The Review of Introduction to Laplace Transform & Its Applications

Rajashri A. Naphade
Applied science & Humanities Department
Mauli Group of Institution College of Engineering & Technology, Shegaon
Dist. -Buldhana (MH), India
E-mail: rajashri.naphade@gmail.com

Ujjvala Y. Gawarguru
Applied science & Humanities Department
Mauli Group of Institution College of Engineering & Technology,Shegaon
Dist. -Buldhana(MH), India
E-mail:ujjvalagawarguru136@gmail.com

Mitali K. Tibdewal
Applied science & Humanities Department
Mauli Group of Institution College of Engineering & Technology,Shegaon
Dist. -Buldhana(MH), India
E-mail:mit.tibdewal24@gmail.com

Rahul M. Jethwani
Applied science & Humanities Department
Mauli Group of Institution College of Engineering & Technology, Shegaon
Dist. - Buldhana (MH), India
E-mail: rahul.jethwani@gmail.com

Abstract— thispaper is simply a review of Introduction to Laplace transform. It deals a Laplace Transform means and how it represents. The paper also includes a brief overview of Laplace Transform and the application of Laplace Transform has been shown. Laplace transform are used in applied mathematics, and especially in the fields of physics and engineering.

Keywords- *laplace transform , properties of Lapalce transform, application of laplace transform.*

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I. INTRODUCTION

The knowledge of Laplace transforms has in recent years become an essential part of mathematical background required of engineers and scientist. This is because the transform methods provide an easy and effective means for the solution of many problems arising in engineering.Laplace transform has advantage for system analysis where initial conditions can be easily included to give system response. Some other applications: transient and steady state analysis of electrical circuits, analysis of impact and mechanical vibrations, and analysis of structural engineering problems such as beams columns etc. The Laplace transform is similar to the Fourier transform. While the Fourier transform of a function is a complex function of a real variable (frequency), the Laplace transform of a function is a complex function of a complex variable. Laplace transforms are usually restricted to functions of t with t > 0. The Laplace transform is connected to the Fourier transform, but where the Fourier transform expresses a function orsignal as a series of modes of vibration (frequencies), the Laplace transform resolves a function into its moments. The Laplace transform is used for solving differential and integral equationsLike the Fourier transform. In physicsand engineering it is used for analysis of linear time-invariant systems as electrical circuits, harmonicoscillators, optical devices, and mechanical systems. The Laplace transform as atransformation from the *time-domain*, in which inputs and outputs are functions of time, to the *frequency-domain*,where the similar inputs and outputs are functions of complex angular frequency, in radians per unit time. TheLaplace transform directly giving the solution of differential equations with given boundary values without the necessity of first finding the general solution and then evaluation from it the arbitrary constants. Moreover, the ready tables of Laplace transforms reduce the problems of solving differential equations to algebraic manipulation[3]

II. HISTORY OF LAPLACE TRANSFORM

The Laplace transform is an integral transform named after its discoverer mathematician and astronomer Pierre-Simon Laplace. It takes a function of a real variable *t* (often time) t a function of a complex variable *s* (frequency).Pierre-Simon Laplace is used a similar transform (now ztransform) in his work on probability theory. The recent widespread use of the transform came about soon after World War II although it had been used in the 19th century by Abel, Lerch, Heaviside and Bromwich. The older history of similar transforms is as follows. From 1744, Leonhard Euler investigated integrals of the form

$$z=\int X(x)e^{ax}dx \quad z(x;a)=\int_0^x e^{at}X(t)dt .$$

Assolutions of differential equations but get trouble very far.Joseph Louis Lagrange was an applaud of Euler and, in his work on integrating probability density functions, investigated expressions of the form

$$\int X(x)e^{-ax}a^x dx$$

Finally,in 1785, Laplace began to using a transformation to solve equations of finite differences, which eventually lead to the current transform

$$S=Ay_s+B\Delta y_s+C\Delta^2 y_s+.......,$$

$$y_s = \int e^{-sx}\varphi(x)dx$$

The Laplace transform is a linear operator that switched a function $f(t)$ to $F(S)$.

DEFINITION OF LAPLACE TRANSFORM

Suppose that f is a real- or complex-valued function of the (time) variable t>0 and s is a real or complex parameter. The Laplace transformation's symbol £ acts on functions $f = f(t)$ and generates a function,

We define the Laplace transform of $f(t)$ as

$$\mathrm{F(s)} = £[f(t)] = \int_0^{\infty} e^{-st} f(t)dt$$

Whenever the limitexists (as a finite number). When it does, the integral issaidtoconverge.Ifthelimit not existsthenintegral is said to diverge and there is no Laplace transform defined for f. The parameter s belongs to some domain on the real line or in the complex plane. In a mathematical and technical sense, the domain of s is quite important. However, in a practical sense, when differential equations are solved, the domain of s is routinely ignored. When s is complex, we will always use the notation $s = x + iy$.

Also, Laplacetransform EquationCan be written as

$£^{-1}[\mathrm{F(s)}] = f(t)$ called as Inverse Laplacetransform.[1]

EXISTENCE CONDITION

Laplace transform does not exist for all functions. If it exists, it is uniquely determined. The following conditions are to be satisfied:

Let $\int_0^{\infty} e^{-st} f(t)dt$ exists form $s > a$, if

1.$f(t)$Ispiecewise continuouson every finite interval

2.$f(t)$ Satisfies the following equality:

$$|f(t)| \leq b.e^{at} \qquad for\ all\ t \geq 0$$

and for someconstants a and b.

Then $£[f(t)]$exists. The function which satisfies the condition 2is known as exponential order.[2]

LAPLACE TRANSFORM OF ELEMENTRY FUNCTION:

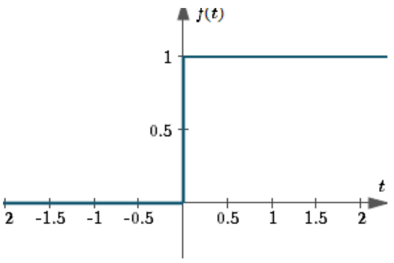
$f(t)$	$£[f(t)]$
1	$\frac{1}{s}$
t^n	$\frac{\Gamma n+1}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sin at$	$\frac{1}{s^2+a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
$\sinh at$	$\frac{1}{s^2-a^2}$

PROPERTIES OF LAPLACE TRANSFORMS

Linearity	$£[af(t)+bg(t)] = aF(s)+bG(s)$
Scaling in time	$£[f(at)] = \frac{1}{a}F(\frac{s}{a})$
First shifting Property	$£[e^{-at}f(t)] = F(s+a)$
Multiplication by t^n	$£[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}F(s)$
Integration Property:	$£[\int_0^t f(u)du] = \frac{F(s)}{s}$
Differentiation Property:	$£[\frac{d}{dt}f(t)] = sF(s) - f(0)$
Frequency integration	$£[\frac{f(t)}{t}] = \int_s^{\infty} F(s)ds$
Convolution	$£[f.g(t)] = \int_0^{\infty} f(t-u)g(u)du$

LAPLACE TRANSFORM OF THE HEAVISIDE STEP FUNCTION

The Heaviside step function is a piecewise continuous function defined by $u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$



$$£[u(t)] = \frac{1}{s}$$

$$£[u(t-a)] = \frac{e^{-as}}{s}$$

$$£[u(t-a)f(t-a)] = e^{-as}F(s)$$

INITIAL VALUE THEOREM:

$$\lim_{t \to 0} f(t) = \lim_{s \to \infty} sF(s)$$

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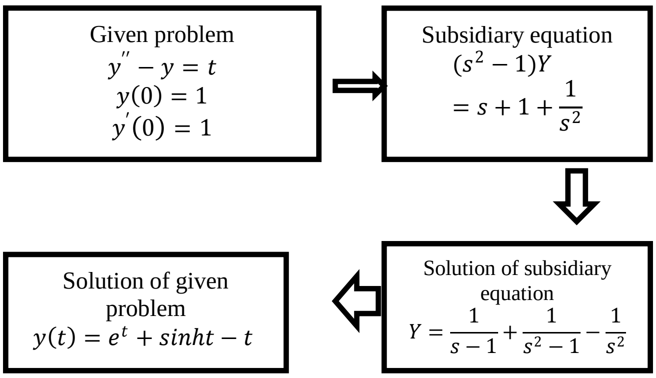
FINAL VALUE THEOREM

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$$

APPLICATION OF LAPLACE TRANSFORM

1.The amazing thing about using Laplace transforms is that we can convert the whole ODE initial value problem (I.V.P) into Laplace transformed functions of s, simplify the algebra, findtransformed solution F(s) the undo the transform to get back to the required solution f as a function of t

Example:Initial value problem: the basic Laplace steps
Solve $y'' - y = t \quad y(0) = 1,\ y'(0) = 1$
$t - spaces - space$

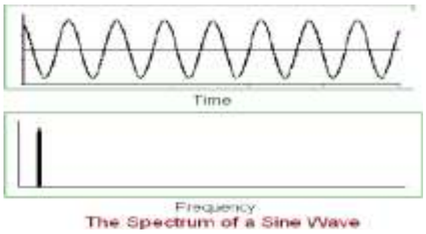


2.Laplace Transform systems have the extremely important property that if the input to the system is sinusoidal, then the output will also be sinusoidal at the same frequency but in general with different magnitude and phase. These magnitude and phase differences as a function of frequency are called as the frequency response of the system.

3. Using the Laplace transform, it is possible to convert a system's time-domain representation into a frequency-domain output/input representation, known as the transfer function. In so doing, it also transforms the governing differential equation into analgebraic equation which is often easier to analyse.

4. Frequency-domain methods are most often used foranalysing LTI single-input/single output (SISO) system

5.In digital signal processing
i) A simple Laplace transform is conducted while sending signals over two-way communication medium (FM/AM stereos, 2 way radio sets, cellular phones.)
ii) When information is sent over medium such as cellular phones, they are first converted into time-varying wave and then it is super imposed on the medium. In this way, the information propagates. Now, at the receiving end, to decipher the information being sent, medium wave's time functions are converted to frequency functions. This is a simple example of Laplace transform in real life.



iii) Laplace transformation is crucial for the study of control systems; hence they are used for the analysis of HVAC (Heating, Ventilation and Air Conditioning) control systems, which are used in all modern building and constructions.

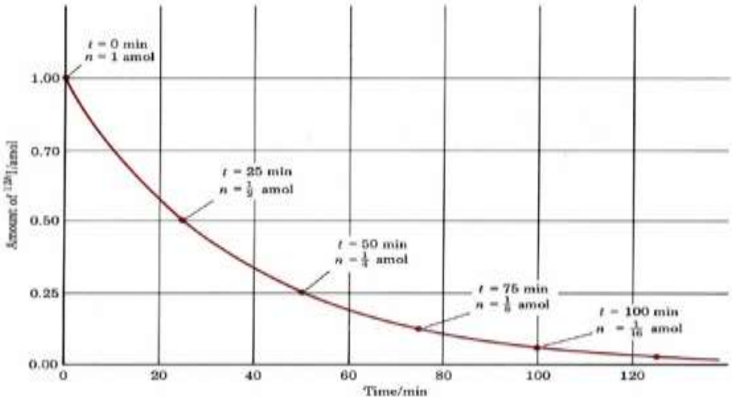
6.Mathematical Modelling of the Road Bumps Using Laplace Transform

i) Traffic engineering is the application of Laplace Transform to the quantification of speed control in the modelling of road bumps with hollow rectangular shape. In many countries the recent practice used for lowering the vehicle speed is to raise road bumps above the road surface. If a hollow bump is used it may be and offers other advantages over road bumps raised above the road surfaces. The method models the vehicle as the classical one-degree-of-freedom system whose base follows the road profile, approximated by Laplace Transform.

7.In nuclear physics

i) In order to get the true form of radioactive decay, a Laplace transform is used.

ii) It makes studying analytic part of Nuclear Physics possible.



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8. Application in Medical field

Laplace transforms can be used in areas such as medical field for blood-velocity/time wave form over cardiac cycle from common femoral artery.

9.Otherapplications

i) Semiconductor mobility
ii) Call completion in wireless networks
iii) Vehicle vibrations on compressed rails
iv) Behavior of magnetic and electric fields above the atmosphere [3][1]

III. CONCLUSION

This paper consists of a brief overview of Laplace transform. A complete description of Laplace transform was mentioned together with the properties and some standard transforms.
Also, it included the description of the use of Laplace transform and its immense potential in converting a time domain function in to its frequency domain function equivalent.

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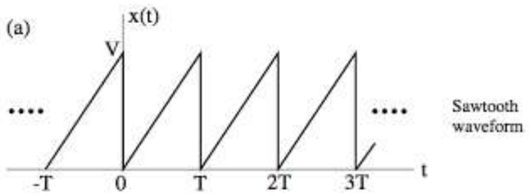

Figure 1. f a TWO-COLUMN figure caption: (a) this is the format for referencing parts of a figure.