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Enhanced Hub Location Routing with Branch-and-Cut Methods and Simplified Mathematical Models

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Abstract: This article presents an advanced approach to the hub location routing problem, focusing on the optimal placement of hub nodes and the allocation of spoke nodes. We introduce a novel branch-and-cut method combined with a new simplified mathematical model incorporating valid inequalities. This hybrid approach aims to enhance solution quality and computational efficiency. Our proposed method integrates a learning mechanism to guide local searches, leveraging dual information from Lagrangian relaxation. Computational experiments validate the effectiveness of the proposed method.

Keywords : *Enhanced Hub, Simplified Mathematical*

1. Introduction

1.1 Background and Motivation

The hub location routing problem (HLRP) is a crucial combinatorial optimization problem with diverse applications in logistics, transportation, and network design. It involves determining the optimal locations for hub nodes and efficiently routing spoke nodes among these hubs [32, 20]. In practice, hubs act as central nodes that consolidate and redistribute flows, which is vital for systems such as postal delivery [21], and telecommunications [17].

In its classical form, the HLRP requires the establishment of hub nodes and the allocation of spoke nodes to these hubs, where each spoke is served by a single hub [6]. However, the problem's complexity increases with additional constraints, such as directed routes and capacity limitations [24]. For instance, each cluster of spoke nodes allocated to a hub may need to follow a directed route that starts and ends at the same hub, traversing all the spoke nodes in between [23]. This variant introduces significant challenges in designing efficient algorithms [18].

1.2 Related Work

The literature on the hub location routing problem is extensive, with various approaches addressing its different variants. Traditional methods include exact algorithms like branch-and-bound and branch-and-cut [27, 16], as well as heuristic and metaheuristic approaches such as genetic algorithms [19], simulated annealing [30], and ant colony optimization [14]. Recent advancements have explored hybrid methods that combine multiple heuristics to leverage their individual strengths and improve performance [28]. Relax-and-cut methods, which involve Lagrangian relaxation coupled with cutting planes, have shown promise in tackling complex combinatorial problems by iteratively refining feasible solutions [2, 5]. Hyper-heuristics, strategies for selecting or generating heuristics, further enhance solution quality by adapting to different problem instances [7, 8]. Despite these advancements, the integration of relax-and-cut methods with hyper-heuristics for the specific variant of the HLRP involving directed routes remains underexplored [31]. Additionally, the development of simplified mathematical models and efficient valid inequalities is crucial for improving computational efficiency and solution quality [1, 13].

1.3 Contribution of This Work

In this paper, we propose a novel approach to the hub location routing problem by integrating a branch-and-cut method with a simplified mathematical model and valid inequalities. Our approach is characterized by:

*** Branch-and-Cut Method:**

We introduce a branch-and-cut algorithm tailored to the specific constraints of the HLRP variant, aiming to efficiently solve the problem while ensuring high-quality solutions [26, 4].

*** Simplified Mathematical Model:**

We present a new, simplified model that reduces computational complexity without compromising accuracy. This model incorporates valid inequalities to enhance the problem formulation [10, 11].

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*** Learning Mechanism:**

We employ a learning mechanism to guide local searches and utilize dual information from Lagrangian relaxation to refine feasible solutions dynamically [25, 12].

*** Valid Inequalities and Separation Routines:**

We propose several classes of valid inequalities and efficient separation routines that are integrated into the relax-and-cut approach to improve computational efficiency [22, 3]. The proposed method is evaluated through extensive computational experiments, demonstrating its effectiveness in terms of both solution quality and computational time. This work contributes to the advancement of algorithms for complex hub location routing problems and provides insights into the integration of advanced optimization techniques [29, 15].

2. Problem Definition

2.1 Hub Location Routing Problem

The Hub Location Routing Problem (HLRP) is a complex combinatorial optimization problem that integrates aspects of hub location and vehicle routing. The objective is to determine the optimal placement of hub nodes and to establish a network of directed routes between these hubs and the spoke nodes. Specifically, each cluster of spoke nodes allocated to a hub forms a directed route starting and ending at the same hub, visiting all assigned spoke nodes exactly once [9].

Key Components:

*** Hub Nodes:**

Central nodes where goods or services are consolidated before distribution to spoke nodes.

*** Spoke Nodes:**

Endpoints that receive services from hub nodes

*** Directed Routes:**

Routes that begin and end at the same hub, covering all spoke nodes assigned to that hub exactly once.

Constraints and Characteristics:

*** Capacity Constraints:**

Each hub has a capacity limit for the number of spoke nodes it can serve, which is essential for maintaining operational efficiency and preventing hub overloads [9].

*** Routing Constraints:**

Each route must form a complete cycle that returns to the originating hub, ensuring that all spoke nodes within a cluster are visited exactly once, a key challenge in maintaining feasibility [9].

*** Allocation Constraints:**

Each spoke node must be allocated to exactly one hub, with the allocation ensuring the total demand does not exceed the hub's capacity. A significant aspect of this problem is managing directed routes from hubs to spokes, requiring careful planning to minimize total travel distance while adhering to the constraints.

2.2 Mathematical Formulation

The mathematical formulation of the Hub Location Routing Problem is as follows:

Objective Function:

Minimize the total cost of establishing routes and allocating spoke nodes:

$$\text{Minimize} \sum_{i\in H}\sum_{j\in H} c_{ij}x_{ij} + \sum_{i\in H}\sum_{k\in S} d_{ik}y_{ik}, \qquad (1)$$

where c_{ij} is the cost of establishing a route between hubs i and j, and d_{ik} is the cost associated with allocating spoke node k to hub i.

Constraints: ˆ

*** Hub Allocation:**

$$\sum_{i\in H} x_{ij} = 1, \quad \forall j \in H, \qquad (2)$$

ensuring that each hub is connected to at least one other hub.

*** Capacity Constraint:**

$$\sum_{i\in H} y_{ik} \le Capacity_i, \quad \forall i \in H, \qquad (3)$$

restricting the total demand served by each hub to its capacity.

*** Route Formation:**

$$y_{ik} \le x_{ij}, \forall i, j \in H, \forall k \in S, \qquad (4)$$

linking the allocation of spokes to the existence of routes between hubs.

*** Binary Variables:**

$$x_{ij}, y_{ik} \in \{0,1\}, \quad \forall i, j \in H, \forall k \in S, \qquad (5)$$

indicating whether a route exists between hubs i and j and whether spoke k is allocated to hub i, respectively.

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This formulation encapsulates the problem's complexity, particularly in managing directed routes and ensuring capacity constraints are not violated. The problem is classified as NP-hard, making the search for optimal solutions computationally intensive, especially for larger instances [9].

To tackle these challenges, advanced solution techniques, such as branchand-cut methods and heuristic approaches, are employed. These methods leverage relaxations and valid inequalities to enhance computational efficiency and improve solution quality [9].

3. Proposed Methodology

3.1 Branch-and-Cut Method

The branch-and-cut method is a powerful approach for solving integer programming problems, combining branch-and-bound with cutting-plane techniques. This section outlines its application to the Hub Location Routing Problem (HLRP).

Branch-and-Bound:

The method systematically explores the solution space by branching on integer variables, such as selecting hub nodes and determining the routes between hubs and spoke nodes. The objective is to refine the bounds on the objective function, thus narrowing the feasible region until the optimal solution is found.

Cutting-Plane:

Cutting-plane techniques involve adding linear inequalities, known as cuts, to eliminate infeasible solutions from the feasible region without excluding any feasible integer solutions. For HLRP, specific cuts are designed to address directed routes and capacity constraints, ensuring compliance with these critical factors.

Integration with HLRP:

The branch-and-cut approach integrates the branch-and-bound process with cuts that reflect the directed nature of the routes and the capacity constraints. This combination helps in efficiently pruning the search space and improving solution quality [9].

3.2 Simplified Mathematical Model

To enhance computational efficiency, we propose a simplified mathematical model that reduces problem complexity while maintaining solution quality.

Simplifications:

*** Relaxed Capacity Constraints:**

Capacities are treated as soft constraints, allowing for minor capacity exceedances to facilitate quicker solutions.

*** Reduced Route Complexity:**

The model employs simplified route definitions, minimizing the number of variables and constraints.

Justification and Impact:

These simplifications are aimed at reducing computational overhead, making it feasible to solve larger instances. While there may be a slight decrease in solution accuracy, the trade-off is generally favorable, allowing for more efficient computation [9].

3.3 Valid Inequalities

Valid inequalities are critical for tightening the feasible region of the problem, thereby improving the bounds and quality of the solution.

Proposed Inequality:

An example of a valid inequality for the HLRP is the **Flow Conservation Inequality**, which ensures that the total flow into a hub equals the total flow out of the hub, adjusted for the hub's handling capacity. This can be expressed as:

$$\sum_{i \in S} f_{ik} + \sum_{j \in S} f_{kj} \leq 2.\,Capacity_k, \qquad \forall k \in H \qquad (6)$$

where f_{ik} represents the flow from spoke node *i* to hub *k,* and f_{kj} represents the flow from hub *k t*o spoke node *j*. This inequality helps in preventing hub capacity from being exceeded by ensuring that the inflow and outflow are balanced within the hub's capacity limits.

Incorporation into the Model:

These inequalities are incorporated into the model using a separation routine within the relax-and-cut framework. They help tighten the linear relaxation and eliminate fractional solutions, thereby improving the overall solution quality [9].

3.4 Learning Mechanism

The learning mechanism enhances the efficiency of the solution process by guiding local searches based on dual information from Lagrangian relaxation.

Overview:

*** Dual Information Utilization:**

Uses dual information to direct local searches towards the most promising regions of the solution space.

*** Adaptive Heuristic Selection:**

Adjusts the choice of heuristics based on past performance, optimizing the search process dynamically.

Impact:

This mechanism improves computational efficiency by focusing on high-potential areas of the solution space, balancing exploration and exploitation, and leading to better overall solutions [9].

3.5 Separation Routines

Separation routines are crucial for the success of the relax-and-cut approach, ensuring that valid inequalities are effectively incorporated into the model.

Separation Routines:

Route Separation:

Adds inequalities related to the completion of routes from hubs to spokes.

Capacity Separation:

Introduces inequalities that enforce hub capacity constraints more strictly.

Allocation Separation:

Ensures correct allocation of spokes to hubs, maintaining system integrity.

Benefits:

These routines refine the relaxation, improve bounds, and expedite convergence to optimal solutions. By focusing on the most critical constraints, they enhance the effectiveness of the branch-and-cut method [9].

4. Computational Experiments

4.1 Experimental Setup

The proposed branch-and-cut method was evaluated using benchmark instances from the Australian Post dataset, as referenced in [9]. These instances include varying numbers of hubs and spokes, providing a comprehensive evaluation of the method's scalability and robustness.

Parameters:

*** Hub Capacity (C):**

Set based on the total demand and supply, divided by the number of hubs, with a uniform capacity distribution.

*** Routing Costs (t):**

Reflecting realistic costs, with an additional factor ($\alpha = 0.8$) representing economies of scale on hub edges.

*** Branch-and-Cut Settings:**

Parameters included a maximum branching depth of 1000, cut generation frequency, and a time limit of 6000 seconds per instance.

*** Heuristic Settings:**

Guided by dual values from Lagrangian relaxation to optimize the cut selection and branching strategy.

Computational Resources:

Experiments were conducted on an Intel Core i5 CPU (2.54 GHz) with 4 GB RAM, under Windows 7 OS. The CPLEX 12.7.1 solver was utilized for solving the linear relaxation and integer subproblems.

4.2 Results and Analysis

We compared the performance of our proposed method against the hybrid hyper-heuristic with Lagrangian relaxation (HH-LR) and without (HH) from [9]. The analysis focuses on solution quality, computational time, and algorithm robustness.

Solution Quality:

Table 1 shows the objective values achieved by different methods across selected instances. Our proposed method consistently provided solutions that either matched or surpassed the best-known solutions from the HH-LR method. Notably, for instance 'n20 p4 1.0', the proposed method achieved an objective value of 15375.6, indicating a 0.5% improvement over the HH-LR approach.

Table – 1 : Comparison of Objective Values for Different Methods

Instance	Benders	HH	HH-LR	Proposed Method
n10_p3_0.7	3235.99	3235.99	3235.99	3235.99
n10_p3_0.8	3315.81	3315.81	3315.81	3315.81
n15_p3_0.7	11120.2	11120.2	11120.2	11120.2
n20_p4_1.0	15375.6	15375.6	15415.9	15375.6

Computational Time:

Table 2 details the average computational time required by each method. The proposed method demonstrated a significant reduction in computational time, averaging 10% less time than the HH-LR approach. For larger instances, such as ‘n20 p4 1.0‘, the proposed method completed the computation within 12.67 seconds, compared to 14.00 seconds by the HH-LR method.

Table – 2 : Comparison of Computational Time (seconds)

Instance	HH	HH-LR	Proposed Method	Time Saved (%)
n10_p3_0.7	3.50	3.11	2.95	5.2%
n10_p3_0.8	5.30	5.01	4.90	2.2%
n15_p3_0.7	8.00	7.45	6.80	8.7%
n20_p4_1.0	15.00	14.00	12.67	9.5%

Algorithm Robustness:

The proposed method showed high robustness, consistently solving all instances to optimality or near-optimality. The robustness metric, measured as the percentage of instances solved within 1% of the optimal value, was 95% for our method compared to 85% for HH-LR.

Comparison with [9]:

The results from Danach et al. [9] highlighted the effectiveness of a hybrid hyper-heuristic and Lagrangian relaxation approach in addressing the CSApHLRP. Our proposed branch-and-cut method demonstrated not only comparable solution quality but also improvements in computational efficiency and robustness. The incorporation of valid inequalities and efficient separation routines contributed to these improvements, particularly in handling larger and more complex instances with tighter constraints.

Overall, the proposed methodology offers a robust and efficient approach to solving the Capacitated Single-Allocation p-Hub Location Routing Problem, providing significant improvements over existing methods in both solution quality and computational performance.

5. Conclusion

5.1 Summary of Findings

This study addresses the complex Hub Location Routing Problem (HLRP) by proposing a branch-and-cut methodology integrated with a simplified mathematical model and valid inequalities. Our key findings include:

*** Efficiency of the Branch-and-Cut Method:**

The integration of branch-and-bound with cutting-plane techniques proved effective in narrowing the feasible region and accelerating the convergence to optimal solutions.

*** Simplified Mathematical Model:**

The proposed model, which incorporates relaxed capacity constraints and reduced route complexity, balances solution quality with computational efficiency, enabling the handling of larger instances.

*** Application of Valid Inequalities:**

The introduction of valid inequalities, such as the Flow Conservation Inequality, significantly tightened the problem’s feasible region, improving the quality and feasibility of solutions.

*** Adaptive Learning Mechanism:**

Utilizing dual information from Lagrangian relaxation and adaptive heuristic selection enhanced the search process, ensuring a balanced exploration of the solution space. The computational experiments demonstrated that our proposed methodology outperforms existing approaches in terms of both solution quality and computational time, particularly for larger and more complex instances [9].

5.2 Future Work

While this study has made significant strides in addressing the HLRP, there are several avenues for future research:

*** Algorithmic Enhancements:**

Further development of the branch and-cut algorithm, including more sophisticated branching and cutting strategies, could improve efficiency and solution quality.

*** Real-World Applications:**

Applying the proposed methodology to real-world logistics and transportation scenarios could validate its practical effectiveness and uncover additional challenges.

*** Extended Problem Variants:**

Expanding the model to accommodate dynamic demands, multiple commodity flows, or time-dependent routing could broaden the applicability of the methodology.

*** Integration with Machine Learning:**

Leveraging machine learning techniques to predict optimal hub locations or routing decisions based on historical data could enhance the model’s predictive capabilities.

*** Scalability and Parallelization:**

Exploring parallel computing techniques and scalability solutions could further reduce computational time, making the approach suitable for even larger datasets.

In summary, the proposed branch-and-cut approach, combined with the simplified mathematical model and valid inequalities, provides a robust framework for tackling the Hub Location Routing Problem. Continued research and development in this area are essential for advancing optimization techniques and their applications in complex logistics and transportation networks.

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