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On the Performance of Hirschman Optimal Transform based LMS Algorithm for Cancellation of ECG Power Line Interference

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Abstract— Power-line interference (PLI) is the most commonly encountered artifact in Electrocardiogram (ECG) during its recording. Several techniques have been proposed for elimination of PLI ranging from conventional notch filters to adaptive filters. As the PLI results in an in-band noise, adaptive filtering offers best solution. In this paper, we present Hirschman Optimal Transform (HOT) based adaptive filter for elimination of the PLI from ECG signals. Simulations and analysis show that HOT based LMS adaptive filter is computationally efficient and has fast convergence compared to LMS, NLMS and DFT based LMS algorithms. Performance of these algorithms was compared with respect to different RMS measures. Comparative test results reveal that HOT based adaptive filter efficiently eliminates the PLI from ECGs.

Keywords- Power-line interference, ECG, LMS, NLMS, HOT.

I. INTRODUCTION

Electrocardiogram (ECG) is the electrical manifestation of the contractile activity of the heart. Unfortunately, ECGs are contaminated by noise and artifacts that can be within the band of interest (0.05-100 Hz) during their recording, of which the most commonly encountered artifact is the power-line interference (PLI) [1]. It is a narrowband noise centered at 50 Hz or 60 Hz depending on the country with a bandwidth of less than 1 Hz, which arises from electromagnetic fields caused by power-line. Sometimes it can also be 16 2/3Hz as this is the frequency used by trains. The power spectrum of the signal provides a clear indication of the presence of PLI as an impulse at 60 Hz as shown in fig1. Under severe conditions, the 60 ±0.2 Hz mains noise interfere with amplitude of up to 50% of full scale deflection (FSD), the peak-to-peak ECG amplitude. Such narrowband noise renders the analysis and interpretation of the low-amplitude ECGs more difficult, since the delineation of low-amplitude waveforms becomes unreliable [2]. Therefore, how to eliminate or reduce the effect of 50/60Hz interference has been one of the most important problems in ECG signal processing.

Several techniques have been proposed [3]-[7] for suppression or elimination of the PLI from ECG signals. Lowpass filtering with bandwidth lower than PLI could smooth and blur the QRS and affect the PQ and ST segments. One of the conventional methods is to pass the signal through a digital notch filter characterised by unit gain at all frequencies except at PLI frequency where its gain is zero [3]. Subtraction procedure has been presented which does not effect the neighbouring to the PL frequency ECG components [4]. Adaptive noise cancelling technique has been proposed,

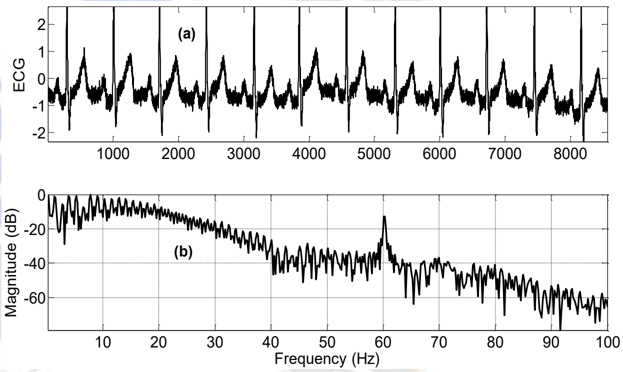

Fig. 1. PLI corrupted ECG in (a) and its power spectrum in (b)

when an auxiliary reference input containing interference alone is available [5],[6]. It has been shown that the adaptive implementation is less complex and introduces less noise, particularly in the ST-segment, into a typical ECG signal [7]. The concepts of adaptive filtering were developed both in time-domain and frequency-domain [8], based on most popular methods being the Least Mean Square (LMS) algorithm and its derivatives. In this paper, Hirshman Optimal Transform (HOT) based frequency domain adaptive filtering method is presented for cancelling the PLI and its performance is compared with LMS, NLMS and DFT based PLI elimination methods.

The rest of the paper is organized as follows. In section II, we briefly review time-domain LMS, NLMS, Transform domain LMS algorithms. Section III presents basics of HOT and HOT based LMS update equation. In section IV, the simulations and experimental results of the presented HOT based PLI elimination. Finally, conclusions are made in section V with a possible scope for future work.

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II. ADAPTIVE FILTERING

Adaptive filters are typically used when noise occurs in the same band as the signal or when the noise band is unknown or varies over time. The basic form of time-domain adaptive filtering is shown in fig. 2.

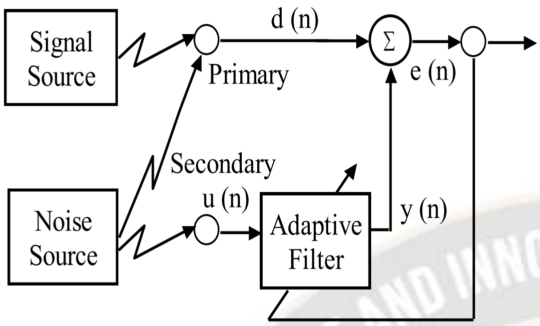

Fig. 2. Basic form of the time-domain adaptive filtering

Different algorithms can be used to adapt the weights **w** of the filter, with a attempt to minimize the mean square error (MSE) performance function.

A. LMS Algorithm

The LMS algorithm makes use of instantaneous estimate of the gradient to search the minimum of the error surface [8]. The complete LMS algorithm is written as three equations.

$y(n)=\mathbf{w}^T(n)\mathbf{u}(n)$: filter output (1)

$e(n)=d(n)-y(n)$: error formation (2)

$\mathbf{w}(n+1)=\mathbf{w}(n)+\mu e(n)\mathbf{u}(n)$: weight vector update (3)

where $\mathbf{u}(n)$ is the filter input at instant *n, e(n)* is the error incurred by the adaptive filter, *d(n)* being the desired output of the filter and μ is the step size used in the weight vector updation, which governs the rate of convergence of the algorithm, with the following bounds.

$$0<\mu<2/\lambda_{\max} \approx 0<\mu<2/\mathrm{tr}[R] \approx 0<\mu<2/S_{\max}$$

where $\lambda_{\max}$ is the largest eigenvalue of input autocorrelation matrix $R=E[\mathbf{u}\mathbf{u}^T]$ and $S_{\max}$ maximum value of the input signal power spectrum. In practice, the exact statistics of **u***(n) and d(n)* are unknown or vary with time. A time-varying step size, $\mu(n)$, if properly computed, can provide stable, robust, and accurate convergence behaviour for the LMS adaptive filter in these situations.

B. NLMS Algorithm

From the weight update equation (3) it is clear that the adjustment is directly proportional to the tap input vector **u** (*n*). Therefore, when **u** (*n*) is large, then the LMS filter suffers from a gradient noise amplification problem. To overcome this difficulty, we may use the normalized LMS filter. In particular, the adjustment applied to the tap weight vector at iteration (*n*+1) is “normalized” with respect to the squared Euclidean norm tap input vector **u**(*n*) at iteration *n.*

So the weight vector update equation, for each iteration, is given as

$$\mathbf{w}(n+1)=\mathbf{w}(n)+\frac{\mu}{\|\mathbf{u}(n)\|^2}\mathbf{u}(n)e(n) \quad (4)$$

With the proper choice of μ , the NLMS adaptive filter can often converge faster than the LMS adaptive filter [9].

C. Transform Domain Adaptive Filters (TDAF)

The concept of adaptive filtering in frequency domain was published in 1978 by Dentino et al, in which in addition to the DFT, other orthogonal transforms such as the DCT and the Walsh-Hadamard Transform (WHT), are also used effectively as a means to improve the LMS algorithm without adding too much computational complexity [10]. The TDAF structure is shown in fig.3.

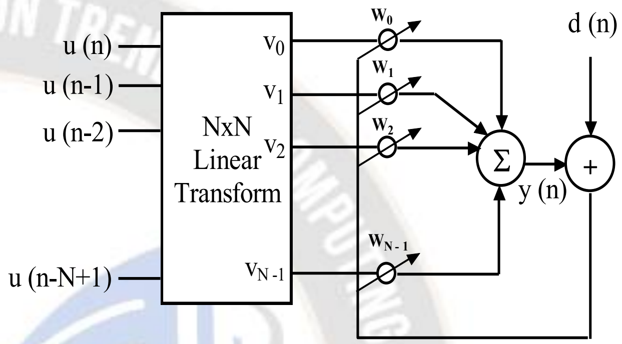

Fig 3. Transform based adaptive filtering

The input signal is pre processed by decomposing the input vector into the orthogonal components, which are in turn used as inputs to a parallel bank of simpler adaptive filters [11]. With an orthogonal transformation, the adaptation takes place in transform domain, as it possible to show that the adjustable parameters are indeed related to an equivalent set of time domain filter co-efficients by means of the same transformation that is used for real time processing. The self orthogonalizing adaptive filtering algorithm for a wide sense stationary environment [12] is described by the following weight vector update equation

$$\mathbf{w}(n+1)=\mathbf{w}(n)+\alpha R^{-1}\mathbf{u}(n)\, e(n) \quad (5)$$

The constant α lies in the range: $0<\alpha<1$ and is given by

$$\alpha=1/(2N) \quad (6)$$

where N is the filter length. An important property of the self orthogonalizing filtering algorithm of eq (5) is that it guarantees a constant rate of convergence, irrespective of input statistics. The transformed outputs form a vector **v***(n)* which is given as

$$\mathbf{v}(n)=\mathit{Transform}\,[\mathbf{u}(n)]=[v_0(n),v_1(n),..........v_{M-1}(n)]^T \quad (7)$$

Here Transform can be any orthogonal transformation

Output is given by

$$y(n)=\mathbf{w}^T(n)\,\mathbf{v}(n) \quad (8)$$

The instantaneous output error is $e(n)=d(n)-y(n)$

Now, replacing $\mathbf{u}(n)$ and R^{-1} with the transformed vector $\mathbf{v}(n)$ and its inverse correlation matrix $\wedge^{-1}$ respectively, eq. (5) becomes

$$\mathbf{w}(n+1)=\mathbf{w}(n)+\alpha\wedge^{-1}\mathbf{v}(n)e(n) \quad (9)$$

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where

$$\wedge = E[\mathbf{v}(n)\mathbf{v}^T(n)] = \; = diag[\lambda_0, \lambda_1, \ldots\ldots\ldots\ldots, \lambda_{M-1}] \quad (10)$$

and the inverse of $\wedge$ is diagonal matrix.

$$\wedge^{-1} = diag[\lambda_0^{-1}, \lambda_1^{-1}, \ldots\ldots\ldots\ldots, \lambda_{M-1}^{-1}] \quad (11)$$

III. HOT Adaptive Filter

The HOT is a discrete unitary transform that uses the orthonormal minimizers of the entropy-based Hirschman uncertainty measure [13]. This measure uses entropy to quantify the spread of discrete-time signals in time and frequency and is different from the energy-based Heisenberg uncertainty measure that is only suited for continuous time signals.

A. HOT Basis Functions

The basis functions that define the HOT are derived using the K-dimensional DFT as the originator signals for $N = K^2$-dimensional HOT basis and K must be an integer. Each of these basis functions must then be shifted and interpolated to produce the sufficient number of orthogonal basis functions that define the HOT.

In general, we have the (unitary) transform relationship [13],

$$H(Kr+l) = \frac{1}{\sqrt{K}} \sum_{n=0}^{K-1} x[Kn+l] e^{-j\frac{2\pi}{K}nr}, 0 \le r, l \le K-1 \quad (12)$$

and it's inverse

$$x(Kn+l) = \frac{1}{\sqrt{K}} \sum_{r=0}^{K-1} x[Kr+l] e^{j\frac{2\pi}{K}nr}, 0 \le n, l \le K-1 \quad (13)$$

In general, the N-point HOT is computationally more efficient than the N-point DFT and increasingly more efficient as $N \to \infty$. A HOT basis sequence of length K^2 is the most compact bases in the time-frequency plane. For a 3^2 - point HOT matrix, we need to start with 3-point DFT and the 9-point HOT matrix H can be derived as follows.

$$H = \begin{pmatrix} I & I & I \\ I & e^{-j2\pi/3} I & e^{-j4\pi/3} I \\ I & e^{-j4\pi/3} I & e^{-j8\pi/3} I \end{pmatrix} \quad (14)$$

where, I is 3x3 identity matrix.

Like the DFT, the HOT is unitary and so the inverse transform can be achieved by simply taking the conjugate transpose and scaling by $\sqrt{K}$.

B. HOT Adaptive Algorithm

Let $\mathbf{u}(n)$ be the input vector to the filter, $\mathbf{u}_H(n)$ is the HOT transform of $\mathbf{u}(n)$, and the filter output is given by

$$y(n) = \mathbf{w}_H^T(n)\mathbf{u}_H(n) \quad (15)$$

The update weight vector equation, for each iteration, is

$$\mathbf{w}_H(n+1) = \mathbf{w}_H(n) + \alpha \wedge^{-1}(n)\, e(n)\, \mathbf{u}_H^*(n) \quad (16)$$

The diagonal matrix $\wedge(n)$ contains the estimated power of the HOT co-efficients and can be updated using recursion

$$\wedge(n) = \wedge(n-1) + \frac{1}{n}[\mathbf{u}_H^*(n-1)\, \mathbf{u}_H(n-1) - \wedge(n-1)] \quad (17)$$

where $\alpha = 1/(2K^2)$ and K^2 is the filter length and * indicates complex conjugation.

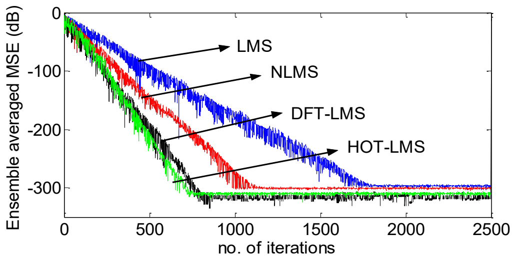

Fig. 4. Learning curves of different adaptive algorithms

IV. Simulations and Experimental Results

Simulations were carried out for HOT based LMS algorithm for 9-tap system identification, with White Gaussian noise. Learning curve of HOT-LMS along with conventional LMS, NLMS, DFT-LMS are shown in fig.4. In general, the transform domain LMS algorithms show better performance with less mean square error over time domain LMS methods due to their ability to decorrelate the samples at the input of the filter. It can be clearly seen that the HOT-LMS converges faster than DFT-LMS. In order to test the efficacy of the HOT-LMS in adaptive noise cancellation, an ECG signal contaminated with PLI, as shown in fig.1(a), is considered. HOT-LMS along with conventional LMS algorithms were applied on the PLI corrupted data. First HOT-LMS was applied on clean ECG data and the result is shown in fig 5(a), where only three cycles of ECG were shown for the sake of clarity. It can be understood that the morphological features of the ECG were not disturbed. Then, HOT-LMS is applied on the PLI corrupted ECG data and the result is depicted in fig. 5(b).

Visual inspection of the filtered output from all the algorithms did not reveal much information about the efficacy of the HOT-LMS. Hence for performance comparison of adaptive PLI filters, the following statistical measures were considered.

1. Root mean square deviation (RMSD): It is the RMS value obtained from the pure ECG signal minus the restored ECG signal that has been processed by the adaptive filter. A smaller RMSD value indicates a better efficacy of the adaptive filter in eliminating PLI and less distortion of signal after the filtering process.

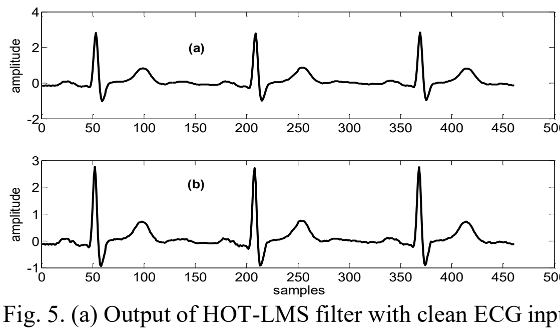

Fig. 5. (a) Output of HOT-LMS filter with clean ECG input
(b) Output of HOT-LMS with PLI corrupted ECG input

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2. *Root mean square error (RMSE):* It is the RMS value of the difference between the restored ECG and the filter output for clean ECG. A smaller RMSE value indicates a lesser distortion of ECG morphology after the filtering operation. RMSE looks similar to RMSD, yet RMSE takes more considerations on the possibility of the ECG morphology distortion after filtering.

3. *Root mean square variation (RMSV):* It is the RMS value of the tiny variation from the original input ECG after it has been blindly processed by the algorithms. The RMSV indicates the degree of variation of the ECG signal processed by the adaptive filter.

The relative RMS statistics for all the algorithms in time and frequency domains were computed and presented in Table I and Table II respectively. The efficacy of HOT-LMS in elimination of PLI is clearly seen from the tables that the method resulted in minimum error statistics.

Table I. Relative RMS statistics for different forms of LMS algorithm in time domain

	RMSV	RMSE	RMSD
LMS	0.0049 ± 2.4 x10^{-4}	0.453 ± 0.027	0.449 ± 0.0272
NLMS	0.0041 ± 3.6 x 10^{-4}	0.452 ± 0.027	0.450 ± 0.0272
DFT-LMS	4.1 x 10^{-5} ± 1.3 x10^{-6}	0.451 ± 0.044	0.448 ± 0.0443
HOT-LMS	2.2 x 10^{-5} ± 8.7 x10^{-8}	0.451 ± 0.027	0.448 ± 0.0270

Table II. Relative RMS statistics for different forms of LMS algorithm in frequency domain

	RMSV	RMSE	RMSD
LMS	0.056 ± 0.0015	2.776 ± 0.021	2.724 ± 0.0209
NLMS	0.045 ± 0.0035	2.773 ± 0.020	2.745 ± 0.0205
DFT-LMS	2.9 x 10^{-4} ± 1.25 x10^{-4}	2.618 ± 0.051	2.718 ± 0.0511
HOT-LMS	2.2 x 10^{-4} ± 8.7 x10^{-8}	2.617 ± 0.035	2.702 ± 0.0357

V. Conclusion

HOT based LMS adaptive filtering for the elimination of PLI from ECG signals has been presented in this paper. The convergence analysis of different adaptive algorithms clearly indicated a faster convergence for HOT-LMS. Some useful statistical measures were evaluated to demonstrate that the HOT-LMS resulted in better filtering of PLI compared to other LMS algorithms. The next goal is to implement these algorithms in FPGA where fast prototyping and reconfigurable computing takes place.

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