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The Review of Introduction to Fourier Series and Fourier Transform

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Abstract— This paper is simply a review of Introduction to Fourier series and Fourier transform. It deals with what a Fourier series means and what it represents. The paper also includes a brief overview of Fourier Transform. The application of Fourier series and Fourier transform has also been shown. Fourier series are used in applied mathematics, and especially in the fields of physics and engineering.

Keywords- *fourier series; periodic functions; fourier transform*

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I. INTRODUCTION

Fourier series are used in the analysis of periodic functions. These periodic functions can be analyzed into their constituent components (fundamental and harmonics) by a process called Fourier analysis. Many of the phenomena studied in engineering and science are periodic in nature e.g. the current and voltage in an alternating current circuit. The Fourier series is concerned with periodic waves. The periodic wave may be rectangular, triangular, saw tooth or any other periodic form. We can call this periodic wave as signal wave can be represented as a sum of series of sinusoidal waves of different frequencies and amplitudes. Examples of the Fourier series for different waveforms are given in figure I.

II. DEFINITION OF FOURIER SERIES

The Fourier series is a specific type of infinite expansion of functions in terms of sines and cosines. The series is defined in trigonometric form as follows:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nt + b_n \sin nt) \qquad (1)$$

Where a_0 ,a_n and b_n are all constants can be represented as a specific trigonometric series. The function f (t) is a periodic function of period 2π.

III. FOURIER SERIES SPECIFIC

A. *Evaluation of Coefficients*

The coefficients a_0, a , b_n of the Fourier Series can be found by using the following formulae,

$$a_0 = \frac{1}{\pi}\int_{c}^{c+2\pi} f(t)\,dt$$

$$a_n = \frac{1}{\pi}\int_{c}^{c+2\pi} f(t).\cos nt \ dt$$

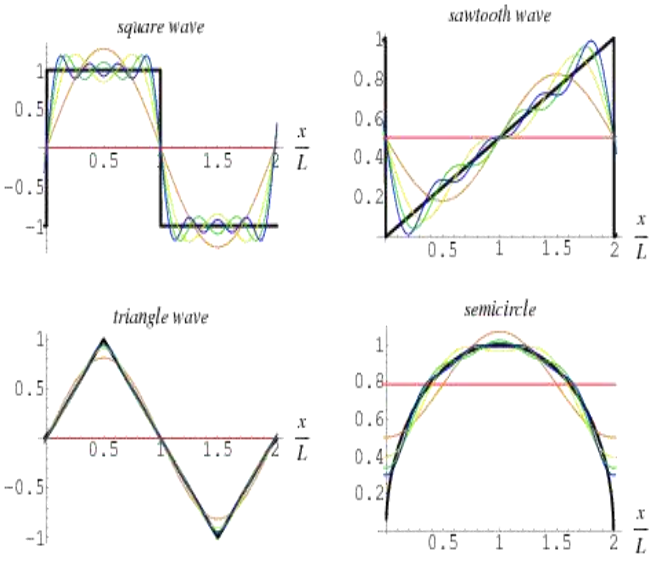

FIG. 1: Fourier series Examples

$$b_n = \frac{1}{\pi}\int_{c}^{c+2\pi} f(t).\sin nt \ dt$$

Thus, a Fourier series represents the sum of a series of trigonometric expressions used in the analysis of periodic functions.

B. *Dirichlet Conditions*

1) f (t) has a finite number of discontinuities within the period 2π
2) f (t) has a finite average value in the period 2π
3) f (t) has a finite number of positive and negative maxima and minima.

If a function is discontinuous, it can still be expressed as Fourier series. For example, if t = p is the point of finite discontinuity, at this point, the series gives the value as,

$$f(p) = \frac{1}{2}[\lim_{x\to p^+} f(t) + \lim_{x\to p^-} f(t)]$$

When these conditions are satisfied the function *f* (*t*) exists. Then these conditions are known as the Dirichlet conditions of the Fourier series.

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C. *Generalised Form*

The above mentioned series (1) is for specific functions that are periodic within the interval (0, 2π).The generalised kind of series may be expressed as,

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\frac{n\pi t}{L} + b_n \sin\frac{n\pi t}{L}\right) \qquad (2)$$

Thus, the above mentioned case is one with L=2π. We will define a Fourier series for functions in the intervals $(0,2L)$, $(0,2\pi)$, $(-L, L)$, $(-\pi, \pi)$.

D. *Complex form of Fourier series*

The Fourier series is defined in its complex form (or imaginary exponential form) as follows:

$$f(t) = \sum_{n=-\infty}^{\infty} A_n\, e^{int} \qquad (3)$$

Where the A_n's are given by the expression

$$A_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(t)\; e^{int}\, dt \qquad (4)$$

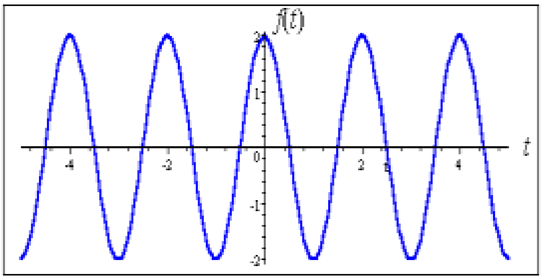
In writing the equality (3), we are supposing that the Dirichlet's conditions are satisfied and further that f (t)is continuous at t. If f (t) is discontinuous at t, the left side of (3) should be replaced by $\frac{[f(t+0)+f(t-0)]}{2}$.

E. *Even Functions*

A Function $y = f(t)$ is said to be even if $f(-t) = f(t)$ for all values of t. The graph of an even function is always symmetrical about the y-axis (i.e. it is a mirror image).

Example of an Even Function $f(t) = 2\cos \pi t$

FIG. 2: Even function Example

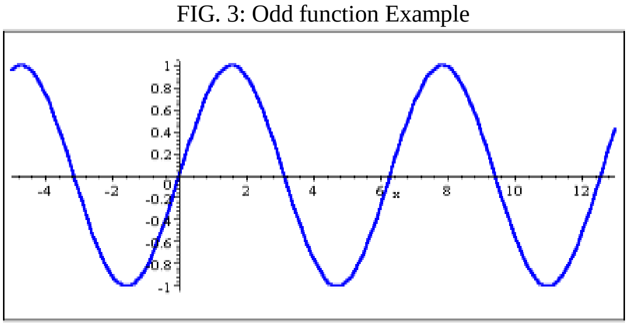


F. *Odd Functions*

A function $y = f(t)$ is said to be an odd if $f(-t) = -f(t)$ for all values of t. The graph of an odd function is always symmetrical about the origin.

Example of an Odd Function: $f(t) = \sin t$

FIG. 3: Odd function Example



In the Fourier series corresponding to an odd function, only sine terms can be present. In the Fourier series corresponding to an even function, only cosine terms can be present.

G. *Periodic Functions*

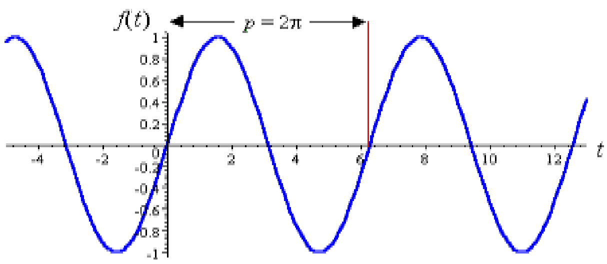
A function $f(t)$ is said to be periodic with period p if $f(t + p) = f(t)$ (for all values of t) and if $p > 0$.

The period of the function $f(t)$ is the interval between two successive repetitions.

Examples of Periodic Functions:

(a)For the function $f(t) = \sin t$, we have

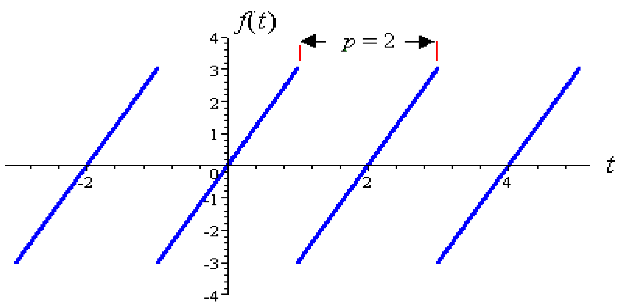
FIG. 4: Sine and Cosine curves



For $f(t) = \sin t$, we have $f(t) = f(t + 2\pi)$. The period is 2π.

(b)For the function $f(t) = 3t$ (for $-1 \le t < 1$)

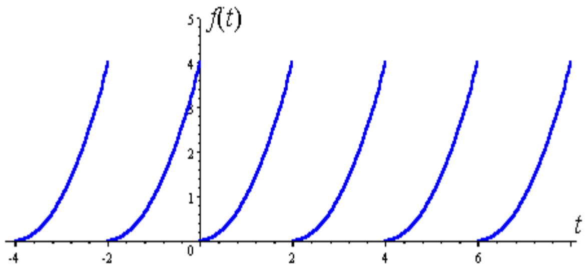
FIG. 5: Straight Lines



For $f(t) = f(t + 2)$, this indicates it is periodic with period 2.

(c)For the functionf (t) = t^2 $(0 \le t < 2)$,*we have*

FIG. 6: Parabolic Waves



For $f(t) = f(t + 2)$, this indicates it is periodic with period 2.

(d)For the function

$$f(t) = \begin{cases} -3 & \text{for } -1 \le t < 1 \\ 3 & \text{for } 1 \le t < 3 \end{cases}$$, we have

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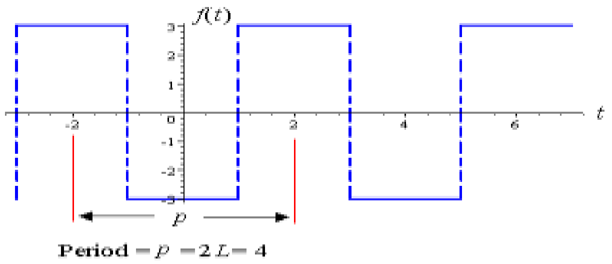
FIG. 7: Square wave



A. *Half Range Fourier Sine or Cosine series*

A half range Fourier sine or cosine series is a series in which only sine terms or only cosine terms are present, respectively. When a half range series corresponding to a given function is desired, the function is generally defined in the interval (0, L) which is half of the interval (-L, L), thus accounting for the name half range and then the function is specified as odd or even, so that it is clearly defined in the other half of the interval namely, (-L, 0). In such a case, we have

For half range sine series,

$$a_n = 0,$$

$$b_n = \frac{2}{L}\int_0^{2L} f(t).\sin\frac{n\pi t}{L}\,dt$$

For half range cosine series, we have

$$b_n = 0,$$

$$a_n = \frac{2}{L}\int_0^{2L} f(t).\cos\frac{n\pi t}{L}\,dt a_n = 0,$$

IV. Fourier Transform

The Fourier transform is applicable to waveforms which are basically a function of time, space or some other variable. The Fourier transform is a mathematical function decomposes a waveform into a sinusoid and hence provides another way to represent a waveform. The Fourier transform is also called a generalization of the Fourier series. This term can applied to both the frequency domain representation and the mathematical function used. The Fourier transform helps to extend the Fourier series to nonperiodic functions, which allows viewing any function as a sum of simple sinusoids.The Fourier series is a method of expressing a periodic function as a sum of sinusoids. The Fourier Transform is an extension of this idea to non-periodic functions as well.

The Fourier transform of a function f(x) is given by:

$$F(s) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{isx}\,dx \tag{5}$$

Which is usually known as the forward transform, and

$$f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} F(s)e^{isx}\,ds \tag{6}$$

Which is the inverse Fourier transform of F(s).

V. Some Application of Fourier Series and Fourier Transform

Fourier analysis is a fundamental tool utilized in all areas of science and engineering.Fourier transforms are used in a huge range of pure and applied science, including information processing, electronics and communications.

- Signal Processing: It may be the most effective application of Fourier analysis.
- Approximation Theory: We use Fourier series to write down a function as a trigonometric polynomial.
- Control Theory: The Fourier series of functions in the differential equation often provides some prediction regarding the behavior of the solution of differential equation. They are helpful to find out the dynamics of the solution.
- Partial Differential equation: We use it to solve higher order partial differential equations by the strategy of separation of variables.
- Fourier analysis finds applications in, Digital processing, Image processing, Digital filtering, Conduction of heat, Wave propagation.

Conclusion

This paper consists of a brief overview of Fourier analysis. A complete description of Fourier series was mentioned together with the evaluation of its coefficients and certain properties and identities. Also, it included the description of the use of Fourier Transform and its immense potential in converting a time domain function into its frequency domain equivalent.

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