Nature of Singularities in (n+2)-Dimensional Gravitational Collapse of Vaidya Space-time in presence of monopole field.

[1]C. S. Khodre , [2]K. D.Patil, [3]S. D.Kohale and [3]P. B.Jikar
[1]Department of Mathematics,
S.D.College of Engineering, Selukate, Wardha (M.S.), India
[2]Department of Mathematics,
B.D.College of Engineering, Sevagram, Wardha (M.S.), India
[1]Department of Mathematics,
S.D.College of Engineering, Selukate, Wardha (M.S.), India
[*]Corresponding Author E-mail: chandrakantkhodre@gmail.com

Abstract:- We study here the nature of singularities on the gravitational collapse of higher-dimensional Vaidya space-time and we analyze the occurrence and nature of the naked singularities formed in the gravitational collapse of Vaidya solution in higher dimensional space-times in the presence of monopole field. Both naked singularities and black holes are shown to be developing as final outcome of the collapse. Earlier work is generalized to higher dimensional space-times to allow a study of the effect of number of dimensions on the possible final outcome of the collapse in terms of a black hole or a naked singularity. No restriction is adopted on the number of dimensions.

Keywords: *Cosmic censorship, naked singularity, gravitational collapse.*
PACS numbers: 04.20Dw, 04.20Cv, 04.70Bw

*****

I. Introduction

The question, which is clearly vital to understanding the final fate of massive collapsing clouds, can be answered only by means of a detailed study of singularities in gravitational collapse phenomena in gravitation theory [1]. The classical spacetime singularities should be smeared out by quantum gravity, and what would really result from such an endless collapse is an extreme strong gravity region, with extreme values of physical parameters such as densities and curvatures, confined to an extraordinarily tiny region of space. If the event horizons of gravity already start developing at an earlier phase during such a collapse, the collapsing star and the eventual fireball as described above gets hidden within the horizon, disappearing from the outside observers in the universe forever. Then we have the formation of a *black hole* in the universe as a result of the gravitational collapse in higher dimensions. On the other hand, if the formation of event horizon gets delayed sufficientlyduring the collapse, the result is the development of a *naked singularity*, or a *visible fire ball*, which can possibly send out massive radiations to faraway observers from near such strong gravity regions.

The investigation on the final fate of gravitational collapse of initially regular distribution of matter is one of the most active field of research in the contemporary general relativity. The physical phenomena in astrophysics and cosmology involve gravitational collapse in a fundamental way. The final fate of a massive star, when it collapses under its own gravity at the end of its life cycle, is one of the most important questions in gravitation theory and relativistic astrophysics today. A sufficiently massive star more than five times the size of sun would undergo a continuous gravitational collapse due to its self gravity, on exhausting its nuclear fuel, without achieving an equilibrium state such as a *neutron star or white dwarf*. The singularity theorems in general relativity then predict that the collapse gives rise to a *space-time singularity*, either hidden within an event horizon of gravity or visible to external universe [2]. The densities and space-time curvatures get arbitrarily high and diverge at these ultra strong gravity regions. Their visibility to outside observers is determined by the casual structure within the dynamically developing collapsing cloud, as governed by the Einstein field equations. When the internal dynamics of the collapse delays the horizon formation, these become visible, and may communicate physical effects to the external universe.

It is assumed that the 4-dimensional space-time of the universe we live in is obtained through a dimensional reduction from higher dimensional space-time. Inspired by work in the string theory and other field theories, there has been considerable interest in recent times to find solutions of the Einstein equation in dimensions greater than four. It is believed that underlying space-time in the large energy limit of the Planck energy may have higher dimensions than the usual four. Higher dimensional gravity theories have been considered as possible avenues to unify the basic forces of nature. 5D Kaluza-Klein [3] theory unifies gravity and electromagnetism and extensions of this have been investigated in [4]. The extra dimensions have been assumed to be small, typically of the order of the Planck length and so Kaluza-Klein Models are highly massive.

Nevertheless, the extra dimensions will not be directly observable in experiments. The success of string theories gave encouragement to search for indirect methods to detect the extra dimensions. Possible effects of the extra dimensions considered as bulk in the standard model have been suggested by Arkani-Hamid, Dimopoulos [5,6]. Higher dimensional space-time is now an active field of research in its attempts to unify gravity with all other forces of nature. It is particularly relevant in cosmology where it is shown that under certain situations, Einstein field equations dictate that as the usual 3D space expands the extra dimensions contract with time via the well- known process of dimensional reduction.

The results on gravitational collapse in higher dimensions are of interest in view of the current possibilities being explored for higher dimensional gravity. In this paper, we shall generalize our previous [7] studies to the case of monopole Vaidya solution in (n+2)-dimensional space-times. We show that the results for gravitational collapse, obtained in four-dimensional monopole space-time are also valid in (n+2)-dimensional monopole space-time. The objective is to fully investigate the situation in the background of higher dimensional space-time.

In section II and III, we give (n+2)-dimensional solution to monopole Vaidya space time. In section IV, we discuss the nature of singularity (visible or invisible), by analyzing the outgoing radial null geodesics emanating from the central singularity when star collapsing. We summarise the paper in V section by some concluding remarks.

II. Monopole Vaidya solution in (n+2)- dimensional space-times

Generalized Vaidya metric in (n+2)-dimensional space-time is given by [8, 9]

$$ds^2 = -Adu^2 + 2dudr + r^2 d\Omega^2 \qquad (1)$$

Where u is the advanced Eddington time coordinate and r is the radial coordinate with $0 < r < \infty$ and

$$A = \left(1 - \frac{m(u,r)}{r^{n-1}}\right) \qquad (2)$$

Where $m(u,r)$ gives the gravitational mass inside the sphere of radius r and

$$d\Omega^2 = d\theta_1^2 + sin^2\theta_1 d\theta_2^2 + sin^2\theta_1 sin^2\theta_2 d\theta_3^2 + \ldots + sin^2\theta_1 sin^2\theta_2 \ldots sin^2\theta_{n-1} d\theta_n^2 \qquad (3)$$

is a metric on n-sphere.

We find that corresponding energy-momentum tensor can be written as [9-11]

$$T_{\mu u} = T_{\mu u}^{(n)} + T_{\mu u}^{(m)}, \qquad (4)$$

and

$$T_{\mu u}^{(n)} = \sigma l_\mu l_u , \qquad (5)$$

also

$$T_{\mu u}^{(m)} = (\rho + P)(l_\mu \eta_u + l_u \eta_\mu) + P g_{\mu u} \qquad (6)$$

In the comoving coordinate $(u, r, \theta_1, \theta_2, \ldots\ldots, \theta_n)$, the two eigen vectors of energy-momentum tensor namely l_μ and η_u are linearly independent future pointing light-like vectors (null vectors) having components

$$l_\mu = \delta_\mu^0, \quad \eta_\mu = \frac{1}{2}\left(1 - \frac{m}{r^{n-1}}\right)\delta_\mu^0 - \delta_\mu^1 , \qquad (7)$$

$$l_\mu \eta^\lambda = -1 , \qquad l_\lambda l^\lambda = \eta_\lambda \eta^\lambda = 0 \qquad (8)$$

Where ρ and P are the energy density and thermodynamic pressure, σ is the energy density corresponding to Vaidya null radiation.

In particular, when $\rho = P = 0$, the solution reduces to higher dimensional Vaidya solution with $m = m(u)$ [12]

Therefore for the general case we consider the EMT of equation (6).

Strong and weak energy conditions for ρ , P and σ are given by

(a) Weak and strong energy condition:

$$\sigma > 0 , \quad \rho \geq 0 , \quad P \geq 0. \qquad (9)$$

(b) Dominant energy conditions:

$$\sigma > 0 , \quad P > 0 , \quad \rho > P \quad . \qquad (10)$$

The non-vanishing component of the Einstein field equations

$$G_{\mu u} = K T_{\mu u} , \qquad (11)$$

for the metric (1) with matter field having stress-energy tensor given by (4) are

$$\rho = \frac{n m'}{k(n-1)r^n} \qquad (12)$$

$$P = \frac{-m''}{k(n-1)r^{n-1}} \qquad (13)$$

and

$$\sigma = \frac{n \dot{m}}{k(n-1)r^n} \qquad (14)$$

Where dot and dash stand for differentiation with respect to u and r respectively.

In order to satisfy the energy conditions (9) and (10), we have the following restrictions on m from the field equations (12) – (14).

(i) $m' \geq 0$, $m'' \leq 0$, (ii) $\dot{m} \geq 0$.

The condition (i) means that the mass function either increases with r or is constant. The second restriction (ii) implies that matter within radius r increases with time.

III. Monopole Vaidya Solution in (n+2) dimensions Space –time

Mass function given by A. Wang in case of 4D is given by

$$m(u,r) = \alpha r \qquad 0 < \alpha < 1$$

Where α is arbitrary constant. (4)

Mass function in (n+2) dimensional Monopole Vaidya solution is given by

$$m(u,r) = \alpha r^{n-1} \qquad 0<\alpha<1 \qquad (15)$$

Monopoles are formed due to guaze symmetry breaking and have many properties of elementary particles. Most of their energy is concentrated in a small region near Monopole score.

Following references [9, 10], we define the mass function of Vaidya space-time in (n+2)-dimension as

$$m(u,r) = g(u) \qquad (16)$$

Where *f* (*v*) and *g*(*v*) are arbitrary functions which are restricted by the energy

Conditions [9,10].

Since the energy momentum tensor linear in terms of mass functions, a linear superposition of particular solutions is also a solution of Einstein's field equation (1) in particular combining the mass functions (15) and (16) ,we obtain the mass function for Monopole Vaidya solution as

$$m(u,r) = \alpha r^{n-1} + g(u) \qquad (17)$$

The physical situation is for $u<0$; the space-time is (n+2)-dimensional Minkowskian monopole field with $g(u)=0$. The radiation is focused into central singularity at $r=0\,, u=0$ of growing mass $g(u)$. At $u=T$, say, the radiation is turned off.

For $u>T$, the exterior space-time settles into (n+2)-dimensional Reissner-Nordstorm monopole field solution.

Hence using mass function (17), (n+2)-dimensional Vaidya monopole space-time

(1) can be written as

$$ds^2 = -\left[1-\left(\frac{\alpha r^{n-1}}{r^{n-1}}+\frac{g(u)}{r^{n-1}}\right)\right]du^2 + 2dudr + r^2 d\Omega^2$$

$$ds^2 = -\left[1-\alpha-\frac{g(u)}{r^{n-1}}\right]du^2 + 2dudr + r^2 d\Omega^2 \quad (18)$$

The above metric may also be called as (n+2)-dimensional generalized Vaidya monopole metric.

IV. Nature of the Singularity

We now find the nature of singularity in both asymptotically flat and cosmological solution. To investigate the nature of singularity, we follow the method given in references [12]. The central singularity is said to be naked, if the radial null geodesic equation admits at least one real and positive root [12, 13].

The outgoing radial null geodesic equation for the metric (18) is given by

$$\frac{dr}{du} = \frac{1}{2}\left[1-\alpha-\frac{g(u)}{r^{n-1}}\right] \qquad (19)$$

In general the above equation does not yield analytic solution for $g(u)$.

However, if one chooses, $g(u) \propto u^{n-1}$, then the equation (17) becomes homogeneous and can be solved in terms of the elementary function [14].

Therefore let us choose

$$g(u) = \lambda u^{n-1} \qquad (20)$$

With the above mass function, Vaidya space-time metric (18) becomes

$$ds^2 = -\left[1-\alpha-\frac{\lambda u^{n-1}}{r^{n-1}}\right]du^2 + 2dudr + r^2 d\Omega^2 \quad (21)$$

To investigate the structure of the collapse, we need to consider the radial null geodesics defined by $ds^2=0$, taking $\dot\theta_1 = \dot\theta_2 = \dot\theta_3 = \cdots \dot\theta_n = 0$ into account.

For metric (21), the radial null geodesics must satisfy the null condition

$$\frac{du}{dr} = \frac{2}{\left[1-\alpha-\frac{\lambda u^{n-1}}{r^{n-1}}\right]} \qquad (22)$$

However, we recall that the coordinate *v* is an advanced time coordinate. It can be observed from equation (22) that this equation has a singularity at $r\to 0,\ u\to 0$. In order to classify the radial and non-radial outgoing non-space like geodesics terminating at this singularity in the past, we need to consider the limiting value of $X=\frac{u}{r}$ along a singular geodesic as the singularity is approached [13].

Thus, for the geodesic tangent to exist uniquely at this point, we must have that

$$X_0 = \lim_{\substack{u\to 0\\ r\to 0}} \frac{u}{r} = \lim_{\substack{u\to 0\\ r\to 0}} \frac{du}{dr} \qquad (23)$$

$$X_0 = \lim_{\substack{u\to 0\\ r\to 0}} \frac{2}{\left[1-\alpha-\frac{\lambda u^{n-1}}{r^{n-1}}\right]} \qquad (24)$$

$$X_0 = \frac{2}{1-\alpha-\lambda X_0^{n-1}} \qquad (25)$$

i.e. $X_0 - \alpha X_0 - \lambda X_0^n = 2$

$$\lambda X_0^n + \alpha X_0 - X_0 + 2 = 0$$

$$\lambda X_0^n - (1-\alpha)X_0 + 2 = 0 \qquad 0<\alpha<1 \qquad (26)$$

The above algebraic equation decides the nature of the singularity. If it has a real and positive root, then there exists future directed radial null geodesics originating from $r=0$ *and* $u=0$. In this case, the singularity will be naked.

ISSN: 2321-8169
491 - 496

If this equation has no positive root, then the singularity will be covered and the collapse ends into a black hole.

To study the equation (26), we shall consider some different values of n , λ , and α .

Case 1: Let us take $n = 2$, and then monopole space-time (21) reduces to four dimensional Vaidya monopole space-time. This four dimensional Vaidya monopole solution admits strong naked singularities.

In particular if one chooses $n = 2$, $\alpha = 0.5$, then equation (26) reduces to

$$\lambda X_0^2 - 0.5X_0 + 2 = 0 \qquad (27)$$

One can easily check that for $\lambda = 0.01$, one of the roots of equation (27) is $X_0 = 4.3845$, which ensures that the singularity is naked.

Case 2: For $n = 3$, the space-time (21) reduces to five dimensional monopole

Vaidya solution given by

$$ds^2 = -\left[1 - \alpha - \frac{\lambda u^2}{r^2}\right] du^2 + 2dudr + r^2[d\theta_1^2 + sin^2\theta_1 d\theta_2^2 + sin^2\theta_1 sin^2\theta_2 d\theta_3^2] \qquad (28)$$

This five dimensional monopole Vaidya solution admits strong naked singularity.

In particular if one chooses $n = 3$, $\alpha = 0.5$, then one of the positive real roots of equation (26) reduces to

$$\lambda X_0^3 - 0.5X_0 + 2 = 0 \qquad (29)$$

If we consider $\lambda = 0.001$, $\alpha = 0.5$, then one of the positive real root of equation (29) is $X_0 = 4.14213$ which shows that five dimensional monopole Vaidya solution has a naked singularity.

For n=4 (i.e. for 6D), the equation (26) becomes ,

$$\lambda X_0^4 - (1 - \alpha)X_0 + 2 = 0 \qquad (30)$$

then the roots of equation (30) obtained for different values of λ in six dimensional monopole Vaidya solution are shown in the following table.

Table 2.1 Values of X_0 for different values of λ in 6D.

λ	X_0			
	α=0	α=0.25	α=0.5	α=0.75
$1 * 10^{-6}$	99.3242	89.9492	77.9892	60.065
$2 * 10^{-6}$	78.6919	71.2007	61.6019	46.9839
$3 * 10^{-6}$	68.6562	62.0808	53.6284	40.5977
$4 * 10^{-6}$	62.3148	56.3177	48.5885	36.548
$5 * 10^{-6}$	57.7979	52.2125	44.9976	33.6534
$6 * 10^{-6}$	54.3486	49.0775	42.2548	31.435
$7 * 10^{-6}$	51.5914	46.5713	40.0617	29.6552
$8 * 10^{-6}$	49.3147	44.5019	38.2505	28.1802
$9 * 10^{-6}$	47.389	42.7513	36.7179	26.9277

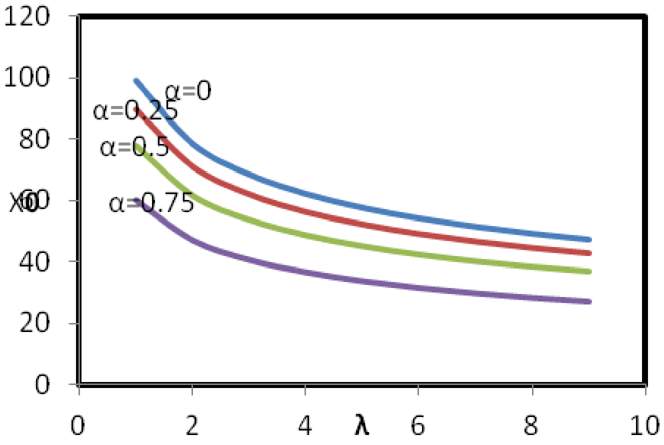

Figure 2.1: Graph of the values of X_0 against the values of λ

From the graph we may observe that initially the value of X_0 is at peak and the values of X_0 decreases when increase the value of λ . It is observe that the peak shifted towards lower values of X_0 for the different values of α . As we increase the values of α the values of X_0 decreases.

For $n = 5$ $(i.e. for\ 7D)$, the equation (2.26) becomes

$$\lambda X_0^5 - (1 - \alpha)X_0 + 2 = 0 \qquad (31)$$

then the roots of equation (31) obtained for different values of λ in seven dimensional monopole Vaidya solution are shown in the following table.

Table 2.2 Values of X_0 for different values of λ in 7D.

λ	X_0			
	α=0	α=0.25	α=0.5	α=0.75
$1 * 10^{-7}$	55.722	51.6427	46.2292	37.4446
$2 * 10^{-7}$	46.773	43.312	38.6935	31.0358
$3 * 10^{-7}$	42.213	39.0669	34.8519	27.7513
$4 * 10^{-7}$	39.247	36.3051	32.3518	25.6032
$5 * 10^{-7}$	37.088	34.2953	30.5319	24.0321
$6 * 10^{-7}$	35.412	32.7342	29.1179	22.8057
$7 * 10^{-7}$	34.052	31.4686	27.9713	21.8064
$8 * 10^{-7}$	32.917	30.4109	27.0128	20.9671
$9 * 10^{-7}$	31.946	29.5067	26.1933	20.2459

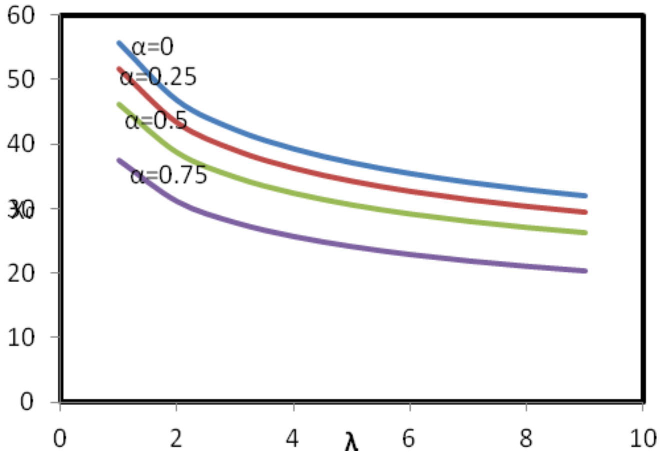

Figure 2.2: *Graph of the values of X_0 against the values of λ .*

If we observe the above graph we see that in seven dimensional space-time , the values of X_0 decrease as we increase the value of **λ .** Also for increasing α the values of X_0 decreases very rapidly.

<u>Case 3</u>:- For constant α ,

if $\alpha = 0.25$ the equation (26) becomes

$$\lambda X_0^n - X_0 + 2 = 0 \qquad (32)$$

then the roots of equation (32) obtained for different values of λ in seven dimensional monopole Vaidya solution are shown in the following table.

Table 2.3 *Values of X_0 for different values of λ for* $\alpha = 0.25$.

λ	X_0			
	n=4	n=5	n=6	n=7
$1*10^{-6}$	99.3242	31.1017	15.4145	9.619
$2*10^{-6}$	78.6919	26.066	13.3569	8.5205
$3*10^{-6}$	68.6562	23.4997	12.2781	7.9333
$4*10^{-6}$	62.3148	21.8299	11.5636	7.5395
$5*10^{-6}$	57.7979	20.6147	11.0367	7.2465
$6*10^{-6}$	54.3486	19.6707	10.6231	7.0148
$7*10^{-6}$	51.5914	18.9054	10.2848	6.8241
$8*10^{-6}$	49.3147	18.2657	10	6.6627
$9*10^{-6}$	47.3889	17.7189	9.7549	6.5231

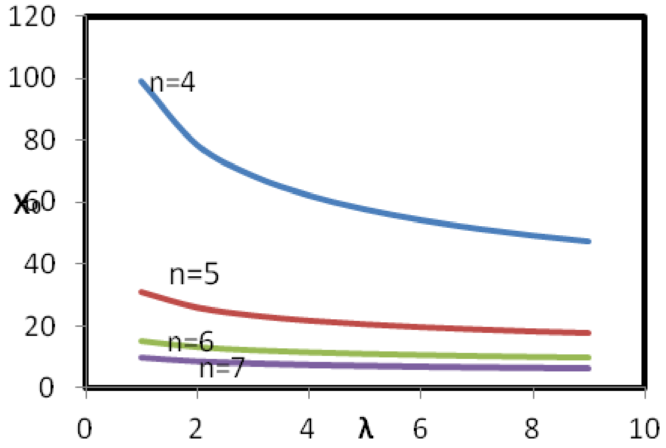

Figure 2.3: Graph of the values of X_0 against the values of λ for fixed value of α .

From the graph we may observe that for fixed α and λ the values of X_0 decreases in higher and higher dimensions. Initially the value of X_0 is at peak then decreases smoothly when increase the value of λ .

For $\alpha = 0.75$, the equation (26) becomes

$$\lambda X_0^n - 0.25X_0 + 2 = 0 \qquad (33)$$

then the roots of equation (33) obtained for different values of λ for $\alpha = 0.75$ monopole Vaidya solution are shown in the following table.

Table 2.4 *Values of X_0 for different values of λ for* $\alpha = 0.75$.

λ λ	X_0			
	n=4	n=5	n=6	n=7
$1*10^{-13}$	13569.4204	1255.4254	300.0827	115.109
$2*10^{-13}$	10769.5055	1055.3617	261.0227	102.3913
$3*10^{-13}$	9407.6921	953.4322	240.5615	95.6044
$4*10^{-13}$	8547.2114	887.1284	227.0178	91.0592
$5*10^{-13}$	7934.3368	838.8844	217.036	87.6805
$6*10^{-13}$	7466.3393	801.4158	209.2045	85.0115
$7*10^{-13}$	7092.2484	771.0421	202.8022	82.8172
$8*10^{-13}$	6783.3753	745.6609	197.4135	80.9613
$9*10^{-13}$	6522.1106	723.9656	192.7783	79.3582

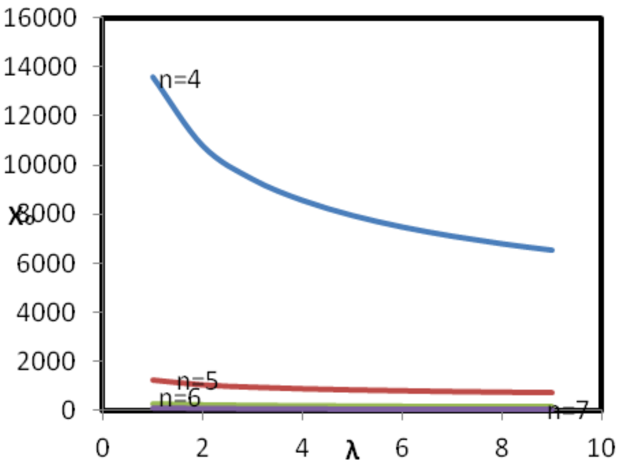

Figure 2.4: Graph of the values of X_0 against the values of λ for fixed value of α .

From the graph we may observe that when we increase the value of α the value of X_0 increases very rapidly and X_0 decreases for higher and higher dimensions of same α.

V. Concluding Remarks

Gravitational collapse of a star is one of the most interesting subjects in gravitational physics. It is well known that singularity formation is inevitable in complete gravitational collapse. It was conjectured that such a singularity should be hidden by horizons if it is formed from generic initial data with physically reasonable matter fields [2].

Many possible counter examples to this conjecture have been proposed over the past four decades[15,16,17], although none of them have proved to be sufficiently generic [18,19]. In these examples, there appears a singularity that is not hidden by horizons. This singularity is called a *naked singularity*.

In the present work shows that naked singularities do occur as the end stage of gravitational collapse in (n+2) dimensional Monopole Vaidya solution and shown that the result in 4-dimensional space-time are also valid in (n+2)-dimensional Monopole Vaidya solution. Imposing some conditions on mass function ,it has been shown that the singularities arising (n+2)-dimensional Vaidya solution in the presence of monopole field are naked in any arbitrary dimensions.

Thus this is observing that the dimension of the space-time does not play any fundamental role in the formation of the naked singularities. Occurrence of naked singularities in higher dimensional Vaidya monopole space-time suggests that the higher dimensional Vaidya monopole solution violates the cosmic censorship hypothesis. These results might be important in the light of the recent proposal that there may exist extra dimensions in the universe.

References

[1] P.S.Joshi, Gravitational collapse:the story so far, pramana journal of phy. Vol.55 (2000) pp.529.

[2] R. Penrose, Gravitational collapse: of The role general relativity,Riv.,Nuovo.Cimento. 1, (1969) pp 252.

[3] Kaluza.T,Sitz-ber Preuss.Akad. Wiss. 33,(1921) pp 966.

[4] Klein O., Z. Phys.37, (1926) pp 895.

[5] K. D. Patil and U. S. Thool, Spherical and non-spherical gravitational collapse in Husain space-time, Int. J. Mod. Phys. D 14 (5) (2005) pp 873-882.

[6] Einstein, Albert, "The Foundation of the General Theory of Relativity", Annalen der Physik (1916).

[7] Einstein, Albert, "Die Feldgleichungen der Gravitation". Sitzungsberichte der Preussischen Akademie der Wissenschaften zu Berlin: 844–847 (1915).

[8] K. D. Patil and U. S. Thool, Spherical and non-spherical gravitational collapse in Husain space-time, Int. J. Mod. Phys. D 14 (5) (2005) pp 873-882.

[9] B. R. Iyer and C. V. Vishveshshwara, The Vaidya solution in higher imensions, Pramana, J. Phys.Vol.32, No.6, (1989) pp 749.

[10] L.K. Patel and N. Dadhich, Exact solutions for null fluid collapse in higher dimensions, gr-qc / 9909068 (1999).

[11] V. Husain, Exact solutions for null fluid collapse,Phys. Rev. D 53, R 1759 (1996).

[12] Anzhong Wang and Yumei Wu, Generalized Vaidya solution, Gen. Relativ. Gravit.Vol.31, No.1, (1999) pp 107.

[13] I. H. Dwivedi and P. S. Joshi, On the nature of naked singularities in Vaidya space-time, Class. Quantum. Grav. 6, (1989) pp 1599.

[14] P. S. Joshi, (Clarendon, Oxford,(1993).

[15] S. G. Ghosh and A. Beesham, Phys. Rev. D 64, 124005 (2001). A. Banerjee, U. Debnath, S. Chakraborty, gr-qc/0211099.

[16] P. S. Joshi and I. H. Dwivedi, Phys. Rev. D 47 5357 (1993).

[17] R. V. Saraykar and S. H. Ghate, Class. Quantum. Grav.16 41(1999).

[18] P.S. Joshi and I. H. Dwivedi, Phys. Rev. D 45 (1992) pp 2147.

[19] S. G. Ghosh and A. Beesham, Higher-dimensional inhomogeneous dust collapse and cosmic censorship, Phys. Rev. D 61, 067502 (2000).

[20] Kishor D. Patil, The final fate of inhomogeneous dust collapse in higherdimensional space-time, Indian J. Phys., Vol. 77, No.3 (2003).