

Numerical Modeling of the Flow Dynamics of Red Blood Cells and Plasma in the Aorta

Ashok Giri^{*1}, Rajesh Johari², and Santosh Kumar³

¹ Research Scholar, Department of Mathematics, Agra College, Agra (Dr. Bhimrao Ambedkar University, Agra), Uttar Pradesh, India

² Professor, Department of Mathematics Agra College Agra, (Dr. Bhimrao Ambedkar University, Agra), Uttar Pradesh, India

³ Professor, Department of T B and Respiratory medicine, S N Medical College, Agra, Uttar Pradesh, India

Abstract: The study of red blood cells (RBCs) and plasma flow rates in the aorta aims to understand the hemodynamic behavior of blood as it circulates through one of the body's largest arteries. RBCs and plasma exhibit different rheological characteristics, with RBCs being more viscous due to their cellular structure, while plasma, primarily made up of water and proteins, is less viscous. This research typically integrates computational modeling and experimental techniques to analyze how these two components interact under varying flow conditions, including shear rates, vessel dimensions, and pressure gradients. The objective is to assess how these interactions influence overall blood flow, which is crucial for understanding circulatory dynamics and addressing cardiovascular conditions like atherosclerosis or aneurysms. Insights from such research are vital for advancing medical treatments, improving diagnostic methods, and optimizing the design of medical devices such as stents and artificial heart valves.

Keyword: RBCs, plasma, Reynolds number, shear rate.

Introduction: The aorta, the largest artery in the human body, plays a critical role in delivering oxygenated blood from the heart to the rest of the circulatory system. Blood is a heterogeneous fluid consisting of plasma, which behaves as a Newtonian fluid, and red blood cells (RBCs), which are non-Newtonian, deformable particles. In the laminar flow regime, the vortex length predicted by low-Reynolds number (Re) models aligns closely with that of laminar flow simulations. This suggests that the proposed model is well-suited for studying blood flow in regions of the arterial tree where both laminar and transitional/turbulent flows coexist [1]. The interaction between RBCs and plasma, combined with the pulsatile nature of blood flow, generates complex hemodynamic patterns in the aorta. These dynamics are influenced by factors such as vessel geometry, blood viscosity, and the mechanical properties of RBCs. Numerical modeling of aortic blood flow has gained prominence in recent years due to its ability to provide detailed hemodynamic insights non-invasively. Studies have shown a strong inverse correlation between temperature and blood viscosity ($r = -0.84$, $p < 0.001$). Additionally, as blood glucose levels rise from 100 to

400 mg/dL, viscosity increases by 25% ($r = 0.59$, $p = 0.002$), leading to a 20% reduction in blood flow rate and a 25% increase in blood pressure (BP) as a physiological compensatory mechanism. These findings highlight the importance of temperature, glucose levels, and viscosity in regulating BP [2]. Globally, approximately 75 million units of blood are collected annually. RBC transfusions are a key intervention for restoring tissue oxygenation when demand exceeds supply. Despite extensive research on blood's respiratory function, the criteria for RBC transfusion remain debated, with clinicians often relying on empirical judgment [3]. This study aims to develop a high-fidelity numerical model to simulate the flow dynamics of RBCs and plasma in the aorta, focusing on their interactions and their impact on overall blood flow. RBCs, or erythrocytes, constitute about 40-45% of blood volume in healthy individuals and are essential for oxygen and carbon dioxide transport. Mass transfer of low-density lipoproteins (LDLs) in the arterial system may contribute to the localization of atherogenesis. To stabilize blood flow, a tapered aortic geometry was employed [4]. Plasma, the liquid component of blood, contains water, electrolytes,

proteins, hormones, and other solutes. The interplay between RBCs and plasma gives rise to unique rheological properties, such as shear-thinning behavior and plasma skimming, which significantly influence blood flow dynamics. Aortic diameter is a key factor in deciding elective ascending aortic replacement to prevent dissection, though recommendations are often based on clinical experience and observations of dissected aortas. Aortic dissection alters the geometry of the thoracic aorta, particularly increasing the diameter of the ascending aorta, with similar geometric changes observed in both spontaneous and retrograde dissection [5]. Blood and plasma viscosity were measured at 37°C and a shear rate of 225/s, with erythrocyte rigidity (Tk) calculated using Dintenfass's method. Routine methods were used to measure blood lipids and glucose. Hyperlipidemic individuals (n = 315) exhibited higher plasma viscosity (1.44 ± 0.13 vs. 1.40 ± 0.12 cP, $p = 0.007$) and blood viscosity (4.51 ± 0.54 vs. 4.35 ± 0.55 cP, $p = 0.013$) compared to normolipidemic subjects (n = 95). Plasma viscosity was positively correlated with LDL cholesterol and inversely correlated with Tk and HDL cholesterol. In multiple regression analysis, LDL and HDL cholesterol together accounted for only 5% of plasma viscosity variability. Blood viscosity was significantly associated with hematocrit, plasma viscosity, and Tk, but not with age. The study concluded that LDL and HDL cholesterol influence plasma viscosity but not blood viscosity, while triglycerides up to 400 mg/dL have minimal effects in healthy subjects at the studied shear rates [6].

Shear Rate (γ') Shear rate is a measure of how quickly adjacent layers of fluid move relative to each other. It describes the rate of deformation of the fluid due to shear stress. Mathematically, shear rate is defined as:

$$\gamma' = \frac{du}{dy} \quad (1)$$

Where:

γ' = shear rate (units: reciprocal seconds, s^{-1})

du = difference in velocity between adjacent fluid layers (m/s)

dy = distance between the layers (m)

The blood, in fact, is a concentrated suspension of cells, mainly red blood cells, in a Newtonian matrix, the

plasma, and consequently its overall behavior is that of a non-Newtonian fluid owing to the action of the cells' membrane on the fluid part. The common practice, however, assumes the blood in large vessels as a Newtonian fluid since the shear rate is generally high and the effective viscosity becomes independent of the former. Even in the aorta, the largest artery of the systemic circulation, owing to the pulsatile and transitional nature of the flow [7]

Shear Stress (τ)

Shear stress is the force per unit area exerted by a fluid (or material) in response to an applied shear force. It is a measure of the internal friction or resistance to flow within the fluid. Mathematically, shear stress is defined as:

$$\tau = \frac{F}{A} \quad (2)$$

Where:

τ = shear stress (Pa, or N/m^2)

F = applied force (Newton's, N)

A = area over which the force is applied (units: square meters, m^2)

Whole blood viscosity (WBV) could be an important factor for the occurrence of aortic valve sclerosis (AVS). WBV was calculated using the hematocrit and total plasma protein at a low shear rate (LSR) and a high shear rate (HSR). AVS was defined as irregular valve thickening and calcification (without evidence of outflow obstruction) documented by a peak transvalvular velocity < 2.5 m/s on echocardiographic examination. [8]

Total Blood Flow in the Aorta

The average cardiac output is 5–6 L/min at rest.

Blood flow in the aorta is approximately 5 L/min in a healthy adult.

Plasma and RBC Flow Rates

Blood consists of plasma (about 55% of blood volume) and formed elements (45%, mostly RBCs). Using these proportions:

Plasma flow rate = 55% of total blood flow
 $\approx 2.75 \text{ L/min}$

RBC flow rate = 45% of total blood flow
 $\approx 2.25 \text{ L/min}$

Factors Affecting Flow Rate

Heart Rate & Stroke Volume: Increased cardiac output (e.g., during exercise) raises both plasma and RBC flow. Hematocrit Levels: Higher RBC percentage (e.g., in polycythemia) increases RBC flow but may reduce plasma flow. Vascular Resistance: Aortic stiffness or narrowing can affect overall blood flow. Plasma serves as the liquid base for whole blood. Whole blood minus erythrocytes (RBCs), leukocytes (WBCs), and thrombocytes (platelets) make up the plasma. Serum, sometimes mistakenly considered synonymous with plasma, consists of plasma without fibrinogen. Plasma contains 91% to 92% of water and 8% to 9% of solids.[9] Blood viscosity is the inherent resistance of the blood stream in the vasculature and is closely related to blood flow rate. An abnormal blood viscosity is associated with decreased tissue perfusion and with the development of atherosclerosis. As a result of increased blood viscosity, stasis occurs secondary to the deterioration of blood flow. [10]

Methodology:

The Reynolds number (Re) is a dimensionless quantity used to predict flow patterns in fluid dynamics. It is defined as:

$$\text{Re} = \frac{\rho v D}{\mu} \quad (3)$$

Where:

ρ is the density of the fluid (kg/m^3),

v is the velocity of the fluid (m/s),

D is the characteristic diameter (m) (for the aorta, this is typically the diameter of the vessel),

μ is the dynamic viscosity of the fluid ($\text{Pa}\cdot\text{s}$).

For blood plasma and red blood cells (RBCs) in the aorta, the flow is a two-phase flow (plasma is the liquid phase, and RBCs are the solid phase). The Reynolds number can be calculated separately for each phase or as an effective Reynolds number for the mixture.

Reynolds Number for Blood Plasma:

Blood plasma is a Newtonian fluid with the following approximate properties:

Density (ρ plasma): $\sim 1025 \text{ kg/m}^3$,

Dynamic viscosity (μ plasma): $\sim 1.2 \times 10^{-3} \text{ Pa}\cdot\text{s}$,

Velocity (v): $\sim 0.3\text{--}0.5 \text{ m/s}$ (average velocity in the aorta),

Diameter (D): $\sim 0.02\text{--}0.025 \text{ m}$ (aortic diameter).

Using these values, the Reynolds number for plasma is:

$$\text{Re}_{(\text{plasma})} = \frac{\rho v D}{\mu}$$

Reynolds Number for RBCs:

RBCs are solid particles suspended in plasma. Their motion is influenced by the surrounding plasma, and their effective Reynolds number can be calculated based on their relative velocity and size. The properties are:

Density (ρ_{RBC}): $\sim 1090 \text{ kg/m}^3$,

Diameter (D): $\sim 0.02\text{--}0.025 \text{ m}$ (aortic diameter).

Relative velocity (v_{RBC}): Slightly different from plasma velocity due to shear-induced migration.

The Reynolds number for RBCs is:

$$\text{Re}_{\text{RBC}} = \frac{\rho v D}{\mu}$$

Governing Equations for Two-Phase Blood Flow

Blood is modeled as a two-phase flow:

- Plasma (Newtonian fluid)
- Red Blood Cells (RBCs - particulate phase)

Continuity Equation (Mass Conservation) [11]**For plasma (fluid phase):**

$$\frac{\partial \alpha_p \rho_p}{\partial t} + \nabla \cdot (\alpha_p \rho_p u_p) = 0 \quad (4)$$

For RBCs (dispersed phase):

$$\nabla \cdot (\alpha_r \rho_r u_r) = 0 \quad (5)$$

Where:

- α_p, α_r : volume fractions of plasma and RBCs ($\alpha_p + \alpha_r = 1$)
- ρ_p, ρ_r : densities of plasma and RBCs
- u_p, u_r : Velocities of plasma and RBCs

Momentum Conservation Equations [12]**For plasma:**

$$\frac{\partial (\alpha_p \rho_p u_p)}{\partial t} + \nabla \cdot (\alpha_p \rho_p u_p \otimes u_p) = -\alpha_p \nabla p + \nabla \cdot (\alpha_p \tau_p) + M_{rp} \quad (6)$$

For RBC

$$\frac{\partial (\alpha_r \rho_r u_r)}{\partial t} + \nabla \cdot (\alpha_r \rho_r u_r \otimes u_r) = -\alpha_r \nabla p + \nabla \cdot (\alpha_r \tau_r) + M_{pr} \quad (7)$$

Where:

- p : common pressure field (incompressible assumption)
- $\tau_p = \mu_p (\nabla u_p + \nabla u_p^T)$: viscous stress tensor for plasma
- τ_r : effective viscous stress tensor for RBCs (often modeled as shear-thinning or viscoelastic)
- $M_{rp} = -M_{pr}$: interaction force (drag, lift, etc.) between plasma and RBCs

Interaction Force (Drag Model) [13][14][15]

The drag force between RBCs and plasma is often modeled as:

$$M_{rp} = K (u_r - u_p) \quad (8)$$

Where:

 K : is the interphase momentum exchange coefficient, often modeled as

$$K = \frac{9\mu_p \alpha_r}{2d_r^2} \quad (9)$$

With d_r being the average RBC diameter ($\sim 8 \mu\text{m}$)**Constitutive Models [16][17][18][19]****For Plasma:**

- Newtonian:

$$\mu_p = \text{constant} \quad (10)$$

For RBCs:

- Non-Newtonian (viscosity depends on shear rate or hematocrit H):

$$\mu_r = \mu_0 \left(1 + \frac{\mu_\infty - \mu_0}{1 + (k\dot{\gamma})^n} \right) \quad (11) \quad \text{Or}$$

$$\mu_{eff} = \mu_p (1 + 2.5\phi + \dots)$$

Pulsatile Flow Boundary Conditions [20][21]

$$u_{inlet}(t) = U_0 (1 + A \sin(\omega t)) \quad (12)$$

Where:

- U_0 : mean inlet velocity
- A : pulsation amplitude
- $\omega = 2\pi f$, $f \sim 1$ Hz (heart rate)

Wall Boundary Conditions

- **No-slip:** $u_p = u_r = 0$
- **Compliance (optional):** if vessel wall motion is considered
- **Navier slip** or deformable wall models (for FSI)

Numerical Modeling of RBC and Plasma Flow in the Aorta

Flow Behavior Plots

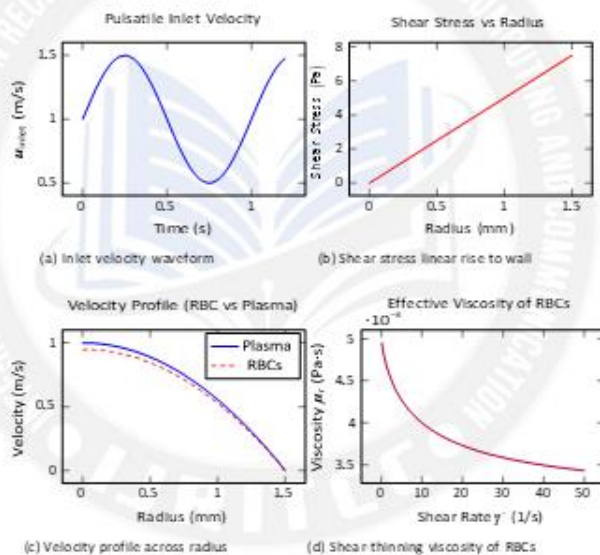


Figure 1: Plots for modeling RBC and plasma dynamics in aorta

Results:

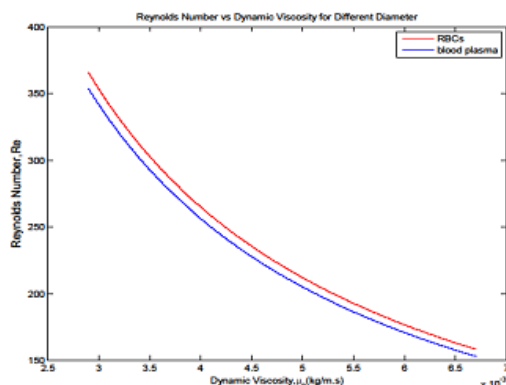


Fig.2 Reynolds number and Dynamic Viscosity of RBCs and blood plasma for different diameter

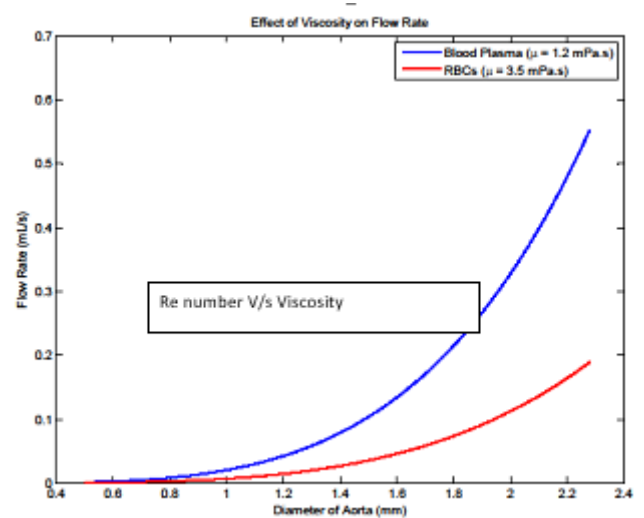


Fig.3 Flow rate V/S aorta diameter

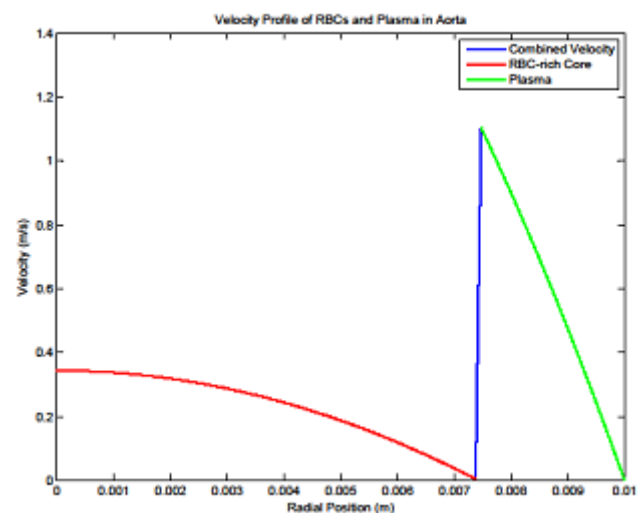


Fig.4 Velocity Profile for RBCs, Blood Plasma and Combined Velocity

Effective Reynolds Number for Two-Phase Flow:

For the mixture of plasma and RBCs, an effective Reynolds number can be calculated using the mixture's effective density (ρ_{eff}) and viscosity (μ_{eff}). The hematocrit (volume fraction of RBCs, typically ~ 0.45 in blood) plays a role in determining these effective properties.

$$Re_{eff} = \frac{\rho_{eff} v D}{\mu_{eff}}$$

Where:

$$\rho_{eff} = (1-H) \rho_{plasma} + H \rho_{RBC},$$

μ_{eff} depends on the hematocrit and shear rate (non-Newtonian behavior).

For example, if $H=0.45$

$$\rho_{eff} = (1-0.45) \times 1025 + 0.45 \times 1090 \approx 1057 \text{ kg/m}^3$$

Assuming $\mu_{eff} \approx 3 \times 10^{-3}$ (typical for blood):

$$Re_{eff} = (1057 \times 0.4 \times 0.025) / 3 \times 10^{-3} \approx 3523$$

The flow behavior of red blood cells (RBCs) and blood plasma in the aorta is influenced by several key factors, including vessel diameter, blood viscosity, and pressure gradients. These factors interact in complex ways to determine the hemodynamic characteristics of the two-phase flow. Below is a detailed analysis of their effects.

Effective viscosity

The effective viscosity of blood depends on the number of red blood cells (RBCs) in the blood, as well as the temperature and other factors.

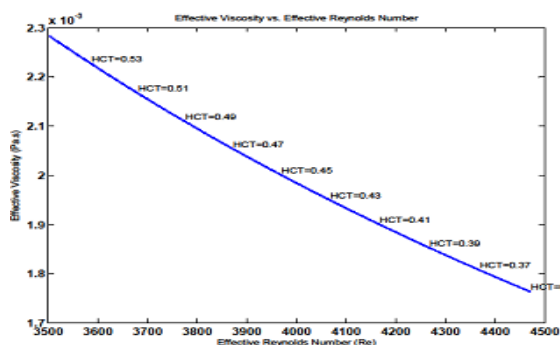


Fig.5 Effective Viscosity and Effective Re

Summary:

Plasma Reynolds number: ~ 8531 (turbulent flow)

RBC Reynolds number: ~ 0.068 (creeping flow)

Effective Reynolds number for blood: ~ 3523

(transitional or turbulent flow). In the aorta, blood flow is generally transitional or turbulent due to the high Reynolds number. This model accurately predicts the critical Reynolds number at which blood flow becomes transitional or turbulent distal an arterial stenosis

CONCLUSION:

Parameter	Effect on Flow Behavior	Clinical Implications
Diameter	Larger diameter: Thicker CFL, lower WSS. Smaller diameter: Thinner CFL, higher WSS.	Aneurysms, stenosis, atherosclerosis.
Viscosity	High viscosity: Increased flow resistance. Low viscosity: Reduced flow resistance.	Polycythemia, anemia, altered hemodynamics.
Pressure Gradient	High pressure: Increased velocity, shear rate, and WSS. Low pressure: Reduced WSS.	Hypertension, heart failure, endothelial dysfunction.

The two-phase flow of RBCs and plasma in the aorta is characterized by complex hemodynamic phenomena, including phase separation, non-Newtonian behavior, and secondary flows. These findings underscore the importance of using two-phase models to accurately predict flow patterns and WSS (wall shear stress), which are critical for understanding the hemodynamic basis of cardiovascular diseases. Future research should focus on patient-specific modeling and the effects of pathological conditions on two-phase flow dynamics. Predict pressure drops in the aorta under physiological and pathological conditions. Analyze the effects of changes in hematocrit, vessel diameter, and flow rate on hemodynamics. Design patient-specific treatments for cardiovascular diseases. The pressure drop in the aorta for two-phase flow of RBCs and plasma depends on the effective viscosity, hematocrit, vessel diameter, and flow rate. This formula provides a framework for analyzing hemodynamic changes in physiological and pathological conditions.

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