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Applications of Laplace Transformation in Engineering Field

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Abstract*:* Laplace transform has important applications in mathematics, physics, engineering and probability theory. Laplace transform makes it easier to solve the increasing complexity of engineering problems for example its applications which make differential equations easy to solve. Here we will discuss about the applications of Laplace in physics. The Laplace transform has the useful property that many relationships and operations over the originals f(t)correspond to simpler relationships and operations over the images F(s).

Keywords*: - Laplace transform; differential equations; Inverse Laplace Transform*

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I. INTRODUCTION

In mathematics, theLaplace transformis a widely used integral transform. It has many important applications in mathematics, physics, engineeringand probability theory. The Laplace transform is related to the Fourier transform, but whereas the Fourier transformer solves a function or signal into its modes of vibration, the Laplace transform resolves a function into. Like the Fourier transform, the Laplace transform is used for solving differential and integral equations. In physics and engineering, it is used for analysis of linear time-invariant systems such as electrical circuits, harmonic oscillators, optical devices, and mechanical systems. In this analysis, the Laplace transform is often interpreted as a transformation from thetime-domain,in which inputs and outputs are functions of time, to thefrequency-domain,where the same inputs and outputs are functions of complex angular frequency, in radians per unit time. Given a simple mathematical or functional description of an input or output to a system, the Laplace transform provides an alternative functional often simplifies the process of analyzing the behavior of the system, or in synthesizing a new system based on a set of specifications. Denoted L{f(t)}, it is a Linear operator on a function $f(t)$ (*original*) with a real argument$t$ ($t \geq 0$) that transforms it to a function$F(s)$ (*image*) with a complex argument s. This transformation is essentiallybijectivefor the majority of practical uses; the respective pairs of $f(t)$ and $F(s)$ arematched in tables. The Laplace transform has the useful property that many relationships andoperations over the originals $f(t)$ correspond to simpler relationships and operations over theimages$F(s)$.

II. HISTORY

The Laplace transform is named in honor of mathematician and astronomer Pierre-SimonLaplace, who used the transform in his work on probability theory. From 1744, Leonhard Euler investigated integrals of the form

$$z = \int X(x).e^{ax}dx$$

$$z = \int X(x).e^{-\lambda}dx$$

as solutions of differential equations but did not pursue the matter very far .Joseph LouisLagrange was an admirer of Euler and, in his work on integrating probability density functions,investigated expressions of the form

$$\int X(x).e^{-ax}.a^{x}dx$$

Which some modern historians have interpreted within modern Laplace transform theory.Thesetypes of integrals seem first to have attracted Laplace's attention in 1782 where he was followingin the spirit of Euler in using the integrals themselves as solutions of equations.However, in1785, Laplace took the critical step forward when, rather than just looking for a solution in theform of an integral, he started to apply the transforms in the sense that was later to become popular. He used an integral of the form

$$\int x^{s}.\phi(s)dx$$

in to aMellin transform, to transform the whole of adifference equation,in order to look for solutions of the transformed equation. He then went on to apply the Laplace transform in thesame way and started to derive some of its properties, beginning to appreciate its potential power.Laplace also recognized thatJoseph Fourier's method of Fourier seriesfor solving thediffusionequationcould only apply to a limited region of space as the solutions were periodic. In 1809,Laplace applied his transform to find solutions that diffused indefinitely in space.

III. FORMAL DEFINITION

The Laplace transform of a function $f(t)$, defined for all real numbers $t \geq 0$, is the function $F(s)$,defined by:

$$L\{f(t)\} = F(s) = \int_{0}^{\infty} f(t).e^{-st}\,dt$$

The parameter $s$is a complex number:

$$s = \sigma - i\omega$$

Withreal numbers σ and ω.

The meaning of the integral depends on types of functions of interest. A necessary condition for existence of the integral is that f must be locally integrable on [0,∞). For locally integrable functions that decay at infinity or are of exponential type, the integral can be understood as a(proper)Lebesgue integral[1]. However, for many applications it is necessary to regard it as aconditionally convergent improper integral at ∞. Still more generally, the integral can beunderstood in a weak senseand this is dealt with below.One can define the Laplace transform of a finiteBorel measureμ by theLebesgue integral.

$$(L\mu)(s) = \int_{[0,\infty)} e^{-st}\, d\mu(t)$$

An important special case is where μ is aprobability measureor, even more specifically, theDirac delta function. Inoperational calculus,the Laplace transform of a measure is often treatedas though the measure came from adistribution function
f. In that case, to avoid potentialconfusion, one often writes

$$(Lf)(s) = \int_0^{\infty} e^{-st}.f(t)dt$$

IV. PROOF OF THE LAPLACE TRANSFORM OF A FUNCTION'S DERIVATIVE

It is often convenient to us the differentiate-on property of the Laplace transform to find the transform of a function's derivative. This can be derived from the basic expression for a Laplace transform as follows [3]:

$$L\{f(t)\} = \int_0^{\infty} f(t).e^{-st}\, dt$$

$$= \left\{_0^{\infty} \frac{f(t)e^{-st}.}{-s} - \int_0^{\infty} \frac{e^{-st}}{-s}.f'(t)dt\right.$$

$$= \frac{-f(0)}{s} + \frac{1}{s}.L\{f'(t)\}$$

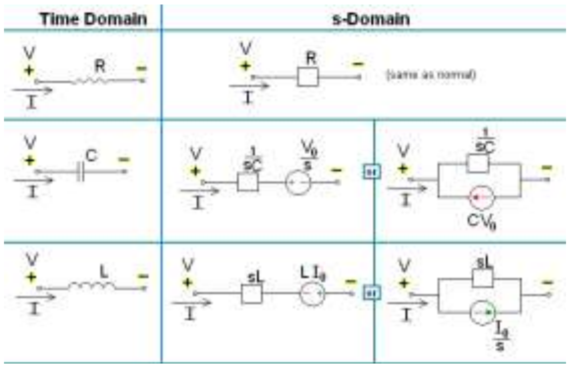
It yields, $L\{\frac{df}{dt}\} = s.L\{f(t)\} - f(0)$

Yielding and in the bilateral case, we have

$$L\left\{\frac{df}{dt}\right\} = s.\int_0^{\infty} f(t).e^{-st}\, dt - s.L\{f(t)\}$$

V. S-DOMAIN EQUIVALENT CIRCUITS AND IMPEDANCES

The Laplace transform is often used in circuit analysis, and simple conversions to the s-Domain of circuit elements can be made. Circuit elements can be transformed into impedances, very similar to phase or impedances.



Note that the resistor is exactly the same in the time domain and the s-Domain. The sources are put in if there are initial conditions on the circuit elements. For example, if a capacitor has an initial voltage across it, or if the inductor has an initial current through it, the sources inserted in the s-Domain account for that. The Laplace transform is used frequently in engineering and physics; the output of a linear timeinvariant system can be calculated by convolving its unit impulse response with the input signal. Performing this calculation in Laplace space turns the convolution into a multiplication; the latter being easier to solve because of its algebraic form. For more information, see control theory[7].

The Laplace transform can also be used tosolve differential equationsand is used extensively inelectrical engineering. The Laplace transform reduces a linear differential equationto analgebraic equation, which can then be solved by the formal rules of algebra. The originaldifferential equation can then be solved by applying the inverse Laplace transform. The Englishelectrical engineer Oliver Heavisidefirst proposed a similar scheme, although without using theLaplace transform; and the resultingoperational calculusis credited as the Heaviside calculus [8, 9].

The following examples, derived from applications in physicsand engineering, will useSIunits of measure. SI is based on meters for distance,kilograms for mass,secondsfor time, and amperesfor electric current.

Example 1: Solving a differential equation

Laplace transform use innuclear physics

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Consider the following first-order, linear differential equation:

$$\frac{dN}{dt} = -\lambda N$$

This equation is the fundamental relationship describing radioactive decay,

$$N = N(t)$$

Whererepresents the number of undecayed atoms remaining in a sample of a radioactive isotope time$t$ (in seconds) and is thedecay constant.We can use the Laplace transform to solve this equation.Rearranging the equation to one side,

$$\frac{dN}{dt} + \lambda N = 0$$

We have next, we take the Laplace transform of both sides of the equation:

$$\left(S.\breve{N}(s) - N_0\right) + \lambda.\breve{N}(s) = 0$$

Where, $\breve{N}(s) = L\{N(t)\}$and

$$N_0 = L\{N(t)\}$$

Solving,$\breve{N}(s) = \frac{N_0}{s+\lambda}$

We findfinally, we take the inverse Laplace transform to find the general solution

$$N\{(t)\} = L^{-1}\{N(s)\} = L^{-1}\left\{\frac{N_0}{s+\lambda}\right\}$$

$$N\{(t)\} = N_0.e^{-\lambda t}$$

Which is indeed the correct form for radioactive decay.

Example 2: Deriving the complex impedance for a capacitor

Laplace transform use in of electrical circuit theory.

The constitutive relation governing the dynamic behavior of a capacitor is the following differential equation:

$$i = C(s.V(s) - V_0)$$

Where,*C* is the capacitance (infarads) of the capacitor,

i=*i*(*t*) is theelectric current(inamperes)through the capacitor as a function of time, and

v=*v*(*t*) is thevoltage(involts) across theterminals of the capacitor, also as a function of time.Taking the Laplace transform of this equation, we obtain

$$I(s) = C(s.V(s) - V_0)$$

Where, $I(s) = L\{i(t)\}$

$$V(s) = L\{V(t)\}$$

And$V_0 = V(t)\ at\ t = o$

Solving for *V* (*s*) we have,

$$V(s) = \frac{I(s)}{sC} + \frac{V_0}{s}$$

The definition of thecompleximpedance*Z* (inohms) is the ratio of the complex voltage

V divided by the complex current*I* while holding the initial state*V* at zero:

$$Z(s) = \frac{V(s)}{I(s)}\ \ at\ Vo = 0$$

Using this definition and the previous equation, we find:

$$Z(s) = \frac{1}{sC}$$

Which is the correct expression for the complex impedance of a capacitor.

CONCLUSIONS

The paper presented the application of Laplace transform in different areas of physics and electrical power engineering. Besides these, Laplace transform is a very effective mathematical tool to simplify very complex problems in the area of stability and control. Laplace transforms have become an integral part of modern science, being used in a vast number of different disciplines. Whether they are being used in electrical circuit analysis, signal processing, or even in modeling radioactive decay in nuclear physics, they have quickly gained popularity among the intellectual community that deals with these subjects on a day to day basis. With the ease of application of Laplace transforms in myriad of scientific applications, many research software's have made it.

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