Image Denoising Using Weighting Approach

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***Abstract*—**Although tremendous work has been done in the image denoising algorithms to tackle the issue of analyzing the correlation among overlapping estimated patches but still correlation among the estimated patches is area of concern in the image processing domain. The Patch based image denoising algorithms filters overlapping image patches and aggregate multiple estimates for the same pixel via weighting. In this paper, we examine the correlation among patches and then estimates them and propose new model to estimate the Mean Squared Error under various weights. This model examines the overlapping information of the patches, and then try to minimize the MSE. We propose a new weighting approach, which can be used into various denoising algorithms. The results show that the Peak Signal to Noise Ratio of algorithm like Expected Patch Log Likelihood (EPLL) can be improved by around 0.2 dB under a range of various noise levels.

Keywords- *Patch based, Mean Squared Error (MSE), Weighting, Bias variance model, Filtering, Weighting, Expected Patch Log Likelihood (EPLL), PSNR, etc*

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I. INTRODUCTION

Image denoising is the most basic image processing problems it aims to recover an image under random additive white Gaussian noise. Various image denoising algorithms are based on image patches. Their denoising methods can be interpreted as an iteration of called Filtering and Weighting process. In this process, local image patches are firstly restored through filtering, and then multiple estimates of the same pixel from overlapping patches are weighted to calculate the final estimate. For the filtering method, advanced patch based image models have been applied to generate the filters, e.g., the sparse coding model [2], the Gaussian Mixture Model [3], and the non-local similarity model [4]. The weighting methods are somewhat easy as compare to filtering methods, either using simple averaging or deriving the weight separately based on certain transform coefficients of the corresponding image patch itself [4], [5].

This type of weighting methods are optimal when the estimates for weighting are independent random variables. However, the estimates can be mostly correlated due to overlapping of the patches, which violates the assumption of independence. Therefore, we may further improve the denoising performance by examine the correlation among the estimates using the overlapping information. Based on the above idea, we describe the F&W process precisely, examine the MSE under various weights, and derive a bias variance model to estimate it accurately. We also show that optimizing the weight under the proposed model yields the minimum MSE with the help of the overlapping information.

We propose a new weighting approach to solve the optimization problem under the bias variance model via Quadratic Programming (QP). We also introduce the proposed weighting approach into the K-SVD algorithm and the EPLL algorithm.

II. LITERATURE REVIEW

In 1984, new method for removing various noises from images was proposed. This filtering scheme is based on replacing the central pixel value by mean value of all pixels inside a sliding window. The new concepts of thresholding which is shown to improve the performance of the generalized mean filter are introduced in this paper. This threshold is derived using a statistical theory. The performance of the proposed filter is compared with that very commonly used median filter by filtering noise from the corrupted real images. The hardware complexity of the two types of filters is compared indicating the advantages of the generalized mean filter [6].

By 1988, two algorithms using adaptive-length median filters are proposed for improving impulse noise removal performance for image processing. The algorithms achieved significantly better image quality than median filters when the images are corrupted by impulse noise. One of the algorithms, when realized in hardware, requires rather simple additional circuitry. Both algorithms can easily be integrated into efficient hardware realizations for median filters [7].

In 1992 Fresh introduced the wavelet and inverse wavelet transforms of self-similar random processes. It showed that, after suitable rescaling, the wavelet transform at a given position becomes a stationary random function of the logarithm of the scale argument in the transform [8]. Appropriate wavelets and their corresponding band-pass filters were selected for image processing. A multichannel optical processing system with two gratings was set up to obtain image representation and image reconstruction [9].

In 2000 for impulsive noise reduction of an image without the degradation of an original signal an adaptive centre weighted median filters was developed. The weight in this filter is decided by the weight controller based on counter propagation networks. This controller classifies an input vector into some cluster according to its feature and gives the weight corresponding to the cluster [10].

A new operator was introduced by Yuksel in year 2006 for removing noise from digital images. The proposed operator was a hybrid filter constructed by combining four centre weighted median filters (CWMF) with a simple adaptive neuro fuzzy inference system (ANFIS). The results showed that the proposed operator significantly outperforms the other operators and efficiently removes noise from digital images without distorting image details [10].

A two-phase median filter based iterative method for removing random-valued noise was proposed in 2010. Simulation results indicated that the proposed method performs better than many well-known methods while preserving its simplicity [11].

III. NOISE MODELS

Noise is present in image either in additive or multiplicative form

A. *Additive Noise Model*

Noise signal that is additive in nature gets added to the original signal to produce a corrupted noisy signal and follows the following model.

$$W(x, y) = s(x, y) + n(x, y) \tag{1}$$

B. *Multiplicative Noise Model*

In this model, noise signal gets multiplied to the original signal. The multiplicative noise model follows the following rule

$$W(x, y) = s(x, y) \times n(x, y) \tag{2}$$

Where, s(x, y) is the original image intensity and n(x, y) denotes the noise introduced to produce the corrupted signal w(x, y) at (x, y) pixel location.

IV. NOISE TYPES

Various types of noise have their own characteristics and are inherent in images in different ways

A. *Gaussian Noise*

Gaussian noise is evenly distributed over the signal. This means that each pixel in the noisy image is the sum of the true pixel value and a random Gaussian distributed noise value. As the name indicates, this type of noise has a Gaussian distribution, which has a bell shaped probability distribution function given by,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{3}$$

B. *Salt and Pepper Noise*

Salt and Pepper is an impulse type of noise and is also referred to as intensity spikes. It is generally caused due to errors in transmission. This is caused generally due to errors in data transmission. It has only two possible values, a and b. The probability of each is typically less than 0.1. The corrupted pixels are set alternatively to the minimum or to the maximum value, giving the image a “salt and pepper” like appearance. Unaffected pixels remain unchanged. For an 8bit image, the typical value for pepper noise is 0 and for salt noise 255. The salt and pepper noise is generally caused by malfunctioning of pixel elements in the camera sensors, faulty memory locations, or timing errors in the digitization process.

C. *Speckle Noise*

Speckle noise [4] [5] is multiplicative noise. This type of noise occurs in almost all coherent imaging systems such as laser, acoustics and SAR (Synthetic Aperture Radar) imagery. The source of this noise is attributed to random interference between the coherent returns. Fully developed speckle noise has the characteristic of multiplicative noise.

V. PROPOSED METHOD

In this section, we first formulate the degradation model of image denoising and describe the F&W process in an analytic way. Then we propose a bias variance model to characterize the correlation of the restored estimates under the F&W process. This new model can estimate the MSE under various weights by exploiting the overlapping information of the restored patches. Therefore, optimizing the weight under this model is nearly equivalent to minimizing the real MSE.

The degradation model of image denoising can be formulated

$$y = x + n$$

Where x denote the clean image, y is its noisy version, and n represents the additive white Gaussian noise with variance σ^2.

In most of the cases, noise can be modelled as Gaussian distribution, and such noise includes, 1) the noise of an image sensor; 2) the shot noise of a photon detector, which is a type of electronic noise that may be dominant. Examine the Gaussian noise level from a single image is a very difficult task we need to decide whether local image variations are due to color, texture and lighting variations of the image itself, or due to the external noise. In the image denoising literature, noise is often assumed to be zero mean additive white Gaussian noise (AWGN). An observed noisy image $A_{(i,j)}$ is expressed as:

$$A_{(i,j)} = A_0(i,j) + N_{(i,j)} \tag{4}$$

D. *Filtering With Local Regions*

Suppose there are m_i local regions (also known as patches) that share pixel. In the *k*-th region, x_i is estimated as

$$x_{i,k} = p_{i,k}^T y + c_{i,k}$$

Where $(p_{i,k}, c_{i,k})$ can be seen as a low-pass global filter. Various denoising algorithms compute $(p_{i,k}, c_{i,k})$ in their own way, but the values are just slightly different. Suppose pixel *i* is at the *l*-th place of the k-th local region, where is a selection matrix, then the patch is

$$\left(\sum + \sigma^2 I\right)^{-1} \left(\sum R_{(i,k)} y + \sigma^2 \mu\right) \tag{5}$$

Where ∑ and μ are the parameter of a Gaussian distribution. In this case, $p_{(i,k)}$ equals to the *l*-th column of

$$\left(\left(\sum + \sigma^2 I\right)^{-1} \sum R_{i,k}\right)^T \tag{6}$$

Due to the property of $R_{i,k}$, $p_{i,k}$ is a sparse vector with nonzero elements only in the *k*-th local region, and its *j*-th element reflects the closeness between x_i and x_j. As illustrated in Fig. 1, the *i*-th element of $p_{i,k}$ is always the largest, and if the local region in **x** contains two smooth areas like in Fig. 1(b), the *j*-th element of $p_{i,k}$ is close to 0 when pixel j is in the other area; $c_{i,k}$ is a bias term of the filter.

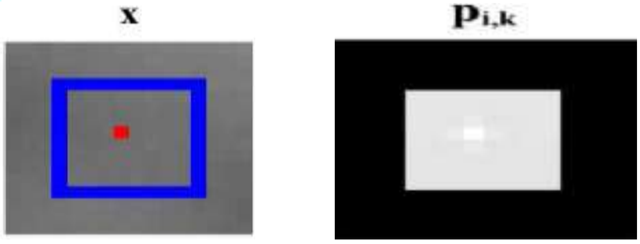

Fig.1
(Filtering with local regions)

E. Weighting to local regions

The estimates m_i are weighted to derive the final estimate of xi as

$$x_i = \sum_{k=1}^{m_i} w_{i,k}\, x_{i,k} \quad (7)$$

Where the weights $w_{i,k}$s are nonnegative and sums to one.

$$x_i = w_i^T (P_i^T y + c_i) \quad (8)$$

All the denoising algorithms in [1], [2], [3], [4], [5], [6] fit the F&W process quite well. As for the Non-local Means algorithm [11], though it can be seem as a weighting algorithm without filtering, the weights actually reflect the closeness among pixels, which is mainly what $p_{i,k}$'s do under the F&W process. Hence, NLM is more proper to be interpreted as a global filtering process with only one estimate for each pixel.

VI. CALCULATION OF MSE

Under the F&W process, we assume (P_i, C_i) is computed exactly using the original filtering method of a denoising algorithm. Therefore, x_i is formulated as a function of w_i, and *MSE* is formulated as

$$MSE\left(\hat{X}(w)\right) = \frac{1}{M}(\hat{x}_i(w_i) - x_i)^2 \quad (9)$$

$$MSE\left(\hat{X}(w)\right) = \frac{1}{M}(w_i^T(P_i^T x + c_i) - x_i + w_i^T P_i^T n)^2 \quad (10)$$

Where M is the number of pixels in x and w is denoted as the concatenation of all w_i's. Since $MSE\left(\hat{X}(w)\right)$ is a random variable depend on noise **n**, we propose a bias-variance model, which estimates it by its expectation under **n.** For mathematical derivation simplicity, we assume that (P_i, C_i)'s are independent. Hence, the expectation can be estimated as

$$\hat{E}\left[MSE\left(\hat{X}(w)\right)\right] = \frac{1}{M}\Sigma\left(Bias^2(\hat{x}_i(w_i)) + Var(\hat{x}_i(w_i))\right) \quad (11)$$

Where Bias$(\hat{x}_i(w_i)) = w_i^T(P_i^T x + c_i) - x_i$ is the bias of $\hat{x}_i(w_i)$ to x_i and Var $(\hat{x}_i(w_i)) = \sigma^2 P_i w_i^T P_i^T n$ is the variance.

In reality, (P_i, C_i)'s are derived from **y**, which makes them still correlated to n. To evaluate the appropriateness of using $\hat{E}\left[MSE\left(\hat{X}(w)\right)\right]$ to $MSE\left(\hat{X}(w)\right)$ approximate, we compute their ratio

$$\gamma(w) = \hat{E}\left[MSE\left(\hat{X}(w)\right)\right] / MSE\left(\hat{X}(w)\right) \quad (12)$$

under various w's. If $\gamma(w)$ is a constant under all w's, then we can conclude that optimizing is equivalent to minimizing the true value of $MSE\left(\hat{X}(w)\right)$. As shown in Table I, under each image and denoising algorithm combination, the values of under an averaging and a uniformly sampled are really close.

VI. SIMULATION RESULTS

Image / σ	$\sigma = 10$		Improvement
	EPLL (db)	WEPLL (db)	
Barbara	33.1583	33.3038	0.1455
Boat	32.5293	32.6480	0.1187
House	35.8943	36.1335	0.2392
Lena	33.5442	33.6748	0.1306
Man	32.0888	32.2169	0.1281
Monarch	32.4719	32.6530	0.1811
Peppers	33.6682	33.8454	0.1772

Table I

PSNR comparison of EPLL and WEPLL

Image / σ	$\sigma = 20$		Improvement
	EPLL (db)	WEPLL (db)	
Barbara	29.4195	29.5525	0.1330
Boat	28.6915	28.7298	0.0383
House	32.3517	32.5210	0.1693
Lena	30.1234	30.2553	0.1319
Man	28.4783	28.5660	0.0876
Monarch	28.3700	28.5649	0.1948
Peppers	29.6488	29.7852	0.1364

Table II

PSNR comparison of EPLL and WEPLL

The above result shows that the improvement is very significant in presence of random noise. Experimental results show that the PSNR gain of EPLL can be improved by about 0.15 dB under a range of noise levels. This work setup a novel model that formulates the selection of weights as an optimization problem.

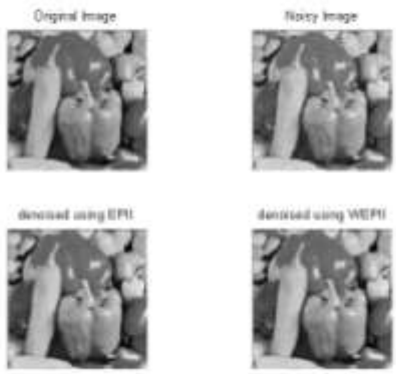

Figure 1. Peppers

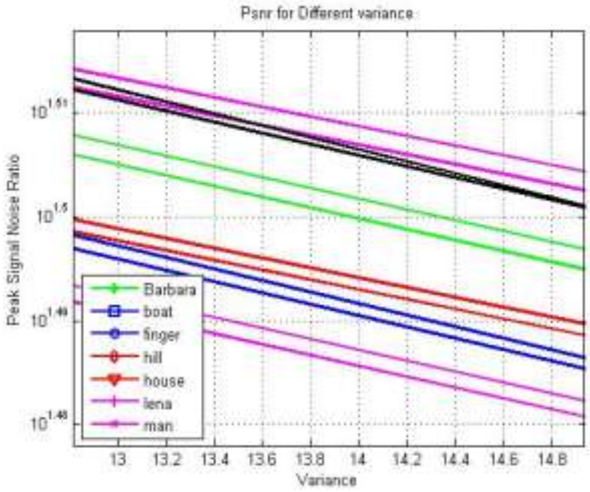

Figure 2. PSNR for different images

V. CONCLUSION

Result shows that the improvement is very significant in presence of random noise, we propose a new model to estimate the MSE accurately by analyzing the correlation among the estimates. Experimental results show that the PSNR gain of EPLL can be improved by about 0.2 dB under a range of noise levels. The 0.2 dB improvement is promising, since it is independent to which image model is used, especially when the gain from designing new image models becomes less and less. This work setup model that formulates the selection of weights as an optimization problem.

VI. REFERENCES

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