Application to Differential Transformation Method for Solving Fourth Order Ordinary Differential Equations

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Abstract: In this paper we present zhou's method (DTM) for solving the initial value problems involving fourth order ordinary differential equations fourth order initial value problems involving fourth order ordinary differential equations we introduce the concept of DTM & applied it to obtain solution of three numerical examples for demonstration. The results are compared with exact solution & DTM method results. There results show that the technique introduced here is accurate & easy to apply.

Keywords*- Ordinary differential equitationszhou's Method (DTM), Initial value problem*

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I. INTRODUCTION

The purpose of this paper is to employ the DTM method on examples of ordinary differential equation of fourth order and compared with result obtain by exact solution by using complimentary function & particular integral. In recent years, Abdel Halim Hassan I. used DTM method for solving differential equations(1), AriKoglu A applied DTM to obtain numerical solution of differential equations (2), Ayaz F has used DTM to find the series solution of system of differential equitation (3), Bert W. has applied DTM on system of linear equation and analysis of its solutions (4), Chen used DTM to obtain the solutions of nonlinear system of differential quotations(5), Chen C.L. has applied DTM technique for steady nonlinear beat conduction problems (6), Duen Y use DTM for Burger's equation to obtain the series solution (7), Using DTM Hassan have find out series solution and that solution compared with DTM method for linear & non linear initial value problems & proved that DTM is reliable tool to find numerical solution (8), Khaled Batiha has been used DTM to obtain the Taylor's series as solution of linear, nonlinear system of ordinary differential equations (9), Kou B has been used to find numerical solution of the free convection problems (10), MontriThangmoon has been used to find numerical solution of ordinary differential equations (11), DTM was first proposed by Zhou & Proved that DTM is an iterative procedure for obtaining analytic Taylor's series solution of differential equations DTM is useful to solve ordinary diff equations. & boundary value problems (12), Bizar J. used for Riccati differential equation (13), Opanuga On numerical solution of systems of ordinary differential equitations by numeriacla analytical method.(14), Edeki, A semi method for solutions of a certain class of second order ordinary differential equations(15), Gbadeyanand Agboola for Dynamic behaviour of a double Rayleigh beam-system due to uniform partially distributed moving load(16).

II. DTM METHOD

A. Basic Definitions & Properties of DTM method

u(t) can be expressed by Taylor's series, then u(t) can be represented as

$$u(t)= \sum_{k=0}^{\infty} \frac{(t-ti)^k}{k!} U(k) =$$

u(t) is called inverse of U(k)

$$\therefore u(t) = \sum_{k=0}^{\infty} \left[\frac{(t-ti)^k}{k!}\right] U(k) = D^{-1} U(k)$$

$$u(t)= \sum_{k=0}^{\infty} \left[\frac{(t-ti)^k}{k!}\right] U(k) + R_{n+1}(t)$$

by Taylor's Series

$$u(k) = \frac{1}{k!}\left[\frac{d^k u(t)}{dt^k}\right] \text{at} \quad t = t_0$$

B.Fundamental Theorems on DTM

Theorem 1:- If z (t) = u(t) $\pm$ v(t) then
z (k) = U (k) $\pm$V(k)

Theorem 2:-If z(t) = 4(t) then $\propto$ u (t) then
z (k) = $\propto$ U(k)

Theorem 3:- Ifz(t) = $\frac{du(t)}{dt}$ then

z(k) = (k+1) U (k+1)

Theorem 4:- If z (t) = $\frac{d^2u(t)}{dt^2}$ then

z(k) = (k+1) (k+2) (k+2)U (k+2)

Theorem 5:- If z (t) = $\frac{d^nu(t)}{dt^n}$ then z(k)=(k+1) (k+2) (k+3)… (k+n) U (K+n)

Theorem 6:- Ifz(t) = u(t) v(t) then

$$z(k) = \sum_{l=0}^{k} V(l)\, U(k-l)$$

Theorem7:- Ifz (t) = t^n them

z(k) = δ (k-n)

$$\delta(k-n) = \begin{cases} 1 \text{ if } k = n \\ 0 \text{ if } k \neq n \end{cases}$$

Theorem8:- If z(t) = $e^{\lambda t}$ them

$$z(k) = \frac{\lambda^k}{k!}$$

Theorem 9:-Ifz(t) = $(1+ t)^n$ then

$$z(k) = \frac{M(n-1).....(n-k+1)}{k!}$$

Theorem 10:- Ifz(t) = sin (wt + ∝) then

$$z(k) = \frac{w^k}{k!} \sin\left(\frac{\pi k}{2} + \propto\right)$$

Theorem 11:-Ifz(t) = cos (wt + ∝) then

$$z(k) = \frac{w^k}{k!} \cos\left(\frac{\pi k}{2} + \propto\right)$$

Theorem 12 :- IfU(k) = D [u(t)]

V(k) = D [v(t)]' & C_1, C_2

Are independent of t, k then

D [c_1,u(t)+c_2 v(t)] = c_1, U(k) + c_2V(k)

Theorem 13:- If z (t) = u(t) v(t)

u (t) = D^{-1} [U (k)]

v (t) = D^{-1} [U (k)]

D [z(t)] = D [u(t) v(t)]

$$= U(k)\, V(k) = \sum_{l=0}^{k} V(l)\, U(k-l)$$

$l = 0$

Theorem 14:- D [u^m (t)] = U^m (k)

= U^{m-1} (k) *U (k)

$$= \sum_{l=0}^{k} U^{m-1}(k)\, U(k-l)$$

III. NUMERICAL EXAMPLES

Example 1.Solve fourth order ordinary differential equation.

u^{iv} (t) = u

The initial conditions are

u(0) = 1, u^1(0)= -1, u^{11} (0) =1, u^{111} (0) =-1

→ Exact solution of

(D^4 – 1)u = 0

A.E (D^4 – 1)

D = ± 1, $\pm i$

C.f = $c_{1e}{}^t$ + c_{2e} -t + e^{0t} [$c_3$cos t + c_4 sin t]

P.I = 0

G.S. = C.F. + P.I

∴u = $c_{1e}{}^t + c_{2e}{}^{-t} + c_3\cos t + c_4\sin t$

$u^I = c_{1e}{}^t - c_{2e}{}^{-t} - c_3 \sin t + c_4 \cos t$

$u^{II} = c_{1e}{}^t + c_{2e}{}^{-t} - c_3 \cos t - c_4 \sin t$

$u^{III} = c_{1e}{}^t - c_{2e}{}^{-t} + c_3 \sin t - c_4 \cos t$

usns initial condition & solving

c_1 =0, c_2 =1

$c_{3=0}$, $c_{4=0}$

u = $\bar{e}^t$ is exact sol^n

$$= 1 - t + \frac{t2}{2!} - \frac{t3}{3!} + \frac{t4}{4!} - \frac{t5}{5!} + \ldots\ldots\ldots$$

Now solution by zhou's Method

u^{iv} (t) = u

U (K) = (k+1) (k+2) (k+3) (k+4)' U (k+4) by then

U (0) = 24 U (4)

U(0) = 1

U(1) = -1

$$U(2) = \frac{1}{2}$$

$$U(3) = \frac{-1}{6}$$

$$U(4) = \frac{1}{24}$$

$$U(5) = \frac{-1}{120} =$$

$$U(6) = \frac{Y2}{(3.4.5.6)} = \frac{U(k)}{(k+1)(k+2)(k+3)(k+4)} = \frac{1}{720}$$

$$U(7) = \frac{-1/6}{4.5.6.7} = \frac{-1}{6x4x5x6x7} = \frac{-1}{5040}$$

$$U(8) = \frac{1/24}{5.6.7.8} = \frac{1}{40320}$$

The Series Sol ^{n} is by Def ^{n}2

$$u(t) = \sum_{k=0}^{n} U(k)\, t^k$$

= U (0) + U(1) t + U(2)t^2

+ U (3) t^3 + U(4) t^4 + U(5) t^5+ ……..

$$= 1 - t + \frac{1}{2}t^2 - \frac{1}{6}t^3 + \frac{1}{24}t^4 - \frac{1}{-120}t^5 + \frac{1}{720}t^6 - \frac{1}{5040}t^7 + \ldots\ldots\ldots$$

Table 1:Numerical result for example 1.

t	Exact Sol ^{n}	D.T.M.	DTM Error
0.1	0.904837418	0.904837418	0.000000000
0.2	0.818730753	0.818730753	0.000000000
0.3	0.740818220	0.740818220	0.000000000
0.4	0.670320046	0.670320046	0.000000000
0.5	0.606530659	0.606530659	0.000000000
0.6	0.548811636	0.548811636	0.000000000
0.7	0.496585303	0.496585303	0.000000000
0.8	0.449328964	0.449328964	0.000000000
0.9	0.406569659	0.406569659	0.000000000
1.0	0.367879441	0.367879441	0.000000000

Example 2. Solve $u^{iv}(t) = e^t$ $0 \angle t \angle 1$
Subject to $u(0) = 3$, $u^1(0) = 1$, $u^{11}(0) = 5$, $u^{111}(0) = 1$

The exact soln by integration & evaluation integration constants is

$u(t) = 2 + 2t^2 + e^t = 2 + 2t^2 + 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \ldots\ldots.$

$= \quad 3 + \frac{5}{2}t^2 + \frac{1}{3!}t^3 + \ldots\ldots.$

Transforming eqn is

$(k+1)(k+2)(k+3)(k+4)\,U(k+4) = 1$

$U(K+4) = \frac{1}{(k+1)(k+2)(k+3)(k+4)}$ Thm 5, 8

$U(0) = 3$

$U(1) = 1$

$U(2) = \frac{5}{2}$

$U(3) = \frac{1}{6}$

$U(4) = \frac{1}{24}$

$U(5) = \frac{1}{120}$

$U(6) = \frac{1}{720}$

$U(7) = \frac{1}{5040}$

$U(8) = \frac{1}{40320}$

Series solution is by **def^{n}2**

$$u(t) = \sum_{k=0}^{n} U(k)\,t^k$$

$= U(0) + U(1)t^1 + U(2)t^2 + U(3)t^3 + U(4)t^4 + \ldots\ldots\ldots\ldots$

$= 3 + t + \frac{5}{2}t^2 + \frac{1}{6}t^3 + \frac{1}{24}t^4 + \frac{1}{120}t^5 + \frac{1}{720}t^6 + \frac{1}{540}t^7 + \frac{1}{40320}t^8 + \ldots\ldots\ldots\ldots..$

Table 2 : Numerical Result for example 2

t	Exact	D.T.M.	DTM Error
0.1	3.125170918	3.125170918	0.000000000
0.2	3.301402756	3.301402756	0.000000000
0.3	3.529858763	3.529858763	0.000000000
0.4	3.811824356	3.811824356	0.000000000
0.5	4.148719618	4.148719618	0.000000000
0.6	4.5421128	4.5421128	0.000000000
0.7	4.993734818	4.993734818	0.000000000
0.8	5.505494756	5.505494756	0.000000000
0.9	6.079496363	6.079496363	0.000000000
1.0	6.71805556	6.71805556	0.000000000

Example 3. $u^{iv}(t) = 3\cos t$ $0 \angle t \angle 1$

With initial conditions
$u(o)=2$, $u^I(o) = 0$, $u^{II}(o) = -2$, $u^{III}(o) = 0$

→Exact soln by integration is

$u = 3\cos t + c_1 t\, {}^3/_6 + c_2 t\, {}^2/_2 + c_3 t + c_4$
where $c_1 = 0$; $c_2 = 1$, $c_3 = 0$, $c_4 = -1$
$u(t) = 3\cos t + t\, {}^2/_2 - 1$

By DTM given D.E. is
$(k+1)(k+2)(k+3)(k+4)\;U(k+4) = \left(\frac{3}{k!}\cos\frac{k\pi}{2}\right)$ thm5, 11

$U(0) = 2$
$U(1) = 0$
$U(2) = -2$
$U(3) = 0$
$U(4) = \frac{1}{8}$
$U(5) = 0$
$U(6) = -{}^1/_{240}$
$U(7) = 0$
$U(8) = \frac{1}{13440}$
$U(9) = 0$
$U(10) = \frac{-1}{1209600}$

The Series solution is

$u(t) = 2 - t^2 + \frac{t4}{8} - \frac{t^6}{240} + \frac{t^8}{13440} - \frac{t^{10}}{1209600} + \ldots\ldots\ldots..$ by then

Table 3 : Numerical result for example 3

t	Exact	D.T.M.	DTM Error
0.1	1.990012496	1.990012496	0.000000000
0.2	1.960199734	1.960199734	0.000000000
0.3	1.91009467	1.91009467	0.000000000
0.4	1.843182982	1.843182982	0.000000000
0.5	1.757477686	1.757477686	0.000000000
0.6	1.656006845	1.656006845	0.000000000
0.7	1.539526562	1.539526562	0.000000000
0.8	1.410120128	1.410120128	0.000000000
0.9	1.269829903	1.269829903	0.000000000
1.0	1.120906911	1.120906911	0.000000000

IV. CONCLUSION

In this work we applied DTM for fourth order ordinary differential equation, it reduces the computational difficulties of other traditional methods (Laplace Transform).
DTM is best for solving initial value problems of fourth order.

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