

# The Gravitational Energy of a Black Hole

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**Abstract** - a certain energy expression for a physical part comes by considering the escape of a gauge boson from the part. The mass of the part at intervals its horizon is found to be doubly its mass as discovered at infinity. This result's vital in understanding gravitative waves in part collisions.

**Keywords:** Black hole; gravitative energy

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## Introduction

What is the energy of a black hole? this is often an issue that seems ought to have an easy answer. it's cheap to conclude that the energy of a part is that that corresponds to its mass as determined by an overseas observer by observation a satellite undergoing associate degree orbiting motion round the part, victimisation the equations of relativity theory. This has been the empirical approach of finding the mass of a planet or a star. The mass obtained during this approach is that the total mass of the system as seen by an overseas observer. For a physical part, it's information superhighway mass obtained from the difference between the constituent mass of the part and its gravitative energy. Since gravitative energy is thought to be negative, thus the constituent mass should be larger than the discovered mass for the part. to grasp the character of mass of a part, it's necessary to understand the energy distribution of the part throughout all area. because the gravitative field of a part extends to infinity, its P.E. extends equally and contributes additionally to its discovered mass. The idea of a part comes from the Schwarzschild resolution to Einstein's equation [1]. A Schwarzschild part encompasses a mass  $M$  and a radius  $RS$  in keeping with an overseas observer stationed at infinity. during this paper, the overall energy expression for a nonrotating part as well as its gravitative energy comes during a easy and physical approach by considering the escape of a gauge boson simply outside the surface of a part during a gedanken experiment kind of like the Hawkingprocess [2].

When a gauge boson of a given energy is emitted simply outside the horizon of a part it'll have zero energy because it

reaches infinity. this suggests the whole energy of the gauge boson is employed to flee the gravitative pull of the part. If the gauge boson comes from the annihilation of a particle of mass  $m$  close to the horizon, then it means that the whole mass of the particle is employed to create the gauge boson break loose the part. This additionally means the energy needed to get rid of a mass  $m$  simply outside the horizon to infinity is solely  $mc^2$ . currently imagine that a mass  $m$  is far from the horizon to infinity terribly slowly by associate degree external agent in order that no K.E. is generated within the method, the energy needed to try and do this is often still  $mc^2$ . Eventually, the mass removed can reach infinity as a free mass. contemplate next a particle of mass  $m$  being made simply outside the horizon and that has sufficient energy to flee to infinity on its own wherever it lands up as a free particle of discovered at infinity is  $M$ , then the initial mass within the part should mass  $m$ . The higher than thought shows that the overall energy needed for this event is solely  $2mc^2$ . As a result, the part can lose energy by an equivalent quantity  $2mc^2$  for every particle of mass  $m$  free at the horizon and discovered at infinity. This energy is freelance of the mass of the part. once a succession of processes during this manner, the whole part is reworked into straight line particles at infinity. If the overall mass of the particles be adequate to  $2M$ ,  $1/2$  that is employed to provide the gravitative energy of those particles, that is additionally the gravitative P.E. of the part itself. this is often an interesting result. so from the purpose of read of an overseas observer, the constituent mass of the part is  $2M$ , although its discovered mass is simply  $M$ . This discovered mass at infinity corresponds to the Arnowitt-Deser-Misner mass [3], that could be a live of the overall energy of a

gravitative system at spatial infinity normally theory of relativity. A part so has the most gravitative energy any system will have. we tend to thus introduce the idea of the horizon mass and state the subsequent theorem on the energy of a part: If  $M$  is that the mass of a black hole at intervals its horizon, then its energy discovered at infinity is  $E = Mc^2$ . allow us to incorporate the higher than result into a mathematical formula. removed from the part, associate degree observer ought to find a degree mass  $M$  and also the spacetime is that the one represented by the Schwarzschild metric. If a gauge boson is emitted at coordinate  $r$  with energy  $\epsilon_r$  and later discovered at infinity, its energy there's given by  $\epsilon_\infty = \epsilon_r \sqrt{1 - \frac{2GM}{rc^2}}$  where  $G$  is that the constant and  $c$  is that the speed of sunshine. The difference between the of the gauge boson at the 2 locations is thus  $\epsilon_r - \epsilon_\infty = \epsilon_r \left[ 1 - \sqrt{1 - \frac{2GM}{rc^2}} \right]$ . The modification within the photon's energy could be a live of the modification of the gravitative P.E. of the part as a operate of the coordinate  $r$ . Next, to explain the entire behavior of the energy of the part itself, we tend to introduce a operate  $f(r)$  interpolating between the surface of the part and infinity in order that the energy of the part additionally becomes a operate of the coordinate  $r$ . This energy expression offers the overall energy of the part contained during a spherical volume from the origin up to the coordinate  $r$  to work out the operate  $f(r)$ , we tend to set the subsequent conditions:

1. the overall energy  $E(r)$  is often positive. so  $f(r)$  should be a positive function between  $RS$  and  $\infty$ .
2. the overall energy  $E(r)$  decreases swimmingly between  $RS$  and  $\infty$ . so its by-product  $dE/dr$  is often negative.
3. At giant distances, the overall energy  $E(r)$  approaches associate degree straight line price. so  $dE/dr \approx$  zero at terribly giant distances.

Taking the by-product  $dE/dr$  in relative atomic mass.(3) and subjecting it to the higher than conditions, we tend to find at giant distances associate degree equation for  $f(r)$ ,  $df(r) = f(r) \frac{dr}{r}$  the answer is found to be  $f(r) = \text{constant} \times r$ .

To determine the constant, we tend to notice at giant distances, the root

1 - in Eq.(3) expands as  $\sqrt{1 - \frac{2GM}{rc^2}} \approx 1 - \frac{GM}{rc^2}$ , the energy of the part ought to approach the straight line price  $Mc^2$  as seen by the distant observer. Thus,  $E(r) \approx f(r)$  giving finally the general energy expression for the part is currently

With this result, we tend to recover the energy of the part within the Schwarzschild horizon as complete earlier by the distant observer. Setting  $r = RS = \frac{2GM}{c^2}$ , we tend to get from relative atomic mass.(7)

The expression given by relative atomic mass.(7) agrees with the analysis of the quasilocal energy of the Schwarzschild resolution by Brown and York [4], and additionally agrees with the calculation of the energy during

a part within the teleparallel equivalent formulation of relativity theory by Maluf [5]. Those developments are however ever a lot of mathematical and framework dependent than this physical approach. The significance of this result's that the overall energy of a part may be found normally theory of relativity while not requiring the utilization of any illusory native gravitative energy density the least bit [6]. Figure one shows the variation of the mass of a part beginning at  $r = RS$  to  $r = 10RS$ , victimisation the mass equivalence of relative atomic mass.(7). As may be seen, the mass decreases quickly from  $2M$  at  $RS$  and levels off to slightly higher than  $M$  at  $10RS$ . At giant distances, the mass is much indistinguishable from its straight line price  $M$ . However, at shut distances, the mass is sort of different from  $M$  as seen by the distant observer. Here the mass operate is defined by standing part collisions. contemplate the subsequent example. once a part of straight line mass  $5M$  collides with a part of straight line mass  $12M$ , the minimum result's a part of straight line mass  $13M$ . This follows from the realm non-decrease theorem for black holes. the realm of a black hole  $A = 4\pi RS^2$  is proportional to the sq. of its straight line mass. Therefore, in keeping with an overseas observer observation the collision, the number of mass radiated away throughout the collision method within the type of gravitative waves is  $(5M + 12M) - 13M = 4M$ .

Without knowing the part energy formula in relative atomic mass.(7), an area observer near the collision method believes that the higher than result's invariably correct. This native horizon observer firmly believes that the horizon mass is that the same because the straight line mass as a result of he cannot sight any measurable changes in particle motions outside a part notwithstanding he were told that the horizon mass is different from the straight line mass. Any particle motion is decided utterly by the Schwarzschild metric supported the straight line mass. so the native horizon observer calculates his own orbit close to the part supported the Schwarzschild metric and without delay concludes that the mass of the part is that the same as once he began from infinity. He cannot justifiably settle for the other result. however with the information of the part energy formula, we will perceive the collision higher. The collision involves a part of horizon mass  $10M$  with a part of horizon mass  $24M$ , leading to a part of horizon mass  $26M$ , once more following the realm non- decrease theorem. thus the overall mass radiated away within the collision method is  $(10M + 24M) - 26M = 8M$ . this is often doubly the number as that complete by the distant observer, and additionally doubly the number complete by the native horizon observer. wherever has the additional mass  $4M$  gone to?

If one believes that gravitative waves are answerable for the difference in mass of the black holes before and once the

collision, then this suggests that an extra energy of {the quantity|the quantity|the number}  $4M$  is needed to permit these waves to propagate from the final part to infinity for the distant observer this is often as a result of once gravitative waves of mass  $4M$  reach infinity they'll have gained P.E. of the equal amount  $4M$ . Energy is inertia. the overall energy lost from the final part is thence  $8M$ , according to our higher than observation. If the native horizon observer was correct, there would be no modification within the P.E. of the gravitative waves the least bit. The gravitative waves during this case cannot propagate off from the part. thus in sleuthing any gravitative signal from a part collision like that projected within the LIGO project, any conclusion regarding the strength of the signals close to its supply ought to be supported the part energy formula. Understanding the collisions of black holes in galaxies is one amongst the outstanding issues in cosmology.

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