\ \gamma\ 1) = \mu_A(0) = 0.9 \geq 0.5 =$ min {$\mu_A(2)$, $\mu_A(1)$}

ii) $\vartheta_A(x\ \gamma\ y\ \gamma\ z) \leq max\{\vartheta_A(x), \vartheta_A(z)\}$

ϑ_A (2 γ 1 γ 1) =$\vartheta_A(0)$ =0.1≤ 0.3 = $max\{\vartheta_A(2)$, $\vartheta_A(1)\}$

2. Prime Condition:

For any x = 2, y = 1 in X and γ ∈ Γ we have (2 γ 1) = 1:

i) $\mu_A(x\ \gamma\ y) \leq max\{\mu_A(x), \mu_A(y)\}$

$\mu_A(2\ \gamma\ 1) = \mu_A(1) = 0.7 \leq 0.7 = max\{\mu_A(2)$, $\mu_A(1)\}$

ii) $\vartheta_A(x\ \gamma\ y) \geq min\{\vartheta_A(x), \vartheta_A(z)\}$

$\vartheta_A(2\ \gamma\ 1) = \vartheta_A(1) = 0.3 \geq 0.3 = min\{\vartheta_A(2), \vartheta_A(1)\}$

Lemma 3.3 Let A = (μ_A, ϑ_A) be a Fermatean fuzzy prime bi-ideals of Γ-BCK algebra X if the inequality (u γ v) ≤ w holds in X for u, v, w ∈ X and γ ∈ Γ then

i) $\mu_A(u\ \gamma\ v) \geq min\{\mu_A(u), \mu_A(w)\}$

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ii) ϑ_A(u γ v) ≤ $max\{\vartheta_A$(u)$, \vartheta_A(w)\}$

Proof: Since A is a Fermatean fuzzy prime bi-ideals, it is by definition a Fermatean fuzzy bi-ideal. Given (u γ v) ≤w, their exists δ∈Γ such that (u γ v) δ w) =0

i) μ_A(u γ v) ≥ $min\{\mu_A$((u γ v)δ w)$, \mu_A(w)\}$

= $min\{\mu_A(0), \mu_A(w)\}$ Since $\mu_A(0)$ ≥ μ_A(u), we have

μ_A(u γ v) ≥ $min\{\mu_A$(u)$, \mu_A(w)\}$

ii) ϑ_A(u γ v) ≤ $max\{\vartheta_A$((u γ v)δ w)$, \vartheta_A(w)\}$

= $max\{\vartheta_A(0), \vartheta_A(w)\}$ Since $\vartheta_A(0)$ ≤ $\vartheta_A(u)$, we have

ϑ_A(u γ v)≤ $max\{\vartheta_A$(u)$, \vartheta_A(w)\}$

Lemma 3.4 Let A = (μ_A, ϑ_A) be a Fermatean fuzzy prime bi-ideals of X. For any a, b, c ∈X and γ ∈ Γ if (a γ b) ≤c then,

i) μ_A(a γ b)≥ $\{\mu_A(c)\}$

ii) ϑ_A(a γ b)≤ $\{\vartheta_A(c)\}$

Proof: Assume (a γ b) ≤ c. Then ((a γ b)δ c) = 0 for some δ ∈ Γ

i) μ_A(a γ b)≥ $min\{\mu_A$((a γ b)δ z)$, \mu_A(c)\}$

=$min\{\mu_A(0), \mu_A(c)\}$

In any Fermatean Fuzzy bi-ideals, $\mu_A(0)$ is upper bound. Thus min $\{\mu_A(0), \mu_A(c)\}$= $\mu_A(c)$

Therefore μ_A(a γ b) ≥ $\mu_A(c)$

ii) similarly ϑ_A(a γ b) ≤ $max\{\vartheta_A(0), \vartheta_A(c)\}$

=ϑ_A(c) Therefore ϑ_A(a γ b) ≤ $\vartheta_A(c)$

Hence μ_A is order-reversing and ϑ_A is order preserving.

Theorem 3.5: Let A = (μ_A, ϑ_A) be a Fermatean fuzzy prime bi-ideals of, X, for any x, y,

$a_1, a_2,\ldots. a_n$ ∈X and γ, δ_1, δ_2,…. δ_n ∈ Γ the condition ((x γ y) δ_1 a_1) δ_2 a_2…… δ_n a_n= 0 implies i)μ_A(x γ y)≥ $min\{\mu_A(a_1), \mu_A(a_2),$ ………………………………………………………………… $, \mu_A(a_n)\}$

A

ii) ϑ_A(x γ y) ≤ $max\{\vartheta_A(a_1), \vartheta_A(a_2),$………………………$, \vartheta_A(a_n)\}$

Proof: Using the property of Fermatean Fuzzy bi-ideals and Lemma 3.4

μ_A(x γ y) ≥ $min\{\mu_A$(((x γ y) δ_1 a_1)

δ_n a_n)$\mu_A(a_1), \mu_A(a_2))\}$

μ_A(x γ y) ≥ $min\{\mu_A(0)$, $\{\mu_A(a_1), \mu_A(a_2),, \mu_A(a_n)\}$

Since $\{\mu_A(0)$ ≥ $\mu_A(a_1)$ for all

We get μ_A(x γ y) ≥ $min\{\mu_A(a_1),$ $\mu_A(a_2),$ $, \mu_A(a_n)\}$

Similarly the result for ϑ_A follows by taking the maximum of non-membership values. **Theorem 3.6:** Every Fermatean Fuzzy prime bi-ideal of a Γ-BCK algebras X is a Fermatean Fuzzy Prime Γ-subalgebra of X.

Proof: Let A = (μ_A, ϑ_A) be a Fermatean fuzzy prime bi-ideals of X, By the property of BCK-algebra, we know that for any x, y, ∈X and γ ∈ Γ, the inequality x γ y≤ X holds.

According to the order property in Lemma 3.4, μ_A(x γ y)≥ $\mu_A(x)$ and ϑ_A(x γ y) ≤ $\vartheta_A(x)$

Now, since A is a Fermatean Fuzzy prime bi-ideal

i) μ_A(x γ y) ≥ $\mu_A(x)$

μ_A(x γ y) ≥ $min\{\mu_A$(x)$, \mu_A(y)\}$(by def)

Since μ_A(x γ y) ≥ $\mu_A(x)$ it

Implies μ_A(x γ y)≥ $min\{\mu_A$(x)$, \mu_A(y)\}$

ii) ϑ_A(x γ y) ≤ $\vartheta_A(x)$

ϑ_A(x γ y) ≤ $max\{\vartheta_A$(x γ y)$, \vartheta_A(x)\}$(by def)

Since ϑ_A(x γ y) ≤ $\vartheta_A(x)$ it

implies ϑ_A(x γ y) ≤ $max\{\vartheta_A$(x)$, \vartheta_A(y)\}$

Therefore A = (μ_A, ϑ_A) satisfies the conditions of a Fermatean Fuzzy Γ- sub-algebra

Hence, Every Fermatean Fuzzy prime bi-ideal is a Fermatean Fuzzy prime Γ- subalgebra of X.

Lemma 3.7 Fermatean fuzzy set A = (μ_A, ϑ_A) is a Fermatean Fuzzy prime bi-ideal of X iff the Fuzzy set μ_A and ϑ_A (where $\vartheta^c = 1 - \vartheta_A$) are Fuzzy prime bi-ideal of X

Proof: Let us suppose that the configuration A = (μ_A, ϑ_A) represents a Fermatean Fuzzy prime bi-ideal of X.

By definition, for the non-membership complement ϑ^c_A

$\vartheta^c_A(0) = 1 - \vartheta_A(0)$

Since $\vartheta_A(0) \leq \vartheta_A(u)$ for all u $\in$X, we have

$1 - \vartheta_A(0) \geq 1 - \vartheta_A(u)$

which implies $\vartheta^c_A(0) \geq \vartheta^c_A(u)$ for any u, v, w $\in$X and $\gamma, \delta \in \Gamma$

$\vartheta^c_A(u\, \gamma\, w) = 1 - \vartheta_A(u\, \gamma\, w)$

Using the bi-ideal property

$\vartheta_A(u\, \gamma\, w) \leq max\, \{\vartheta_A((u\, \gamma\, v\delta\, w), \vartheta_A(v)\}$

$\vartheta^c_A(u\, \gamma\, w) \geq 1 - max\, \{\vartheta_A((u\, \gamma\, v\delta\, w), \vartheta_A(v)\}$

$= \min\, \{1 - \vartheta_A((u\, \gamma\, v\delta\, w), 1 - \vartheta_A(v)\}$

$= \min\, \{\vartheta^c_A(u\, \gamma\, v\delta\, w), \vartheta^c_A(v)\}$

Hence, it is verified that ϑ^c_A satisfies the conditions of a fuzzy Prime bi-ideal of X. In contrast Assume that μ_A and ϑ^c_A are prime bi-ideal of X. $\forall$ u $\in$X, we have

$\mu_A(0) \geq \mu_A(u)$

And $\vartheta^c_A(0) \geq \vartheta^c_A(u)$

Now $\vartheta^c_A(0) \geq \vartheta^c_A(u)$

$\Rightarrow$ 1-$\vartheta_A(0) \geq$ 1-$\vartheta_A(u)$

Which gives $\vartheta_A(0) \leq \vartheta_A(u)$

Since μ_A and ϑ^c_A are prime bi-ideal for any u, v, w $\in$X and $\gamma, \delta \in \Gamma$

$\mu_A(u\, \gamma\, v) \geq min\{\mu_A(u\, \gamma\, v\, \delta\, w, \mu_A(v)\}$ and

$\vartheta^c_A(u\, \gamma\, v) \geq min\{\vartheta^c_A(u\, \gamma\, v\, \delta\, w, \vartheta^c_A(v)\}$

1-$\vartheta_A(u\, \gamma\, w) = \vartheta^c_A(u\, \gamma\, v)$

$\geq min\{\vartheta^c_A(u\, \gamma\, v\, \delta\, w, \vartheta^c_A(v)\}$

$= \min\, \{1 - \vartheta_A((u\, \gamma\, v\delta\, w), 1 - \vartheta_A(w)\}$

$=1-\max\{\vartheta_A(u\, \gamma\, v\delta\, w), \vartheta_A(w)\}$

Thus 1-ϑ_A(u γ w) $\geq$1-max$\{\vartheta_A(u\, \gamma\, v\delta\, w), \vartheta_A(v)\}$ Hence A = (μ_A, ϑ_A) is a Fermatean Fuzzy prime bi-ideal of X.

Definition 3.8: Let f: X→ Y be a Γ-Homomorphism of Γ-BCK-algebras. For any Fermatean fuzzy set B= (μ_B, ϑ_B) in Y, we define a new Fermaten fuzzy set A=F^{-1}(B)= (μ_A, ϑ_A) in X, called the pre-image of B, defined as $\mu_A(u)= \mu_B(f(u))$

$\vartheta_A(u)= \vartheta_B(f(u))$

Theorem 3.9: Consider f: X→ Y be a Γ-Homomorphism of Γ-BCK-algebras. If B= (μ_B, ϑ_B) is a Fermatean fuzzy prime bi-ideal of Y, then the pre-image F^{-1}(B) = A = (μ_A, ϑ_A) is a Fermatean fuzzy prime bi-ideal of X

Proof: Let B= (μ_B, ϑ_B) be a Fermatean Fuzzy prime bi-ideal of Y. We define its pre-image A= (μ_A, ϑ_A) in X as, $\mu_A(u)= \mu_B(f(u))$ and $\vartheta_A(u)= \vartheta_B(f(u))$ for all u $\in$X

i) Condition for zero element:

$\mu_A(0) = \mu_B(f(0)\; = \mu_B(0')$

Since B is a bi-ideal of Y, $\mu_B(0') \geq \mu_B(f(u))$

$=\mu_A(u)$

$\Rightarrow \mu_A(0) \geq \mu_A(u)$

Similarly $\vartheta_A(0) = \vartheta_B(f(0))$

$= \vartheta_B(0')$

Since B is a bi-ideal of Y

$\vartheta_B(0') \leq \vartheta_B(f(u))$

$= \vartheta_A(u)$

$\Rightarrow \vartheta_A(0) \leq \vartheta_A(u)$

ii) Bi-ideal condition for membership (μ_A): for any u, v, w ∈ X and γ, δ ∈ Γ

$\mu_A(u\, \gamma\, w) = \mu_B(f(u\, \gamma\, w))$

$=\mu_B(f(\text{u})\ \gamma\ \text{f(w))}$

Since B is a Bi-ideal in Y:

μ_B (f(u) γ f(w) $\geq min$ $\{\mu_B((f(\text{u})\ \gamma\text{f(v)}\delta\ \text{F(w)}, \mu_B(f(v))\}$ Since f is a Γ-Homomorphism

$\mu_A(\text{u}\ \gamma\ \text{w}) \geq min\ \{\mu_B f((\text{u}\ \gamma\ \text{v})\delta\ \text{w}), \mu_B(f(v))\}$

$\mu_A(\text{u}\ \gamma\ \text{w}) \geq min\ \{\mu_A f((\text{u}\ \gamma\ \text{v})\delta\ \text{w}), \mu_A(v)\}$

iii) Bi-ideal condition for non-membership ϑ_A:

For any u, v, w ∈X and γ, δ ∈ Γ

$\vartheta_A(\text{u}\ \gamma\ \text{w}) = \vartheta_B(f(\text{u}\ \gamma\ \text{w}))$

$=\vartheta_B(f(\text{u})\ \gamma\ \text{f(w))}$

Since B is a Bi-ideal in Y:

ϑ_B (f(u) γ f(w) $\leq$ min $\{\vartheta_B((f(\text{u})\ \gamma\text{f(v)}\delta\ \text{f(w)}, \vartheta_B(f(v))\}$

$\vartheta_A(\text{u}\ \gamma\ \text{w}) \leq max\ \{f((\text{u}\ \gamma\ \text{v})\delta\ \text{w}), \vartheta_B(f(v))\}$

$\vartheta_A(\text{u}\ \gamma\ \text{w}) \leq max\ \{\vartheta_A((\text{u}\ \gamma\ \text{v}\delta\ \text{w}), \vartheta_A(v)\}$

iv) Prime Property:

To show that A is prime, for any u_1, u_2 ∈ X

$\mu_A(u_1\ \gamma\ u_2) = \mu_B(f(u_1\ \gamma\ u_2))$

$=\mu_B(f(u_1)\ \gamma\ \text{f(}\ u_2))$

Since B is a fuzzy prime Bi-ideal in Y

μ_B (f(u_1) γ f(u_2) = max $\{\mu_B((f(u_1), \mu_B(f(u_2))\}$

$\Rightarrow$ $\mu_A(u_1\ \gamma\ u_2) = max\{\mu_A(u_1)\ , \mu_A(u_2)\}$ Similarly for non-membership

$\vartheta_A(u_1\ \gamma\ u_2) = \vartheta_B(f(u_1\ \gamma\ u_2))$

$=\vartheta_B(f(u_1)\ \gamma\ \text{f(}\ u_2))$

$= min\ \{\mu_B((f(u_1), \mu_B(f(u_2))\}$

$\vartheta_A(u_1\ \gamma\ u_2)\ = min\{\vartheta_A(u_1)\ , \vartheta_A(\ u_2)\}$

Since all the conditions are satisfies, the pre-image F^{-1}(B)= A is a Fermatean fuzzy prime bi-ideal of X.

Theorem 3.10 Consider f: X→ Y be an epimorphism of Γ-BCK algebras. If A = (μ_A, ϑ_A) is a Fermatean Fuzzy prime bi-ideal of X, then its homomorphic image f(A) = B = (μ_B, ϑ_B) is a Fermatean Fuzzy prime bi-ideal of Y.

Proof: Since f is an epimorphism for any y ∈Y, there exists x∈X such that f(x)=y. The image B= (μ_B, ϑ_B) is defined by $\mu_B(y) = \mu_B(x)\ and\ \vartheta_B$ (y)= $\vartheta_B(x)$

$\mu_B(0') \geq \mu_B(y)$ and $\vartheta_B(0') \leq \vartheta_B(y)$

$\mu_B((y_1\ \gamma\ y_3) \geq min\ \{\mu_B((y_1\ \gamma\ y_2)\delta\ Y_3)), \mu_B((y_2)\}$

$\vartheta_B((y_1\ \gamma\ y_3) \leq max\ \{\vartheta_B((y_1\ \gamma\ y_2)\delta\ y_3)), \vartheta_B((y_2)\}$

To prove that B is a prime ideal, we need to show the intersection property. For any

$\mu_B((y_1\ \gamma\ y_2) = \mu_B(f(x_1)\ \gamma\ \text{f(}\ x_2))$

$= \mu_B(f(x_1\ \gamma\ x_2))$

$= \mu_A(x_1\ \gamma\ x_2)$

Since A is fuzzy prime ideal in X.

i) $\mu_A(x_1\ \gamma\ x_2)\ = max\{\mu_A(x_1)\ , \mu_A(\ x_2)\}$

$\Rightarrow$ $\mu_B((y_1\ \gamma\ y_2)\ = max\{\mu_B(y_1)\ , \mu_B(y_2)\}$

ii) $\vartheta_B((y_1\ \gamma\ y_2)\ =\vartheta_B(f(x_1)\ \gamma\ \text{f(}\ x_2))$

$=\vartheta_B(f(x_1\ \gamma\ x_2))$

$=\vartheta_A(x_1\ \gamma\ x_2)$

Since A is fuzzy prime,

$\vartheta_A(x_1\ \gamma\ x_2)\ = min\{\vartheta_A(x_1)\ , \vartheta_A(\ x_2)\}$

$\Rightarrow$ $\vartheta_B(y_1\ \gamma\ y_2)\ = min\{\vartheta_B(y_1)\ , \vartheta_B(\ y_2)\}$

Since B satisfies both the bi-ideal and prime properties, f(A)=B is a Fermatean Fuzzy prime bi-ideal of Y.

Conclusion:

Fermatean Fuzzy prime bi-ideal in Γ-BCK algebras have been introduced in this study. The constraint 0≤ $(\mu_A(x))^3 + (\vartheta_A(x))^3 \leq 1$ was used to discuss the structural properties.The work's primary findings are ; Fermatean Fuzzy prime bi-ideals in Γ-BCK algebras were introduced and their features were described.Demonstrated that these ideals' pre-image and homomorphic image maintain their primary bi-ideal characteristics within the Γ-BCK structure.

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