Extension of Stoke's Theorem

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Abstract – This paper makes an attempt to understand the relation between Green's theorem which is applied on two dimesional vector field and Stoke's theorem which generalizes in three-dimensional vector field. C is the curve in xy-plane and our region D becomes surface S in the xy-plane whose boundary is the curve C . We know by Stoke's theorem

$$\int F.dr = \iint (\nabla \times F).N\, ds$$

N is outward normal to the surface .But in two dimensional vector field F = P i +Q j+ 0 k When the surface is on xy-plane N = k

Microscopic circulation = $\int$ F.dr = $\iint (\nabla \times F).k\, dxdy$

$$\nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & 0 \end{vmatrix} = 0 + 0 + k\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)$$

$$\int F.dr = \iint k\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right).k\, dxdy$$

So Stoke 's theorem is equivalent to Green's theorem when the surface is on xy-plane . **Key words** : Three dimensional vector field, Stoke's theorem , Angle of inclination

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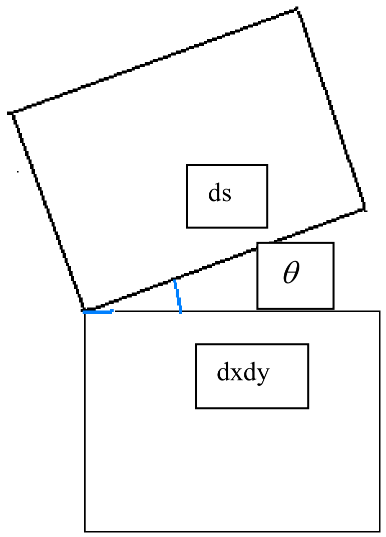
I. Introduction

I have tried to prove a different mathematical expression for Stoke's theorem when the surface is inclined to xy –plane or yz-plane or zx plane by an acute angle θ .

.If S is an open two sided surface bounded by a simple closed curve C on X-Y plane and if the vector F over the surface S has continuous derivatives on X-Y plane then

$$\oint F.dr = \oiint_S N.(\nabla \times F)\frac{dxdy}{\cos\theta}$$

Where N is the outword normal vector to the elementary surface ds . θ is the acute angle between the surface and X-Y plane .



dxdy = ds cos θ

ds = $\dfrac{dxdy}{\cos\theta}$

Similarly if the projection will be taken on Y-Z plane then

ds = $\dfrac{dydz}{\cos\theta}$

Similarly if the projection will be taken on X-Z plane then

ds = $\dfrac{dxdz}{\cos\theta}$

Example : Verify Stoke's theorem for F = (2x-y) i - yz ^{2} j -y ^{2} z k where S is the upper half surface of the surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is the boundary of the surface .

Solution : The boundary C of S is a circle in the X-Y Plane of radius 1 and centre at the origin . Let R be the projection of S on the X-Y-plane . Let x= cost ,y =sint ,z = 0 ; $0 \le t \le 2\pi$ be the para- metric equations of C .
Then

$$\oint F.dr = \oint (2x-y)dx - yz^2 dy - y^2 zdz$$

$$= \int_0^{2\pi} (2\cos t - \sin t)(-\sin t\ dt)$$

$$= \pi$$

Also $\nabla \times F = \begin{vmatrix} i & j & k \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 2x-y & -yz^2 & -y^2 z \end{vmatrix} = $ k

Then $\iint_s (\nabla \times F).N \dfrac{dxdy}{\cos\theta}$

$$= \iint k.N \frac{dxdy}{\cos\theta}$$

(But k .N = $|k||N| \cos\theta$ where N unit out word normal to the surface.)

$$= \iint |k||N| \cos\theta \frac{dxdy}{\cos\theta}$$

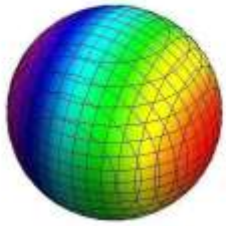
$$= 4\iint dxdy$$

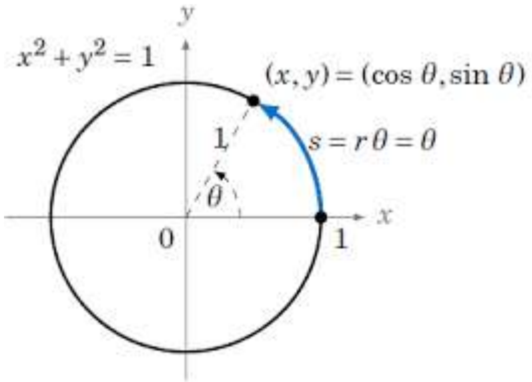
where inner integral varies from y = 0 to y = $\sqrt{1-x^2}$ and outer integral varies from 0 to 1 .

$$= 4 \int_0^1 \sqrt{1-x^2}\, dx$$

$$= 4 \int_0^{\frac{\pi}{2}} \cos\theta . \cos\theta\ d\theta = 4\ \frac{1}{2} \text{B} \left(\frac{3}{2}, \frac{1}{2}\right)$$

$$= \pi$$





II. CONCLUSION

In my view of extension of Stoke's theorem I have tried to prove a new relationship of line integral and surface integral when the surface is inclined to X-Y plane or Y-Z plane or Z-X plane by an acute angle θ .

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