

Availability Modeling of Two Units System subject to Degradation Perfect Repair Post Failure is Feasible Using RPGT

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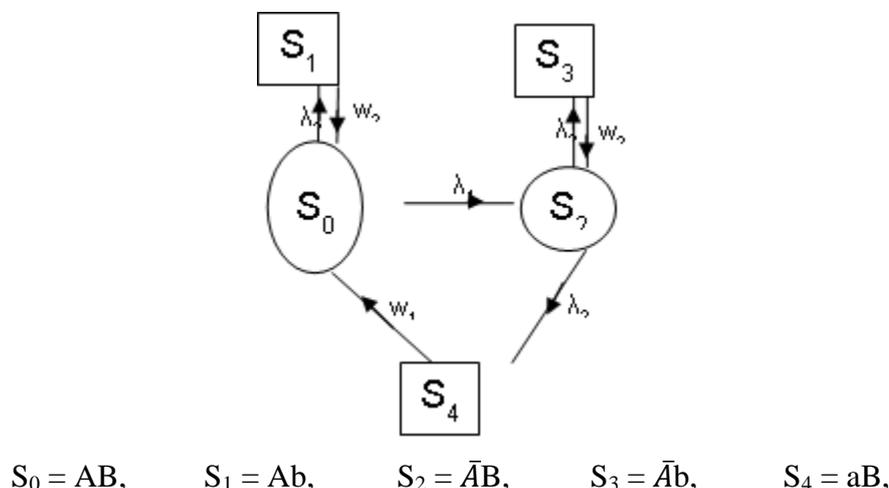
1 Introduction: -In this chapter, the reliability model for availability analysis of two unit system out of which one unit undergoing degradation after complete failure is done. Here the main unit 'A' is repairable only of the complete failure and is in degrade state post repair. Unit 'B' may have direct failure and repair of which is considered perfect. Such situation occurs in MILK PLANT, where the main unit 'A' is milk supply unit and unit 'B' is cooling the milk. This situation may occur in a number of processing industries. The failure rates and repair rates are taken exponential. There is a single server (repairman), who inspects and repairs the units on each failure. On each repair unit under goes degradation, if the server reports that unit is not repairable then it is replaced by a new one, which follows a general distribution other unit have perfect repair. Using above model expression for four parameters namely Mean Time to System Failure, Availability, Busy Period of the Server and Number of Server's Visits have been determined. Using derivatives it is proved that Availability and MTSF increase with increase in repair rates and decrease with increase in failure rates while Busy Period of the Server and Number of Server's Visits increase with increase in failure rates and decrease with increase in repair rates which are in agreement with the hypothesis. Thus, this work focuses on increasing availability of the units which is helpful to the manufacturer in particular and common man in general. Fuzzy concept is used to determine failure/working state of a unit. Taking failure rates exponential, repair rates general and taking into consideration various probabilities, a transition diagram of the system is developed to find Primary circuits, Secondary circuits & Tertiary circuits and Base state. Problem is solved using RPGT to determine system parameters. System behavior is discussed with the help of graphs and tables. Particular and special cases are discussed. In this post repair is feasible.

2 Assumptions and Notations: - The following assumptions and notations are taken:

1. For one of the units repair is imperfect and repaired system is not good as new one post repair.
2. Replacement of Un-repairable unit and repair facility is immediate.
3. No two units can fail simultaneously.

3 Transition Diagram of the System

Taking into consideration the above assumptions and notations the Transition Diagram of the system is given in Figure



The possible transitions between states alongwith transition time c.d.f.'s are shown in Fig. Primary, Secondary and Tertiary Circuits associated with the system are given in Table

Primary, Secondary & Tertiary Circuits at the various

Vertex i	Primary Circuits	Secondary Circuits	Tertiary Circuits
0	(0,1,0)	Nil	Nil
	(0,2,4,0)	(2,3,2)	Nil
1	(1,0,1)	(0,2,4,0)	(2,3,2)
2	(2,0,2)	(0,1,0)	Nil
	(2,4,0,2)	Nil	Nil
3	(3,2,3)	(2,4,0,2)	(0,1,0)
4	(4,0,2,4)	(0,1,0)	Nil
		(2,3,2)	(0,1,0)

Primary, Secondary, Tertiary Circuits w.r.t. the Simple Paths (Base-State '0')

Vertex j	$(0 \xrightarrow{S_r} j): (P0)$	(P1)
0	$(0 \xrightarrow{S_1} 0): (0,1,0)$	Nil
	$(0 \xrightarrow{S_2} 0): (0,2,9,0)$	(2,3,2)
1	$(0 \xrightarrow{S_1} 1): (0,1)$	Nil
2	$(0 \xrightarrow{S_1} 2): (0,2)$	(2,3,2)
3	$(0 \xrightarrow{S_1} 3): (0,2,3)$	(2,3,2)
4	$(0 \xrightarrow{S_1} 4): (0,2,4)$	(2,3,2)

Transition Probability and the Mean sojourn times.

Transition Probabilities

$q_{i,j}^{(t)}$	$P_{ij} = q_{i,j}^{*(t)}$
$q_{0,1} = \lambda_2 e^{-(\lambda_2 + \lambda_1)t}$	$p_{0,1} = \lambda_2 / (\lambda_2 + \lambda_1)$
$q_{0,2} = \lambda_1 e^{-(\lambda_1 + \lambda_2)t}$	$p_{0,2} = \lambda_1 / (\lambda_1 + \lambda_2)$
$q_{1,0} = w_2 e^{-w_2 t}$	$p_{1,0} = 1$
$q_{2,3} = \lambda_2 e^{-(\lambda_2 + \lambda_3)t}$	$p_{2,3} = \lambda_2 / (\lambda_2 + \lambda_3)$
$q_{2,4} = \lambda_3 e^{-(\lambda_2 + \lambda_3)t}$	$p_{2,4} = \lambda_3 / (\lambda_2 + \lambda_3)$

$q_{3,2} = w_2 e^{-w_2 t}$	$p_{3,2} = 1$
$q_{4,0} = w_1 e^{-w_1 t}$	$p_{4,0} = 1$

Mean Sojourn Times

$R_i(t)$	$\mu_i = R_i^*(0)$
$R_0^{(t)} = e^{-(\lambda_1 + \lambda_2)t}$	$\mu_0 = 1/(\lambda_1 + \lambda_2)$
$R_1^{(t)} = e^{-w_2 t}$	$\mu_1 = 1/w_2$
$R_2^{(t)} = e^{-(\lambda_2 + \lambda_3)t}$	$\mu_2 = 1/(\lambda_2 + \lambda_3)$
$R_3^{(t)} = e^{-w_2 t}$	$\mu_3 = 1/w_2$
$R_4^{(t)} = e^{-w_1 t}$	$\mu_4 = 1/w_1$

4 Evaluation of Parameters: - The Mean time to system failure and all the key parameters of the system under steady state conditions are evaluated, applying Regenerative Point Graphical Technique (RPGT) and using '0' as the base-state of the system as under:

The transition probability factors of all the reachable states from the base state ' $\xi = 0$ ' are:

Probabilities from state '0' to different vertices are given as

$$V_{0,0} = (0,1,0) + [(0,2,4,0)/(1-(2,3,2))] = p_{0,1}p_{1,0} + [p_{0,2}p_{2,4}p_{4,0}/(1-p_{2,3}p_{3,2})] = 1$$

$$V_{0,1} = (0,1) = p_{0,1} = (\lambda_2/\lambda_1 + \lambda_2)$$

$$V_{0,2} = (0,2)/[1-(2,3,2)] = p_{0,2}/(1-p_{2,3}p_{3,2}) = [\lambda_1(\lambda_2 + \lambda_3)/\lambda_3(\lambda_1 + \lambda_2)]$$

$$V_{0,3} = (0,2,3)/[1-(2,3,2)] = p_{0,2}p_{2,3}/(1-p_{2,3}p_{3,2}) = [\lambda_1\lambda_2/\lambda_3(\lambda_1 + \lambda_2)]$$

$$V_{0,4} = (0,2,4)/[1-(2,3,2)] = p_{0,2}p_{2,4}/(1-p_{2,3}p_{3,2}) = [\lambda_1/\lambda_1 + \lambda_2]$$

$$D = (V_{0,0}\mu_0 + V_{0,1}\mu_1 + V_{0,2}\mu_2 + V_{0,3}\mu_3 + V_{0,4}\mu_4) = [(2w + 3\lambda)/3w\lambda]$$

MTSF(T_0): The regenerative un-failed states to which the system can transit(initial state '0'), before entering any failed state are:

'i' = 0,2 taking ' $\xi = 0$ '

$$MTSF(T_0) = \left[\sum_{i,sr} \left\{ \frac{\left\{ \text{pr} \left(\xi \xrightarrow{sr} i \right) \right\} \mu_i}{\prod_{m=1 \neq \xi} \left\{ 1 - V_{m1m1} \right\}} \right\} \right] \div \left[1 - \sum_{sr} \left\{ \frac{\left\{ \text{pr} \left(\xi \xrightarrow{sr} \xi \right) \right\}}{\prod_{m=2 \neq \xi} \left\{ 1 - V_{m2m2} \right\}} \right\} \right]$$

$$= (V_{0,0}\mu_0 + V_{0,2}\mu_2) / (V_{0,0}\mu_0 + V_{0,1}\mu_1 + V_{0,2}\mu_2 + V_{0,3}\mu_3 + V_{0,4}\mu_4)$$

Availability of the System: The regenerative states at which the system is available are 'j' = 0,2 and the regenerative states are 'i'

= 0 to 4 taking ' $\xi = 0$ ' the total fraction of time for which the system is available is given by

$$A_0 = \left[\sum_{j,sr} \left\{ \frac{\left\{ \text{pr} \left(\xi \xrightarrow{sr} j \right) \right\} f_{j,\mu_j}}{\prod_{m=1 \neq \xi} \left\{ 1 - V_{m1m1} \right\}} \right\} \right] \div \left[\sum_{i,sr} \left\{ \frac{\left\{ \text{pr} \left(\xi \xrightarrow{sr} i \right) \right\} \mu_i^1}{\prod_{m=2 \neq \xi} \left\{ 1 - V_{m2m2} \right\}} \right\} \right]$$

$$= [\sum_j V_{\xi,j}, f_j, \mu_j] \div [\sum_i V_{\xi,i}, f_j, \mu_i^1]$$

$$= (V_{0,0}f_0\mu_0 + V_{0,2}f_2\mu_2) / (V_{0,0}\mu_0 + V_{0,1}\mu_1 + V_{0,2}\mu_2 + V_{0,3}\mu_3 + V_{0,4}\mu_4)$$

Proportional Busy Period of the Server: The regenerative states where server 'j' = 1,3,4 and regenerative states are 'i' = 0 to 4,

taking $\xi = 0$, the total fraction of time for which the server remains busy is

$$B_0 = \left[\sum_{j,sr} \left\{ \frac{\left\{ \text{pr} \left(\xi \xrightarrow{sr} j \right) \right\} n_j}{\prod_{m=1 \neq \xi} \left\{ 1 - V_{m1m1} \right\}} \right\} \right] \div \left[\sum_{i,sr} \left\{ \frac{\left\{ \text{pr} \left(\xi \xrightarrow{sr} i \right) \right\} \mu_i^1}{\prod_{m=2 \neq \xi} \left\{ 1 - V_{m2m2} \right\}} \right\} \right]$$

$$= [\sum_j V_{\xi,j}, n_j] \div [\sum_i V_{\xi,i}, \mu_i^1]$$

$$= (V_{0,1}\mu_1 + V_{0,3}\mu_3 + V_{0,4}\mu_4) / (V_{0,0}\mu_0 + V_{0,1}\mu_1 + V_{0,2}\mu_2 + V_{0,3}\mu_3 + V_{0,4}\mu_4)$$

Expected Fractional Number of Inspections by the repair man in unit time: The regenerative states where the repair man does this job $j = 1, 3, 4$ the regenerative states are $i = 0$ to 4 , Taking ' ξ ' = '0', the number of visit by the repair man is given by

$$V_0 = \left[\sum_{j, sr} \left\{ \frac{\{pr(\xi^{sr} \rightarrow j)\}}{\prod_{k_1 \neq \xi} \{1 - V_{k_1 k_1}\}} \right\} \right] \div \left[\sum_{i, sr} \left\{ \frac{\{pr(\xi^{sr} \rightarrow i)\} \mu_i^1}{\prod_{k_2 \neq \xi} \{1 - V_{k_2 k_2}\}} \right\} \right]$$

$$= [\sum_j V_{\xi,j}] \div [\sum_i V_{\xi,i}, \mu_i^1] = (V_{0,1}\mu_1 + V_{0,3}\mu_3 + V_{0,4}\mu_4) / (V_{0,0}\mu_0 + V_{0,1}\mu_1 + V_{0,2}\mu_2 + V_{0,3}\mu_3 + V_{0,4}\mu_4)$$

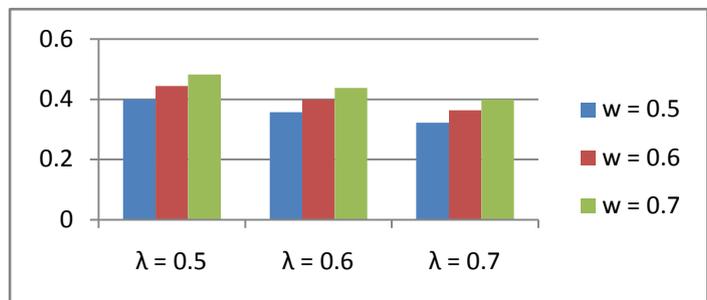
5 Particular Cases

MTSF (T_0) = $[(1/\lambda_1 + \lambda_2) + \{(\lambda_2/\lambda_2 + \lambda_3)/(\lambda_3/\lambda_1 + \lambda_2)\}(1/\lambda_2 + \lambda_3)]/D = [2w/(2w + 3\lambda)]$

MTSF Table

T_0	$w = 0.5$	$w = 0.6$	$w = 0.7$
$\lambda = 0.5$	0.400000	0.444444	0.482759
$\lambda = 0.6$	0.357143	0.400000	0.437500
$\lambda = 0.7$	0.322581	0.363636	0.400000

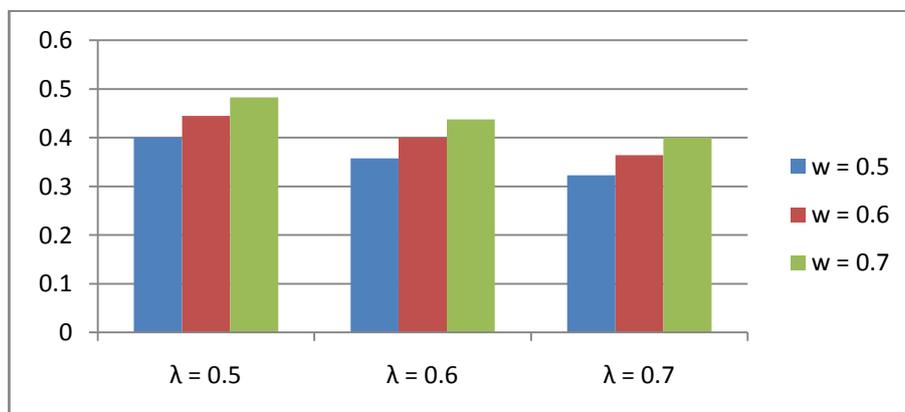
MTSF Graph



Availability of the System (A_0) Availability of the System Table

A_0	$w = 0.5$	$w = 0.6$	$w = 0.7$
$\lambda = 0.5$	0.400000	0.444444	0.482759
$\lambda = 0.6$	0.357143	0.400000	0.437500
$\lambda = 0.7$	0.322581	0.363636	0.400000

Availability of the System Graph



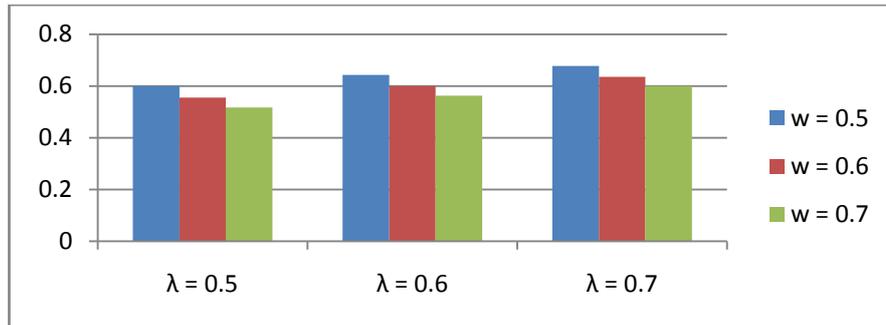
Busy Period of the Server (B_0)

= $[(\lambda_2/\lambda_1 + \lambda_2)(1/w_2) + \{(\lambda_1\lambda_2)/\lambda_3(\lambda_1 + \lambda_2)\}(1/w_2) + (\lambda_1/\lambda_1 + \lambda_2)(1/w_1)]/D = [3\lambda/(2w + 3\lambda)]$

Busy Period of the Server Table

B_0	$w = 0.5$	$w = 0.6$	$w = 0.7$
$\lambda = 0.5$	0.600000	0.555556	0.517241
$\lambda = 0.6$	0.642857	0.600000	0.562500
$\lambda = 0.7$	0.677414	0.636363	0.600000

Busy Period of the Server Graph

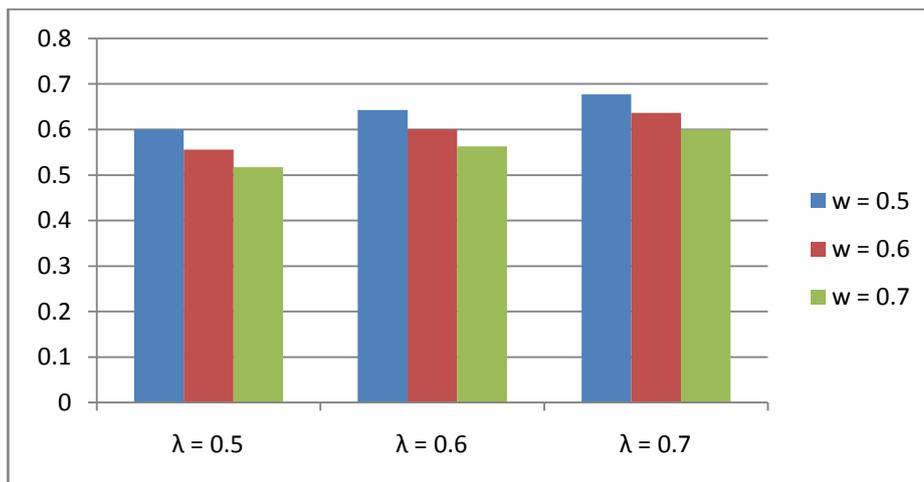


Expected Number of Server's Visits (V_0)

Expected Number of Server's Visits Table

V_0	$w = 0.5$	$w = 0.6$	$w = 0.7$
$\lambda = 0.5$	0.600000	0.555556	0.517241
$\lambda = 0.6$	0.642857	0.600000	0.562500
$\lambda = 0.7$	0.677414	0.636363	0.600000

Expected Number of Server's Visits Graph



Profit Function

$$= A_0R_0 - (B_0R_1 + V_0R_2) = A_0R_0 - B_0R_1 - V_0R_2$$

Where A_0 = Availability of System, B_0 = Busy Period of Server

V_0 = Expected Number of Inspection by the Repair Man, R_0 = Revenue

R_1 = Busy Period per Unit, R_2 = Rate of calling for a visit

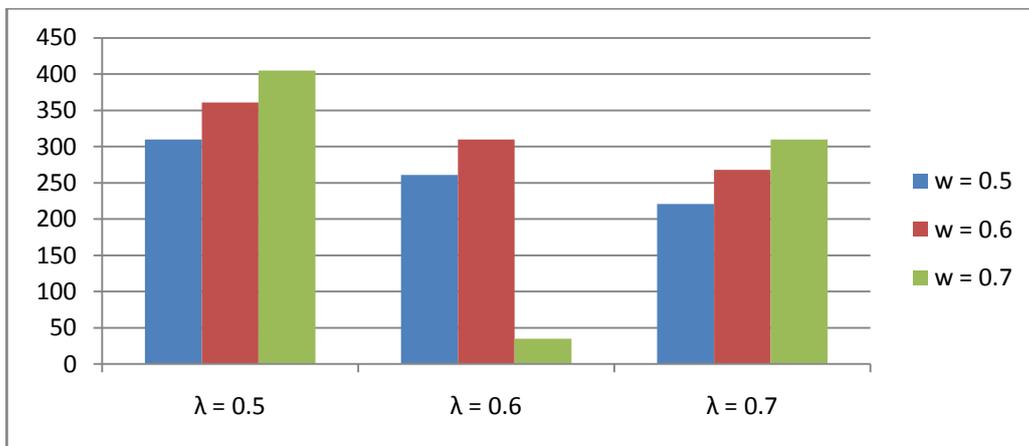
$R_0 = 1000, R_1 = 50, R_2 = 100$

$$\text{Profit} = [2000w(w+\lambda)/(2w^2+3\lambda w+4\lambda^2)] - 50 - [100w^2(2w^2+3\lambda w+4\lambda^2)] - 100 - [200w(w+\lambda)/2(w+\lambda)^2 + (2\lambda+w^2)]$$

	w = 0.5	w = 0.6	w = 0.7
$\lambda = 0.5$	310.0000	361.111066	405.17285
$\lambda = 0.6$	260.71445	310.00000	35.0125
$\lambda = 0.7$	220.9689	268.18155	310.000

Profit Table

Profit Function Graph



6 Conclusion: using these tables and graph we observe that the result obtained using Regenerative Point Graphical Technique is same as obtained other techniques.using RPGT, the results very easily and quickly without writing any state equations and without any lengthy procedures. Such situations occur in Milk Plant, Power Plant and many others industries. It is proved that Availability and MTSF increase with increase in repair rates and decrease with increase in failure rates while Busy Period of the Server and Number of Server's Visits increase with increase in failure rates and decrease with increase in repair rates. Results derived in corollary match with the results obtained by other researchers and practically possible and other results may also be obtained when there is no repair or no failures.