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897 - 905

Two-Person Zero-Sum Game using MAX-MIN Approximation

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Abstract:-In this paper we discuss Two-Person Zero-Sum Game with Epsilon-Delta fuzzy payoff matrix is defined and obtained optimal fuzzy strategies by using MAX-MIN approximation. Criterion of Fuzzy Minimax-Maximin is established. Some examples are given.

Keywords: Epsilon-delta fuzzy number; Fuzzy payoff matrix; Fuzzy saddle point; Fuzzy two person zero sum game, MAX-MIN approximation.

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1. Introduction

The mathematical analysis of competitive problems is based upon 'Minimax-Maximin Criterion' of J. Von Neuman. The simplest type of competitive situations is two-person zero-sum game [4-6]. These games involve only two players; they are called *zero-sum* games because one player wins whatever the other player loses [7].

Game theory is a type of decision theory in which one's choice of action is imprecise after taking into account all possible alternatives available to an opponent playing the same game. Fuzzy games are intended to model conflict situation with imprecise information, Payoffs strategies etc. In this paper, we have proved 'Fuzzy Minimax-Maximin criterion' by using Epsilon–delta fuzzy number [1] as cell entries of payoff matrix. We use MAX-MIN approximation [2] for establishing the Minimax-Maximin criterion.

In section 2 we give basic notions which are used in the sequences. In section 3 fuzzy two person zero sum game is discussed. In section 4 results are elaborated by empirical examples.

2. Preliminaries

Basic notions of Two-Person Zero-Sum Games

- A two-person game is characterized by the strategies of each player and the payoff matrix.
- The payoff matrix shows the gain (positive or negative) for player 1 that would result from each combination of strategies for the two players. *Note that the matrix for player 2 is the negative of the matrix for player 1 in a zero-sum game.*
- The entries in the payoff matrix can be in any units as long as they represent the *utility (or value)* to the player.
- There are two key assumptions about the behavior of the players. The first is that both players are *rational*. The second is that both players are *greedy* meaning that they choose their strategies in their own interest (to promote their own wealth).
- The definition of a two-person zero-sum game in normal form amounts to defining sets of strategies A and B of players I and II respectively, and of the pay-off function H of player I, defined on the set $A\times B$ of all situations (the pay-off function of player II is $-H$ by definition). Formally, a two-person zero-sum game Γ is given by a triplet $\Gamma=\langle A, B, H\rangle$. Play consists in the players choosing their strategies $a \in A$, $b \in B$, after which player I obtains the sum $H(a, b)$ from player II. Such a definition of a two-person zero-sum game is sufficiently general to include all variants of two-person zero-sum games,

ISSN: 2321-8169
897 - 905

including dynamic games and positional games provided that the sets of strategies and the pay-off function are properly described. A rational choice of actions (strategies) of the players in the course of a two-person zero-sum game is based on a minimax principle:

If *max a∈A, inf b∈B, H (a, b) =min b∈B, sup a ∈AH (a, b)* (1)

or

sup $a \in A$, inf $b \in B$, $H(a, b) =$ inf $b \in B$, sup $a \in A$ $H(a, b)$ (2)

the game Γ has optimal strategies (ϵ-optimal strategies, respectively) for both players. The common value of both parts of equation (2) is called the value of the game Γ. However, equations (1) or (2) may not be valid even in the simplest cases. Eg. In a matrix game with payoff matrix

Player B

Player A $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

The following equalities are valid:

$$\max_i \min_j a_{ij} = -1, \min_j \max_i a_{ij} = 1$$

Definition 1. [8] A *fuzzy number A* is a subset of the real line $\mathbb{R}$ with membership function $A : \mathbb{R} \to [0,1]$ such that *A* is normal, *A* is fuzzy convex and upper semi-continuous i.e. α-cut is closed for all $\alpha \in [0,\ 1]$ .Support of *A* is bounded. If left hand curve and right hand curve are straight lines then the fuzzy number is called triangular fuzzy number.

Definition 2. [1] If *r* is a real number then ε-δ fuzzy number $r_{\varepsilon,\delta}$ is the triangular fuzzy number for some $\varepsilon, \delta \in \mathbb{R}$, $(\varepsilon, \delta > 0)$ is a fuzzy set $r_{\varepsilon,\delta} : \mathbb{R} \to [0,1]$ defined by

$$r_{\varepsilon,\delta}(x) = \begin{cases} \dfrac{x-(r-\varepsilon)}{\varepsilon}, & \text{if } r-\varepsilon < x \le r, \\ \dfrac{x-(r+\delta)}{-\delta}, & \text{if } r < x \le r+\delta, \\ 0, & \text{otherwise.} \end{cases}$$

A triangular fuzzy number $A = (l, m, n)$ in above notation is denoted by $A = m_{m-l,n-m}$. Also, $r_{\varepsilon,\delta} = (r-\varepsilon, r, r+\delta)$.

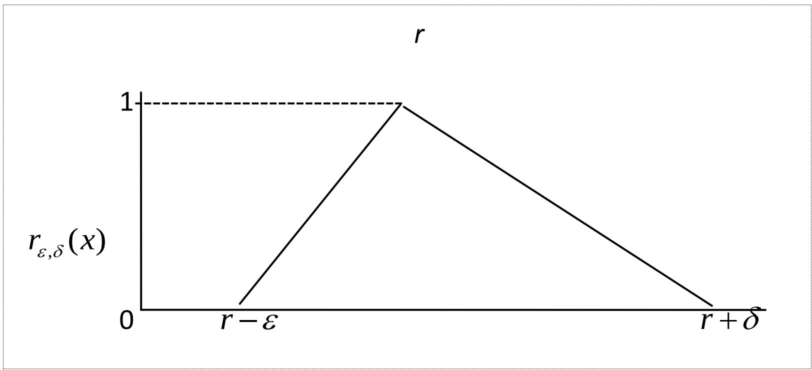


Fig.1. Membership functions of the fuzzy number $r_{\varepsilon,\delta}$.

The *support* of ε-δ fuzzy number $r_{\varepsilon,\delta}$ is $\left(r-\varepsilon(1-\alpha), r+\delta(1-\alpha)\right), r\in ¡\ ,\varepsilon,\delta\in ¡$ and $\varepsilon,\delta>0$. The α-*cut* of $r_{\varepsilon,\delta}$ is denoted by $(r_{\varepsilon,\delta})_{\alpha} = [r-\varepsilon(1-\alpha), r+\delta(1-\alpha)], \alpha\in(0.1]$.

The above notation for triangular fuzzy number is simple and may be considered as a family of functions of three parameters.

Definition 3 [2] Let $r_{\varepsilon_1,\delta_1}$ and $s_{\varepsilon_2,\delta_2}$ be any two epsilon-delta fuzzy numbers where $r\le s$. If $r-\varepsilon_1\le s-\varepsilon_2$ and $r+\delta_1\le s+\delta_2$ then we define

$$(r_{\varepsilon_1,\delta_1}\wedge s_{\varepsilon_2,\delta_2}) = r_{\varepsilon_1\vee\varepsilon_2,\delta_1\wedge\delta_2} \text{ and. } r_{\varepsilon_1,\delta_1}\vee s_{\varepsilon_2,\delta_2} = s_{\varepsilon_1\wedge\varepsilon_2,\,\delta_1\vee\delta_2}$$

If $s-\varepsilon_2<r-\varepsilon_1$ and $r+\delta_1\le s+\delta_2$ then we define

$$(r_{\varepsilon_1,\delta_1}\wedge s_{\varepsilon_2,\delta_2}) = r_{\varepsilon_2-(s-r),\delta_1} \text{ and } (r_{\varepsilon_1,\delta_1}\vee s_{\varepsilon_2,\delta_2}) = s_{\varepsilon_1+(s-r),\delta_2}\ .$$

If $r-\varepsilon_1\le s-\varepsilon_2$ and $s+\delta_2<r+\delta_1$ then

$$(r_{\varepsilon_1,\delta_1}\wedge s_{\varepsilon_2,\delta_2}) = r_{\varepsilon_1,\delta_2+(s-r)} \text{ and } (r_{\varepsilon_1,\delta_1}\vee s_{\varepsilon_2,\delta_2}) = s_{\varepsilon_2,\delta_1-(s-r)}\ .$$

If $s-\varepsilon_2<r-\varepsilon_1$ and $s+\delta_2<r+\delta_1$ then

$$r_{\varepsilon_1,\delta_1}\wedge s_{\varepsilon_2,\delta_2} = r_{\varepsilon_2-(s-r),\,\delta_2+(s-r)} \text{ and } (r_{\varepsilon_1,\delta_1}\vee s_{\varepsilon_2,\delta_2}) = s_{\varepsilon_1+(s-r),\delta_1-(s-r)}\ .$$

The above evaluations are triangular approximations of max and min operations on fuzzy numbers. Many researchers have used Dubois and Prade approximation of max and min operations given below

$$r_{\varepsilon_1,\delta_1}\wedge s_{\varepsilon_2,\chi_2} = (r\wedge s)_{\varepsilon_1\vee\varepsilon_2,\,\delta_1\vee\delta_2} \text{ and } r_{\varepsilon_1,\delta_1}\vee s_{\varepsilon_2,\chi_2} = (r\vee s)_{\varepsilon_1\wedge\varepsilon_2,\,\delta_1\vee\delta_2}\ .$$

Addition or subtraction of the term $|r-s|$ may be considered as a correcting factor and thereby gives better approximation.

3. Fuzzy two person zero sum game

The sets of the possible feasible strategies of player I are two fuzzy sets *A* and *B* on S_1 and S_2 respectively. The two payoff functions P1 (for player I) and P2 (for player II) from $S_1\times S_2\to[0,1]$ are fuzzy.

Epsilon-delta Payoff matrix

Definition 4. A quantitative measure of satisfaction of a person expressed in terms of epsilon-delta fuzzy numbers is called epsilon-delta payoff

Suppose player A has m activities and player B has n activities. Then a fuzzy
Payoff matrix can be formed by adopting following rules.

The player A’s fuzzy payoff matrix

 ISSN: 2321-8169
897 - 905

Player B

Player A

Strategies	1	2	…	j	…	n
1	$r^{11}_{\varepsilon_{11},\delta_{11}}$	$r^{12}_{\varepsilon_{12},\delta_{12}}$	….	$r^{1j}_{\varepsilon_{1j},\delta_{1j}}$	…..	$r^{1n}_{\varepsilon_{1n},\delta_{1n}}$
2	$r^{21}_{\varepsilon_{21},\delta_{21}}$	$r^{22}_{\varepsilon_{22},\delta_{22}}$	…..	$r^{2j}_{\varepsilon_{2j},\delta_{2j}}$	……	$r^{2n}_{\varepsilon_{2n},\delta_{2n}}$
: :	: :	: :	…..	: :	……	: :
i	$r^{i1}_{\varepsilon_{i1},\delta_{i1}}$	$r^{i2}_{\varepsilon_{i2},\delta_{i2}}$	……	$r^{ij}_{\varepsilon_{ij},\delta_{ij}}$	…….	$r^{in}_{\varepsilon_{in},\delta n}$
: :	: :	: :	…..	: :	……	: :
m	$r^{m1}_{\varepsilon_{m1},\delta_{m1}}$	$r^{m2}_{\varepsilon_{m2},\delta m2}$	……	$r^{mj}_{\varepsilon_{mj},\delta_{mj}}$	…….	$r^{mn}_{\varepsilon_{mn},\delta_{mn}}$

We denote above fuzzy payoff matrix as follows

$$A_{E,D}=\begin{bmatrix} r^{11}_{\varepsilon_{11},\delta_{11}} & r^{12}_{\varepsilon_{12},\delta_{12}} & \cdots & r^{1n}_{\varepsilon_{1n},\delta_{1n}} \\ r^{21}_{\varepsilon_{21},\delta_{21}} & r^{22}_{\varepsilon_{22},\delta_{22}} & \cdots & r^{2n}_{\varepsilon_{2n},\delta_{2n}} \\ \vdots & \vdots & \vdots & \vdots \\ r^{m1}_{\varepsilon_{m1},\delta_{m1}} & r^{m2}_{\varepsilon_{m2},\delta_{m2}} & \cdots & r^{mn}_{\varepsilon_{mn},\delta_{mn}} \end{bmatrix}_{m\times n} \text{,where } [A]=\begin{bmatrix} r^{11} & r^{12} & \cdots & r^{1n} \\ r^{21} & r^{22} & \cdots & r^{2n} \\ \vdots & \vdots & \vdots & \vdots \\ r^{m1} & r^{m2} & \cdots & r^{mn} \end{bmatrix}_{m\times n},$$

$$E=[\varepsilon_{ij}]=\begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \cdots & \varepsilon_{1n} \\ \varepsilon_{21} & \varepsilon_{22} & \cdots & \varepsilon_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \varepsilon_{m1} & \varepsilon_{m2} & \cdots & \varepsilon_{mn} \end{bmatrix}_{m\times n} \text{ and } D=[\delta_{ij}]=\begin{bmatrix} \delta_{11} & \delta_{12} & \cdots & \delta_{1n} \\ \delta_{21} & \delta_{22} & \cdots & \delta_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \delta_{m1} & \delta_{m2} & \cdots & \delta_{mn} \end{bmatrix}_{m\times n}$$

Fuzzy Minimax-Maximin Criterion

Theorem 4. *Let, P be the payoff matrix for a two person zero sum game where* $P: X \to [0,1]$.

If $\vee_i\left(\wedge_j r^{ij}_{\varepsilon_{ij},\delta_{ij}}\right)=\underline{r}$ *and* $\wedge_j\left(\vee_i r^{ij}_{\varepsilon_{ij},\delta_{ij}}\right)=\overline{r}$ *then* $\overline{r}\geq\underline{r}$

$$i.e.\ \wedge_j\left(\vee_i r^{ij}_{\varepsilon_{ij},\delta_{ij}}\right)\geq\vee_i\left(\wedge_j r^{ij}_{\varepsilon_{ij},\delta_{ij}}\right).$$

Proof. We have

$$\vee_i r^{ij}_{\varepsilon_{ij},\delta_{ij}}\geq r^{ij}_{\varepsilon_{ij},\delta_{ij}} \text{ and } \wedge_j r^{ij}_{\varepsilon_{ij},\delta_{ij}}\leq r^{ij}_{\varepsilon_{ij},\delta_{ij}},\ \forall i,j$$

Let maximum be obtained at $i=i^*$ and minimum be obtained at $j=j^*$. Therefore,

$$r^{i^*j}_{\varepsilon_{i^*j},\delta_{i^*j}}\geq r^{ij}_{\varepsilon_{ij},\delta_{ij}}\geq r^{ij^*}_{\varepsilon_{ij^*},\delta_{ij^*}},\ \forall i,j$$

 ISSN: 2321-8169
897 - 905

$$\bigwedge_j\left(r^{i^*j}_{\varepsilon_{i^*j},\delta_{i^*j}}\right)\ge r^{ij}_{\varepsilon_{ij},\delta_{ij}}\ge\bigvee_i\left(r^{ij^*}_{\varepsilon_{ij^*},\delta_{ij^*}}\right),\forall i,j$$

$$\bigwedge_j\left(\bigvee_i r^{ij}_{\varepsilon_{ij},\delta_{ij}}\right)\ge\bigvee_i\left(\bigwedge_j r^{ij}_{\varepsilon_{ij},\delta_{ij}}\right)$$

Fuzzy Saddle Point

Definition 5. If

$$\bigwedge_j\left(\bigvee_i r^{ij}_{\varepsilon_{ij},\delta_{ij}}\right)=\bigvee_i\left(\bigwedge_j r^{ij}_{\varepsilon_{ij},\delta_{ij}}\right)$$

i.e.If $\overline{r}=\underline{r}=r^{ij}_{\varepsilon_{ij},\delta_{ij}}$, Then the fuzzy game has a saddle point at the cell (i,j).

4. Fuzzy optimal strategies

If fuzzy payoff $r^{ij}_{\varepsilon_{ij},\delta_{ij}}$ is a saddle point the players have fuzzy optimal strategies in pure strategies: Player I have ith and player II have jth fuzzy optimal strategies respectively.

Value of Fuzzy Game

The fuzzy payoff $r^{ij}_{\varepsilon_{ij},\delta_{ij}}$ at saddle point (i,j) is called value of fuzzy game.

Fair Fuzzy Game

A fuzzy game is said to be fair game if saddle point $r^{ij}_{\varepsilon_{ij},\delta_{ij}}=0_{\varepsilon,\delta}$

Example 6. Consider two-person zero-sum game which represents fuzzy payoff to the player A. Find the optimal strategy if any

Player B

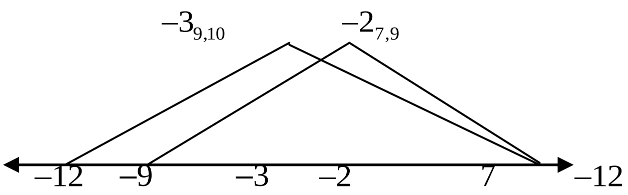
		I	II	III	Row Minimum
Player A	I	$-3_{9,10}$	$-2_{7,9}$	$6_{5,9}$	$-3_{9,9}$
	II	$2_{9,7}$	$0_{4,8}$	$2_{9,7}$	$0_{9,7}$
	III	$5_{13,11}$	$-2_{6,8}$	$-4_{7,9}$	$-4_{11,9}$
Column Maximum		$5_{12,11}$	$0_{4,9}$	$6_{5,9}$	

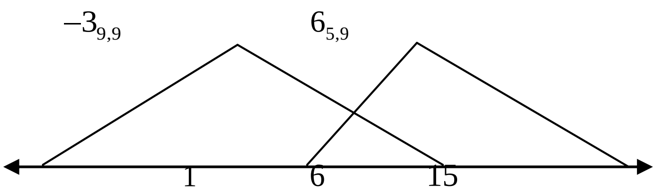
Maximin Value $(\underline{r})=0_{9,7}$

Minimax Value $(\overline{r})=0_{7,9}$

Solution: Row Minimum

$$I:\left(-3_{9,10}\wedge-2_{7,9}\right)\wedge 6_{5,9}=-3_{9,9}$$





ISSN: 2321-8169
897 - 905

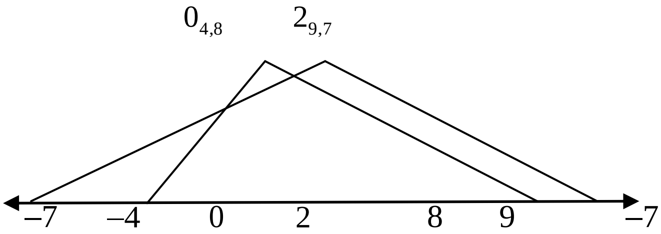
No Crossing

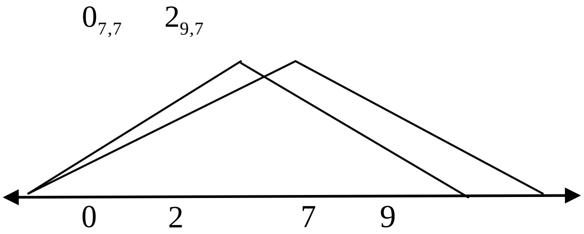
$-3_{9,10} \wedge -2_{7,9} = -3_{9,9}$

No Crossing

$-3_{9,9} \wedge 6_{5,9} = -3_{9,9}$

$II : \left(2_{9,7} \wedge 0_{4,8}\right) \wedge 2_{9,7} = 0_{9,7}$





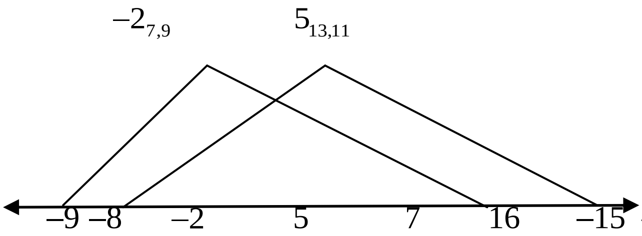
Left Crossing

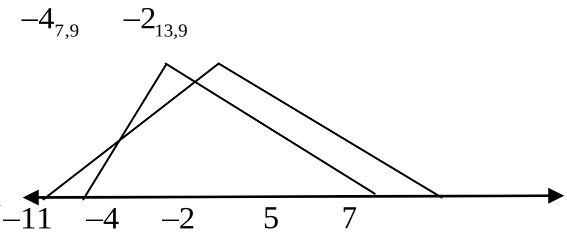
$2_{9,7} \wedge 0_{4,8} = 0_{7,7}$

No Crossing

$0_{7,7} \wedge 2_{9,7} = 0_{9,7}$

$III : \left(5_{13,11} \wedge -2_{7,9}\right) \wedge -4_{7,9} = -4_{11,9}$





No Crossing

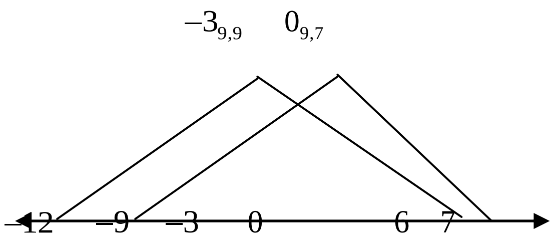
$5_{13,11} \wedge -2_{7,9} = -2_{13,9}$

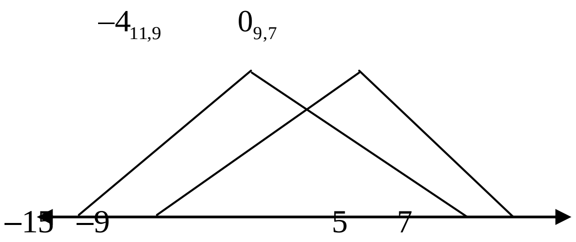
Left Crossing

$-2_{13,9} \wedge -4_{7,9} = -4_{11,9}$

Maximin

$\left(-3_{9,9} \vee 0_{9,7}\right) \vee -4_{11,9} = 0_{9,7}$





No Crossing

$-3_{9,9} \vee 0_{9,7} = 0_{9,7}$

No Crossing

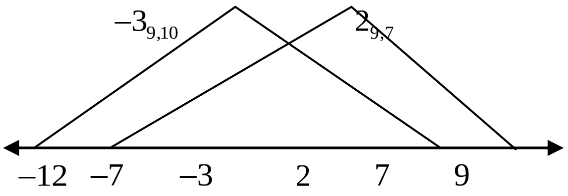
$-4_{11,9} \vee 0_{9,7} = 0_{9,7}$

In general player A selects the strategies that maximize his minimum gain.

 ISSN: 2321-8169
897 - 905

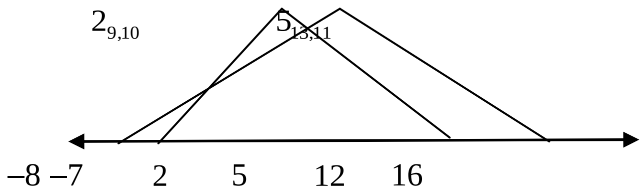
Column Maximum

$$I:\left(-3_{9,10}\vee 2_{9,7}\right)\vee 5_{13,11}=5_{12,11}$$



No Crossing

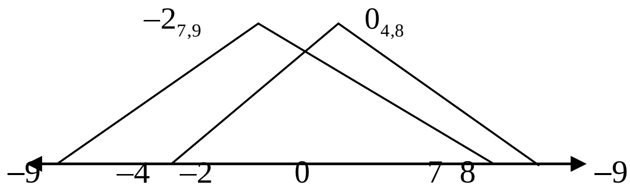
$-3_{9,10}\vee 2_{9,7}=2_{9,10}$



Left Crossing

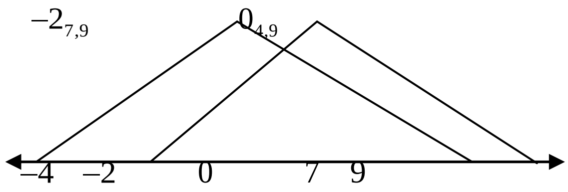
$2_{9,10}\vee 5_{13,11}=5_{12,11}$

$$II:\left(-2_{7,9}\vee 0_{4,8}\right)\vee -2_{7,9}=0_{4,9}$$



No Crossing

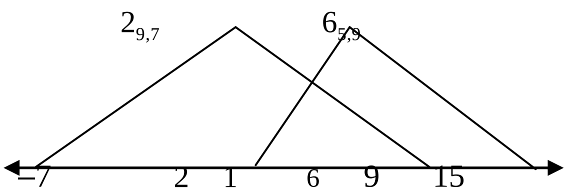
$-2_{7,9}\vee 0_{4,8}=0_{4,9}$



No Crossing

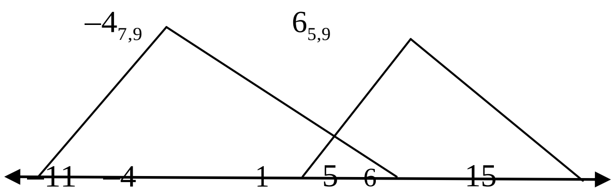
$0_{4,9}\vee -2_{7,9}=0_{4,9}$

$$III:\left(6_{5,9}\vee 2_{9,7}\right)\vee -4_{7,9}=6_{5,9}$$



No Crossing

$6_{5,9}\vee 2_{9,7}=6_{5,9}$

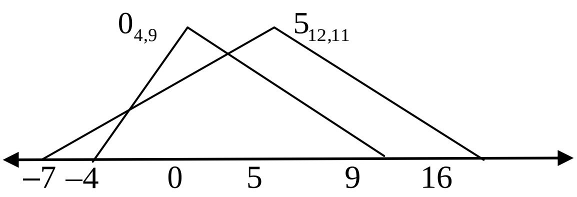


No Crossing

$6_{5,9}\vee -4_{7,9}=6_{5,9}$

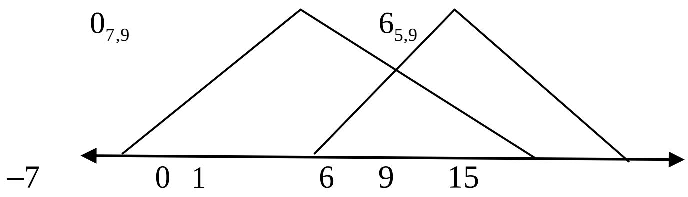
Minimax:

$$\left(5_{12,11}\wedge 0_{4,9}\right)\vee 6_{5,9}=6_{5,9}$$



Left Crossing

$5_{12,11}\vee 0_{4,9}=0_{7,9}$



No Crossing

$0_{7,9}\vee 6_{5,9}=0_{7,9}$

Player B selects the strategies that minimize his maximum loss.

In this case equality holds,

ISSN: 2321-8169
897 - 905

$$\underset{j}{\wedge}\left(\underset{i}{\vee} r^{ij}_{\varepsilon_{ij},\delta_{ij}}\right) = 0_{9,7} \le \underset{i}{\vee}\left(\underset{j}{\wedge} r^{ij}_{\varepsilon_{ij},\delta_{ij}}\right) = 0_{7,9}$$

i.e. $\overline{r} = \underline{r}$.

Thus, fuzzy optimal strategies of player I and II is 2 and fuzzy game have saddle point $0_{8,9}$.

Example7. Consider following fuzzy two-person zero-sum game

Player A \ Player B	I	II	III	Row Minimum
I	$3_{1,2}$	$-4_{2,3}$	$8_{1,2}$	$-4_{2,2}$
II	$-8_{2,1}$	$5_{6,1}$	$-6_{2,2}$	$-8_{6,1}$
III	$6_{2,2}$	$-7_{1,2}$	$6_{2,2}$	$-7_{3,2}$
Column Maximum	$6_{1,3}$	$5_{1,3}$	$8_{1,5}$	

Maximin Value $(\underline{r}) = -4_{2,2}$

Minimax Value $(\overline{r}) = 5_{1,2}$

As discussed in above example, $\underset{j}{\wedge}\left(\underset{i}{\vee} r^{ij}_{\varepsilon_{ij},\delta_{ij}}\right) = 5_{1,2}, \underset{i}{\vee}\left(\underset{j}{\wedge} r^{ij}_{\varepsilon_{ij},\delta_{ij}}\right) = -4_{2,2}$

Also, $\underset{j}{\wedge}\left(\underset{i}{\vee} r^{ij}_{\varepsilon_{ij},\delta_{ij}}\right) \ge \underset{i}{\vee}\left(\underset{j}{\wedge} r^{ij}_{\varepsilon_{ij},\delta_{ij}}\right)$

Such fuzzy games are called fuzzy games without saddle point.

Example 8. Consider following fuzzy two-person zero-sum game

Player A \ Player B	I	II	Row Minimum
I	$-1_{11,12}$	$6_{9,10}$	$-1_{11,10}$
II	$2_{11,14}$	$4_{8,9}$	$2_{11,11}$
III	$-2_{14,11}$	$-6_{10,9}$	$-6_{10,9}$
Column Maximum	$2_{11,14}$	$6_{9,10}$	

Maximin Value $(\underline{r}) = 2_{9,10}$

Minimax Value $(\overline{r}) = 2_{11,14}$

As discussed in above example, $\underset{j}{\wedge}\left(\underset{i}{\vee} r^{ij}_{\varepsilon_{ij},\delta_{ij}}\right) = 2_{11,14} = \underset{i}{\vee}\left(\underset{j}{\wedge} r^{ij}_{\varepsilon_{ij},\delta_{ij}}\right) = 2_{9,10} \neq 0_{\varepsilon,\delta}$

The fuzzy optimal solution to the fuzzy game is given by

1) The fuzzy optimal strategy for player A is II and the fuzzy optimal strategy for player B is I
2) The value of the game is $2_{9,10}$ for player A and $-2_{10,9}$ for player B.
3) Also, $\underset{j}{\wedge}\left(\underset{i}{\vee} r^{ij}_{\varepsilon_{ij},\delta_{ij}}\right) = \underset{i}{\vee}\left(\underset{j}{\wedge} r^{ij}_{\varepsilon_{ij},\delta_{ij}}\right) = 2_{9,10} \neq 0_{\varepsilon,\delta}$,the fuzzy game is not fair.

5. Conclusion

In the present paper we considered a fuzzy two-person zero-sum game with fuzzy payoffs having cell entries epsilon-delta fuzzy numbers. We defined the ordering of $a_{\varepsilon_1,\delta_1}$ and $b_{\varepsilon_2,\delta_2}$. The optimal strategies are obtained.

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