

The Review of Introduction & Application of Complex Number in Engineering

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Abstract—this paper is simply a review of Introduction to complex number. The application of complex number has also been shown. Complex numbers are used in applied mathematics, and especially in the fields of physics and engineering.

Keywords-complex number, application of complex number in engineering field.

INTRODUCTION

A complex number is a number comprising a real part and an imaginary part. It can be written in the form $a+ib$, where a and b are real numbers, and i is the standard imaginary unit with the property $i^2 = -1$. The complex numbers contain the ordinary real numbers, but extend them by adding in extra numbers and correspondingly expanding the understanding of addition and multiplication.

HISTORY OF COMPLEX NUMBERS:

Complex numbers were first conceived and denoted by the Italian mathematician Gerolamo Cardano, who called them "fictitious", during his attempts to find solutions to cubic equations. This ultimately led to the fundamental theorem of algebra, which shows that with complex numbers, a solution exists to every polynomial equation of degree one or higher. Complex numbers thus form an algebraically closed field, where any polynomial equation has a root. The rules for addition, subtraction and multiplication of complex numbers were developed by the Italian mathematician Rafael Bombelli. A more abstract formalism for the complex numbers was further developed by the Irish mathematician William Rowan Hamilton.

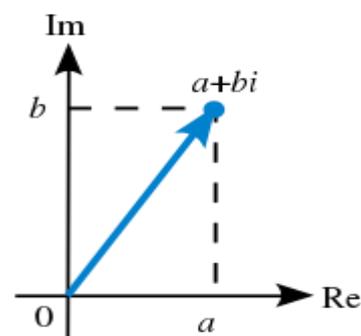
COMPLEX NUMBER INTERPRETATION:

A number in the form of $a+ib$ where x and y are real numbers and $i = \sqrt{-1}$ is called a complex number.

Let $z = a+ib$

X is called real part of z and is denoted by $R(z)$

Y is called imaginary part of z and is denoted by $I(z)$



CONJUGATE OF A COMPLEX NUMBER:

A pair of complex numbers $x+iy$ and $x-iy$ are said to be conjugate of each other.

PROPERTIES OF COMPLEX NUMBERS ARE:

1. If $x_1+iy_1 = x_2+iy_2$ then $x_1-iy_1 = x_2-iy_2$
2. Two complex numbers x_1+iy_1 and x_2+iy_2 are said to be equal, if $R(x_1+iy_1) = R(x_2+iy_2)$
 $I(x_1+iy_1) = I(x_2+iy_2)$
3. Sum of the two complex numbers is
 $(x_1+iy_1) + (x_2+iy_2) = (x_1+x_2) + i(y_1+y_2)$
4. Difference of two complex numbers is
 $(x_1+iy_1) - (x_2+iy_2) = (x_1-x_2) + i(y_1-y_2)$
5. Product of two complex numbers is

$$(x_1 + iy_1)(x_2 + iy_2) = x_1x_2 - y_1y_2 + i(y_1x_2 + y_2x_1)$$

6. Division of two complex numbers is

$$(x_1 + iy_1) / (x_2 + iy_2) = (x_1x_2 + y_1y_2) + i(y_1x_2 - y_2x_1) / (x_2^2 + y_2^2)$$

7. Every complex number can be expressed in terms of

$$r(\cos\theta + i\sin\theta)$$

$$R(x + iy) = r\cos\theta$$

$$I(x + iy) = r\sin\theta$$

$$r = \sqrt{x^2 + y^2} \text{ and } \theta = \tan^{-1}(y/x)$$

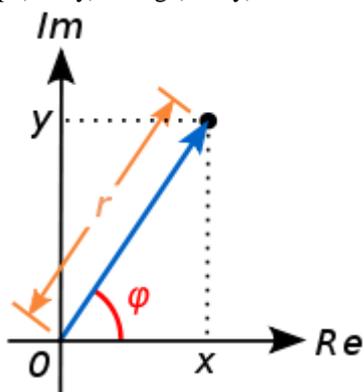
REPRESENTATION OF COMPLEX NUMBERS IN PLANE

The set of complex numbers is two-dimensional, and a coordinate plane is required to illustrate them graphically. This is in contrast to the real numbers, which are one-dimensional, and can be illustrated by a simple number line. The rectangular complex number plane is constructed by arranging the real numbers along the horizontal axis, and the imaginary numbers along the vertical axis. Each point in this plane can be assigned to a unique complex number, and each complex number can be assigned to a unique point in the plane.

Modulus and Argument of a complex number:

The number $r = \sqrt{x^2 + y^2}$ is called modulus of $x + iy$ and is written by $\text{mod}(x + iy)$ or $|x + iy|$

$\Phi = \tan^{-1}(y/x)$ is called amplitude or argument of $x + iy$ and is written by $\text{amp}(x + iy)$ or $\text{arg}(x + iy)$



Application of imaginary numbers:

For most human tasks, real numbers (or even rational numbers) offer an adequate description of data. Fractions such as $2/3$ and $1/8$ are meaningless to a person counting stones, but essential to a person comparing the sizes of different collections of stones. Negative numbers such as -3 and -5 are meaningless when measuring the mass of an object, but essential when keeping track of monetary debits and credits. Similarly, imaginary numbers have essential concrete applications in a variety of sciences and related areas such as signal processing, control theory, electromagnetism, quantum mechanics, cartography, vibration analysis, and many others.

The size of the quanta typically varies from system to system. Under certain experimental conditions, microscopic objects like atoms or electrons exhibit wave-like behavior, such as interference. Under other conditions, the same species of objects exhibit particle-like behavior such as scattering. This phenomenon is known as wave-particle duality.

In Computer Science.

1. Arithmetic and logic in computer system

Arithmetic and Logic in Computer Systems provides a useful guide to a fundamental subject of computer science and engineering. Algorithms for performing operations like addition, subtraction, multiplication, and division in digital computer systems are presented, with the goal of explaining the concepts behind the algorithms, rather than addressing any direct applications. Alternative methods are examined, and explanations are supplied of the fundamental materials and reasoning behind theories and examples.

2. Rectifying Software engineering in 21st century

This technological manual explores how software engineering principles can be used in tandem with software development tools to produce economical and reliable software that is faster and more accurate. Tools and techniques provided include the Unified Process for GIS application development, service-based approaches to business and information technology alignment, and an integrated model of application and software security. Current methods and future possibilities for software design are covered.

3. Fractals

A fractal is a never-ending pattern. Fractals are infinitely complex patterns that are self-similar across different scales. They are created by repeating a simple process over and over in an ongoing feedback loop. Driven by recursion, fractals are images of dynamic systems – the pictures of Chaos. Geometrically, they exist in between our familiar dimensions. Fractal patterns are extremely familiar, since nature is full of fractals. Fractals are produced using an iteration process. This is where we start with a number and then feed it into a formula. We get a result and feed this result back into the formula, getting another result. And so on and so on. Fractals start with a complex number. Each complex number produced gives a value for each pixel on the screen. The higher the number of iterations, the better the quality of the image.

A US company called Fractal Antenna Systems, Inc. makes antenna arrays that use fractal shapes to get superior performance characteristics, because they can be packed so close together

In Electrical Engineering:

Complex numbers are used in a variety of fields, one of them is electrical engineering. As soon as AC circuits are analyzed, it turns out that complex numbers are the natural way to do this. Complex equations and their graphs are used to visualize electrical and fluid flow in the real world.

The voltage produced by a battery is characterized by one real number (called potential), such as $+12$ volts or -12 volts. But the "AC" voltage in a home requires two parameters. One is a potential, such as 120 volts, and the other is an angle (called phase). The voltage is said to have two dimensions. A 2-dimensional quantity can be represented mathematically as either a vector or as a complex number (known in the engineering context as phasor).

In the vector representation, the rectangular coordinates are typically referred to simply as X and Y . But in the complex number representation, these same components are referred to as real and imaginary. When the complex number is purely

imaginary, such as a real part of 0 and an imaginary part of 120, it means the voltage has a potential of 120 volts and a phase of 90° , which is physically very real.

Application in electronics engineering

In signal processing, complex analysis and Fourier analysis go hand in hand in the analysis of signals, and this by itself has tons of applications, e.g., in communication systems (your broadband, wifi, satellite communication, image/video/audio compression, signal filtering/repair/reconstruction etc). Basically, if you search for applications of signal processing, those are the applications that are indirectly the applications of complex analysis. Although most engineers will tell you that complex analysis is not necessary to "understand" signal processing, But it is found that it is very helpful in going beyond simply blindly applying the Fourier transform etc., to a stage where one truly understands what is going on.

The second application area is control theory, specifically in the analysis of stability of systems and controller design. Here the word "system" is used generically, and does not necessarily refer to an electrical system. For e.g., one could use it to (try to) understand stock market movement, chemical processes/reactions. Also, control theory is used heavily in robotics, and by extension, so is complex analysis.

Information that expresses a single dimension, such as linear distance, is called a scalar quantity in mathematics. Scalar numbers are the kind of numbers students use most often. In relation to science, the voltage produced by a battery, the resistance of a piece of wire, and current through a wire are scalar quantities. When electrical engineers analyzed alternating current circuits, they found that quantities of voltage, current and resistance were not the familiar one-dimensional scalar quantities that are used when measuring DC circuits. These quantities which now alternate in direction and amplitude possess other dimensions that must be taken into account. In order to analyze AC circuits, it became necessary to represent multi-dimensional quantities. In order to accomplish this task, scalar numbers were abandoned and complex numbers were used to express the two dimensions of frequency and phase shift at one time.

In mathematics, i is used to represent imaginary numbers. In the study of electricity and electronics, j is used to represent imaginary numbers so that there is no confusion with i , which in electronics represents current. It is also customary for scientists to write the complex number in the form $a+jb$.

In electrical engineering, the Fourier transform is used to analyze varying voltages and currents. The treatment of resistors, capacitors, and inductors can then be unified by introducing imaginary, frequency-dependent resistances for the latter two and combining all three in a single complex number called the impedance. This approach is called phasor calculus. This use is also extended into digital signal processing and digital image processing, which utilize digital versions of Fourier analysis (and wavelet analysis) to transmit, compress, restore, and otherwise process digital audio signals, still images, and video signals.

Complex numbers are used a great deal in electronics. The main reason for this is they make the whole topic of analyzing and understanding alternating signals much easier.

Applications in Fluid Dynamics

In fluid dynamics, complex functions are used to describe potential flow in two dimensions. Fractals. Certain fractals are plotted in the complex plane, e.g. the Mandelbrot set. Fluid Dynamics and its sub disciplines aerodynamics, hydrodynamics, and hydraulics have a wide range of applications. For example, they are used in calculating forces and moments on aircraft, the mass flow of petroleum through pipelines, and prediction of weather patterns. The concept of a fluid is surprisingly general. For example, some of the basic mathematical concepts in traffic engineering are derived from considering traffic as a continuous fluids.

Relativity

In special and general relativity, some formulas for the metric on spacetime become simpler if one takes the time variable to be imaginary. (This is no longer standard in classical relativity, but is used in an essential way in quantum field theory.) Complex numbers are essential to spinors, which are a generalization of the tensors used in relativity.

Applied mathematics

In differential equations, it is common to first find all complex roots r of the characteristic equation of a linear differential equation and then attempt to solve the system in terms of base functions of the form $f(t) = e^{rt}$.

In Electromagnetism:

Instead of taking electrical and magnetic part as a two different real numbers, we can represent it as in one complex number

In Civil and Mechanical Engineering:

The concept of complex geometry and Argand plane is very much useful in constructing buildings and cars. This concept is used in 2-D designing of buildings and cars. It is also very useful in cutting of tools. Another possibility to use complex numbers in simple mechanics might be to use them to represent rotations.

CONCLUSION

This paper consists of a brief overview of complex analysis. Complex numbers are used in a huge range of pure and applied science, including signal processing, control theory, fluid dynamics, special and general relativity, Computer Science, electrical engineering, civil and mechanical Engineering.

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