The Review of Introduction to Linear Algebra & Its application in Engineering field

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Abstract—This paper is simply a review of the introduction to linear algebra. It deals with matrix and vectors which are useful in all branches of engineering. The paper also includes a brief over view of matrix representation and vector representation. The4 application of linear algebra has also been shown below. Linear algebra also used in applied mathematics and in fields of physical and engineering.

Keywords*-Linear algebra, matrix,vector*

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I. INTRODUCTION

Matrices are mathematical objects which appear in connection with linear equations, linear transformations, and also in conjunction with bilinear and quadratic forms, which were important in geometry, analysis, number theory, and physics. Matrices as rectangular arrangement of numbers in columns and rows appeared around 200 BC in Chinese mathematics, but there they were merely abbreviations for systems of linear equations. Matrices are useful on operations as addition, subtraction,and especially multiplication; more important, when it is shown what use they are to be put to. Matrices were introduced obliquely as abbreviations of linear transformations by Gauss in his Disquisitiones mentioned earlier, but now in a significant way. Gauss undertook a deep study of the arithmetic theory of binary quadratic forms,

$f(x, y) = ax2 + bxy + cy2$. He called two forms $f(x, y)$ & $F(X, Y) = AX2 + BXY + CY2$ "equivalent" if they yield the same set of integers, as x, y,X, and Y range over all the integers (a, b, c and A, B, C are integers). According to Gauss that there exists a linear transformation T of the coordinates (x, y) to (X, Y) with determinant = 1 that transforms f (x, y) into F (X, Y). The linear transformations ere represented as rectangular arrays of numbers—matrices, although Gauss did not use matrix terminology. He also defined indirectly the product of matrices (for the 2 × 2 and 3 × 3 cases only); he thought about the composition of the corresponding linear transformations also show up in projective geometry, founded in the seventeenth century and described analytically in the early nineteenth. In attempts to extend Gauss'work on quadratic forms, Eisenstein and Hermite tried to construct a general arithmetic theory of forms f (x1, x2,...,xn) of any degree in any number of variables. In this connection they too introduced linear transformations, denoted them by single letters—an important idea—and studied them as independent entities, defining their addition and multiplication (composition).He noted that they "comport themselves as single entities" and recognized their usefulness in simplifying systems of linear equations and composition of linear transformations. He defined the sum and product of matrices for suitable pairs of rectangular matrices, and the product of a matrix by a scalar, a real or complex number. He also introduced the identity matrix and the inverse of a square matrix, and showed how the latter can be used in solving n × n linear systems under certain conditions. In his 1858 paper "A memoir on the theory of matrices" Cayley proved the important Cayley–Hamilton theorem that a square matrix satisfies its characteristic polynomial. The proof consisted of computations with 2 × 2 matrices, and the observation that he had verified the result for 3 × 3 matrices. He noted that the result applies more widely. formal proof of the theorem in the general case of a matrix of any degree." Hamilton proved the theorem independently (for n = 4, but without using the matrix notation) in his work on quaternions. Cayley used matrices in

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another paper to solve a significant problem, it is also called Cayley–Hermite problem, which asks for the determination of all linear transformations leaving a quadratic form in n variables invariant. Cayley advanced considerably the important idea of viewing matrices as constituting a symbolic algebra. He use a single letter to represent a matrix was a significant step in the evolution of matrix algebra. But his papers of the 1850s were little noticed outside England until the 1880s. During the years nearly 1820s–1870s ,deep work on matrices was done on the continent, by Cauchy, Jacobi, Jordan, Weierstrass, and others. They created what may be called the spectral theory of matrices: their classification into types such as symmetric, orthogonal, and unitary; results on the nature of the eigenvalues of the different types of matrices; and, above all, the theory of canonical forms for matrices—the determination, among all matrices of a certain type, of those that are canonical in some sense. An important example is the Jordan canonical form, introduced by Weierstrass (and independently by Jordan), who showed that two matrices are similar if and only if they have the same Jordan canonical form. Spectral theory originated in the eighteenth century in the study.

II. DEFINITION

Linear algebra is the branch of mathematics that deals with matrices, vectors and vector spaces,and systems of linear equations.A vector is any quantity that is represented by both an amount and a direction, like velocity orforce, like an arrow of a certain length pointing a certain way. A vector can be represented by anordered pair, triple, or more, like (3, 2, -1). This would stand for an arrow pointing from (0, 0, 0) to
(3, 2, -1). We can find the length and direction using the ordered triple (3, 2, -1).
A matrix (the plural is matrices) is like a table or array in which numbers are arranged in rows andcolumns. We can think of the rows of the matrix as vectors, and we can also think of the columns ofa matrix as vectors.A system of linear equations is probably familiar to you, at last to some degree. It consists ofmore than 1 equation in which none of the variables are raised to any power other than 1, andsolution of the system will have to make all of the equations true. Here's a simple example:

$3x + 2y = 5$

$2x + 5y = 3$

If we organize the equations so that the variables are in the same order, we can look at theCoefficients (the numbers that are multiplied by the variables) and the constants (the numbers that are not multiplied by anything) as a matrix, like this:

3	2	5
2	5	3

III. OBJECTIVES

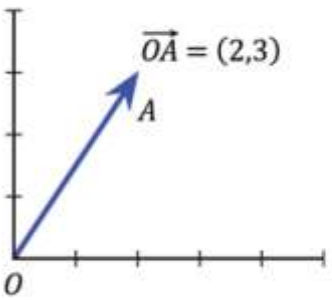
Most of the mathematicians define *Linear Algebra* **a**s the branch of mathematics which deals withthe study of vectors, vector spaces and linear equations. Newly defined mathematics also depend on linear transformations and systems of vector matrix. Analytic geometry operates thetechniques learned at the time of study of linear algebra, for analytically computing complexgeometrical shapes. Along science, engineering and mathematics, linear algebra has extensive applications in the natural and the social sciences. Linear algebra now hasbeen extended to consider *n*-dimensional space. Even if it is very difficult to imagine
vectors in *n*-space, such n-dimensional vectors are extremely useful in representing data,whichcan easily summarize and manipulate data efficiently in this framework, when data are
*well-arranged*as a list of *n* componentsSince the students taking pre-calculus have very little bit knowledge about the subject ofmatrices, it has become very useful to give the subject matter in depth. In 1989, theNCTM recognized the need for greater emphasis on linear algebra and stated that "matricesand their applications" should receive "increased attention" in high school curriculum. Itshould be recognized that linear algebra is as important as calculus to scientists andengineers.In *linear algebra* we had been studies sets of linear equations and their transformation properties. It ispossible to consider the analysisof rotations in space, selected curve fitting techniques,differential equation solutions, as well as many other problems in science and engineering field usingtechniques of linear algebra. Two tools are extensively used in linear algebra which are :*The Matrix* and *The Determinant.* Solution to a vector matrix model equation is observedas one of the most important of 'central problems' of linear algebra.Study of vectors in 2-dimensional as well as 3- dimensional space is veryimportant for design engineers. A course was specially designed to provide theengineering and engineering technology students with all the necessary mathematical toolsthat are important and necessary for a four-year of engineering program. The course included thecoverage of a wide variety of introductory topics such as complex numbers, partialfractions, determinants, Taylor and Maclaurin Theorems, etc. Vectors, vector spaces, scalarproducts and vector products are considered very important. About weeklongdiscussion is incorporated in the course engineering to confirm that the students obtain a very strongfoundation that pertains to vector operations. Matrices and Matrix operations both are coveredextensively. Eigenvalues and Eigenvectors, Diagonalization of Matrices are consideredessential base for subsequent engineering courses and as such several homeworkexercises are necessarily assigned in this area.Eigenvectors are extremely important while producing engineering models whether it be asatellite or a jet engine.. Eigenvalues can be used to explainseveral parts of musicalperformances. It is very well known that frequencies are vital in music performance.Tuning of an instrument means that their frequencies are matched. Because of the frequencysome musicis pleasing to the human ear. A study of eigenvalues explains why certain sounds are pleasing to the humanear while certain others sounds are not also Musicians may not study eigenvalues. When number of people sing in harmony, thefrequency matters a lot.Engineers utilize Ordinary differential equations as solutions toseparable, exact and homogeneous equations are discussed in great detail. Bernoulli andRiccati Equations are stressed in particular because of their importance in subjects suchas Fluid Mechanics, Thermodynamics, Heat Transfer, etc. Discussion of second andhigher order differential equations follows, however third and higher order equations arenot discussed at

length.Laplace Transform has been recognized as *the mathematical signature of an engineer*.
This technique, in addition to the *z - transform* is heavily utilized while designing analogas well as discrete / digital control system components. Extra emphasis is placed while discussing topics as Transfer Function, Initial and Final Value Theorems. While discussing transfer functions,theimportance of *natural frequency* is stressed. Study of *Tacoma Narrows Bridge Disaster* is considered extremely important and some studentgroups have created a web site pertaining to this disastrous event. Also, the 1831 breakdown of a bridge in Manchester in England is discussed that was the result of conflicts betweenfrequencies. Soldiers marching in step had caused the bridge to *oscillate* at its own *natural frequency*. This resulted in the building up of *large oscillations* , which ultimatelyresulted in the collapse of the bridge. Soldiers are therefore required to break cadence while crossing a bridge.The Fourier series is another important mathematical tool engineers often used. About three weeks are spent discussing the Fourier Integral, the Fourier Transform alsoThe FourierSine and Cosine Transforms. In addition several assignments reinforce the mathematicalknowledge essential for engineers. A short list is given below.
Linear Difference Equations and the Fibonacci Sequence.
Erratic Behavior of a Sequence Generated by a Difference Equation.The Row-Reduced Echelon Form of a Matrix and its Rank. Theorem on theRank of Matrix Product ABC. Consistency of Augmented Coefficient Matrices, Solution by Back Substitution andCramer's Rule.A One-Way Traffic Flow Problem. Forces in Bridge Struts Verifying and using the Cayley-Hamilton Theorem. Diagonalization of a Matrix.
Orthogonal Vectors Computed by the Gram-Schmidt Method. Reduction of aQuadratic Form to Standard Form.The Hubble Space Telescope and Quadratic Forms. Dynamical Systems andLogging Operations to supply a sawmill.First Order Linear Differential Equations : Direction Fields and Integral Curves.Direction Fields and Isoclines.The Limit Cycle of the van der Pol Equation. Period of Oscillation of NonlinearPendulum.Erratic Behavior of a Sequence Generated by a Difference Equation .Laplace Transforms : Solution to a Third Order Initial Value Problem. Solving an Equation with the Heaviside Step Function in the NonhomogeneousTerm. Solving an Equation with the Dirac Delta Function in theNonhomogeneous Term. Eigenvalues / Eigenvectors : Chebyshev Approximation, Legendre Approximationand Bessel Function Approximation.Fourier Series : Plotting Partial Sums, Examining the Gibbs Phenomenon

IV. APPICATION

One of the things we can do with a vector is to find its length if we know its coordinates. For example, suppose we have a vector represented by (2,3):

1	2	1	8
2	1	1	7
3	1	2	11



We can find the length of this vector by using the Pythagorean Theorem: If *c* is the length of the hypoteneuse (the long side) of a right triangle and *a* and *b* are the lengths of the other sides, then*c*^2 = *a*^2 + *b*^2. In fact, the length of 'A' is the square root of 2^2 + 3^2, which is the square root of We can extend this into 3 dimensions or more and the length of a vector in any number of dimensions, we square each coordinate, add up all those squares, and take the square root of thatsum. Let's see it written out:

$\vec{A} = \sqrt{x^2 + y^2 + } \dots \dots \dots$

The length of a vector can represent the speed of a moving object or the size of a force, or manyother things other than simple distance.Now, let's look at another system of equations, along with its augmented matrix, a matrix of allthe numbers in the system. Suppose I need to create a mixture for dog food that has 8 grams ofcarbohydrate, 7 grams of protein, and 11 grams of fat. I have 3 raw ingredients: *x* has 1 gram of carbohydrate, 2 grams of protein, and 3 grams of fat per ounce; *y* has 2 grams of carbohydrate, 1gram of protein, and 1 gram of fat per ounce; *z* has 1 gram of carbohydrate, 1 gram of protein, and2 grams of fat per ounce. We can write this as a system of equations like this:

$x + 2y + z = 8$
$2x + y + z = 7$
$3x + y + 2z = 11$

We can solve the system of equations by performing elementary row operations on the matrix.
There are 3 kinds of elementary row operations:
- Multiply a row through by a number that is not 0.
-Switch the positions of 2 rows.
- Add a multiple of 1 row to another

V. CONCLUSION

The paper consist of brief over view of Linear Algebra. A complete description of linear algebra was mentioned with the help of Matrix and Vector representation. Also it included representation of matrix ,vector and Linear differential equation and its application in fields of physical engineering.

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