Parametric Optimization of Average Coefficient of Drag using Taguchi Method

Mohammed Shihab Patel, Mrunalini Deshmukh and Shantiaum Khatavkar
Department of Civil Engineering
Shree L R Tiwari College of Engineering
Thane-401107, Maharashtra, India
shihabpatel91@gmail.com

Abstract - This paper deals with minimization of average coefficient of drag by optimizing flow parameters. Taguchi's Method of optimization is implemented. This method of optimization is a statistical method aimed to minimize the average coefficient of drag. The study for minimization is focused on three parameters, viz length, kinematic viscosity and free stream velocity. Accordingly a suitable orthogonal array was selected and average coefficient of drag was calculated followed by computation of signal to noise ratio. The manual method of calculation was validated with minitabs software and with the help of graph, optimum parameters value were then obtained. The variations of parameters of study were done arbitrarily.

Index Terms – *Reynolds Number, Coefficient of drag, free stream velocity, optimization, taguchi method.*

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I. INTRODUCTION

Taguchi method is a statistical method developed by Taguchi and Konishi [1]. Initially it was developed for improving the quality of goods manufactured (manufacturing process development), later its application was expanded to many other fields in Engineering, such as Biotechnology [2] etc. Professional statisticians have acknowledged Taguchi's efforts especially in the development of designs for studying variation. Success in achieving the desired results involves a careful selection of process parameters and bifurcating them into control and noise factors. Selection of control factors must be made such that it nullifies the effect of noise factors. Taguchi Method involves identification of proper control factors to obtain the optimum results of the process. Orthogonal Arrays (OA) are used to conduct a set of experiments. Results of these experiments are used to analyze the data and predict the quality of components produced. The Taguchi method is best used when there are an intermediate number of variables (3 to 50), few interactions between variables, and when only a few variables contribute significantly.

We have made an attempt to minimize the energy loss per meter length of the spillway. Energy loss per meter length of the spillway is product of density (ρ), energy thickness (δ_e)and free stream velocity (U).

II. DESIGN OF EXPERIMENTS

Classical experimental design methods are too complex and are not easy to use. A large number of experiments have to be carried out when the number of process parameters increase. To solve this problem, the Taguchi method uses a special design of orthogonal arrays to study the entire parameter space with only a small number of experiments. Three parameters are selected as controlling factors. They are density, energy thickness and free stream velocity.

A. Selection of Orthogonal Array

Orthogonal arrays are special standard experimental design that requires only a small number of experimental trials to find the main factors effects on output. Before selecting an orthogonal array, the minimum number of experiments to be conducted is to be fixed based on the formula below

$$E= 1+ P (L – 1) \quad (1)$$

E = Number of experiments to be conducted
P = Number of parameters
L = Number of levels

In this work,
P = 3 and L = 3, Hence E = 1+ 4 (3-1) = 9

The following standard orthogonal arrays are commonly used to design experiments:

2-Level Arrays: L4, L8, L12, L16, L32
3-Level Arrays: L9, L18, L27
4-Level Arrays: L16, L32

In this work L9 is sufficient. It would require a total of 27 experiments to optimize the parameters. Taguchi experimental design of experiments suggests L9 orthogonal array, where 9 experiments are sufficient to optimize the parameter s. Based on main factor, the variables are assigned at columns, as stipulated by orthogonal array. The performance parameter (output) is noted for each experimental run for analysis. Once the orthogonal array is selected the experiments are selected as per the level combinations.

B. S/N Ratio

In Taguchi's design method the design parameters (factors that can be controlled by designers) and noise factors (factors that cannot be controlled by designers, such as environmental factors) are considered influential on the product quality. The Signal to Noise (S/N) ratio is used in this analysis which takes both the mean and the variability of the experimental result into account. The S/N ratio depends on the quality characteristics of the product/process to be optimized. Usually, there are three categories of the performance characteristics in the analysis of the S/N ratio; that is, the lower-the-better, the higher-the-better, and the nominal-the-better.

The S/N ratio for each response is computed differently based on the category of the performance characteristics and hence regardless of the category the larger S/N ratio corresponds to a better performance characteristic. In the present study, the main concern of optimization is the coefficient of drag. Hence, the parametric study of average coefficient of drag using controlled factors like length, kinematic viscosity and free stream velocity are the lower-the-better performance characteristics. The mathematical relation for the lower-the-better performance characteristic is given by:

$$\text{S/N ratio } (\eta) = -10 \log [ny^2] \text{ (where n = 1)} \qquad (2)$$

Once all of the S/N ratios have been computed for each run of an experiment, Taguchi advocates a graphical approach to analyse the data. In the graphical approach, the S/N ratios and average responses are plotted for each factor against each of its levels. These S/N ratios were verified with the mini tabs software.

C. *Effect of parameters on study of average coefficient of drag*

The coefficient of drag assuming Blasius velocity profile is given by the following mathematical relation

$$C_D = \frac{1.328}{\sqrt{\frac{UL}{\nu}}} \qquad (3)$$

Where, L is length of flat plate kept at zero angle of incidence in 'm', ν is the kinematic viscosity in 'cm^2/s'and U is the free stream velocity in 'm/s'.

D. *Minitab Software*

Minitab is statistical analysis software. It can be used for learning about statistics as well as statistical research. Statistical analysis computer applications have the advantage of being accurate, reliable, and generally faster than computing statistics and drawing graphs by hand. Minitab is relatively easy to use once you know a few fundamentals.

III. RESULTS

The following variations in the parameters were selected. Table I show the parameters value considered for the energy loss per meter length of the spillway.

Table I
Parameters of Study for optimization

Parameter/Level		1	2	3
A	Length (m)	20	30	40
B	Kinematic viscosity (cm^2/s)	0.154	0.156	0.16
C	Free Stream Velocity (m/s)	25	30	35

Calculation of S/N ratio

As stated before, L9 orthogonal array is used to conduct the experimental values. The following table II tabulates the S/N ratio values for different combination of levels.

Table II
Experimental Values of E_L and S/N calculated manually

Exp. No	A	B	C	L	ν	U	C_D	S/N
1	1	1	1	20	0.154	25	0.023306	-32.6505
2	1	2	2	20	0.156	30	0.021413	-33.3863
3	1	3	3	20	0.16	35	0.020077	-33.9458
4	2	1	2	30	0.154	30	0.017371	-35.2033
5	2	2	3	30	0.156	35	0.016186	-35.8167
6	2	3	1	30	0.16	25	0.019396	-34.2455
7	3	1	3	40	0.154	35	0.013928	-37.1221
8	3	2	1	40	0.156	25	0.016586	-35.6048
9	3	3	2	40	0.16	30	0.015334	-36.2867

Equation (2) and (3) were used to calculate the values of S/N ratio and C_D respectively.

A. Response Table for S/N ratio

After this Response table and graph is generated. Response table consist of mean S/N ratio for each level of factors length, kinematic viscosity and free stream velocity. In simple terms it gives average values for each level of factors. Overall mean S/N ratio is also calculated. Response table also provide rank to the factors. This is done with the help of delta statistics calculation. Delta is nothing but the value obtained by taking difference of highest average (Max. mean) and lowest average (Min mean) of each factor. After this, rank is assigned on these deltas values.

Rank 1 is assigned to the highest delta value, Rank 2 is assigned to second highest delta value and Rank 3 is assigned to the lowest delta value. Following table I*V* shows the mean S/N ratio for each factor, Delta values and Ranks calculated.

Table I*V*
Response Table of E_L for Signal to Noise Ratio: - Lower-the-better

Level	L	ν	U
1	-33.3276*	-34.992	-34.1669*
2	-35.0885	-34.9359	-34.9587
3	-36.3379	-34.826*	-35.6282
Delta (max-min)	3.0103	0.165993	1.46128
Rank	1	3	2

After the Response table generation, graph is plotted. This is called as Main effect plot for S/N ratio. Maximum value of each factor is chosen from the graph. This can also be found out from response table by selecting highest value of each factor which is shown by asterisk (*) sign. From both by Response table and by Graph, the optimum combination obtained is A1B3C1.

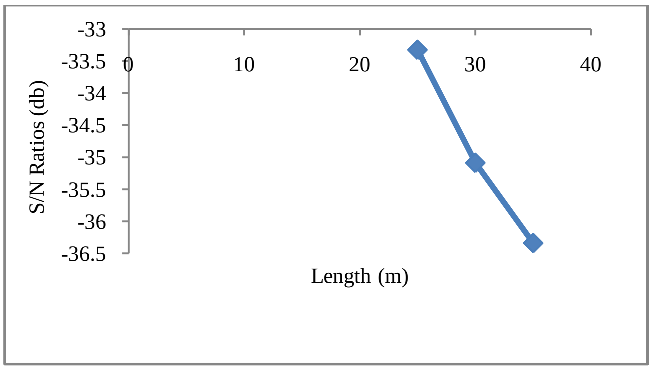

Fig. 1 Main effect plot of length for S/N ratios

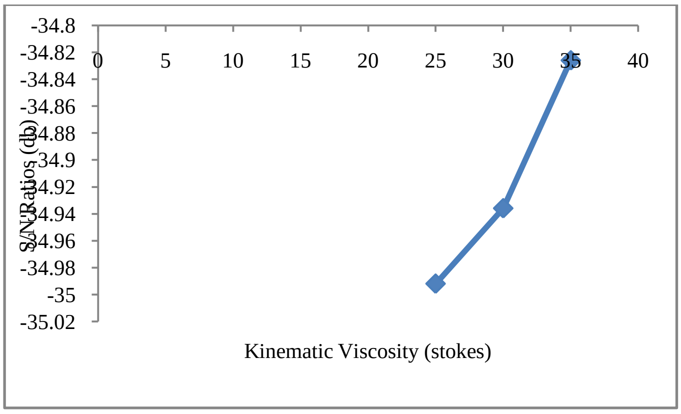

Fig. 2 Main effect plot of energy thickness for S/N ratios

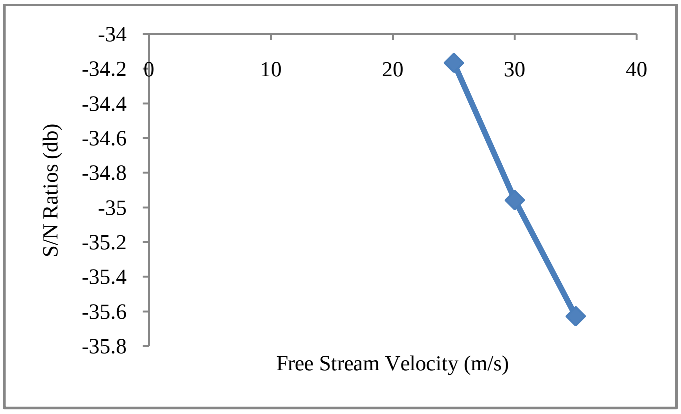

Fig. 3 Main effect plot of free stream velocity for S/N ratios

B. ANNOVA

After obtaining the rank of factors through Response table, Analysis of Variance is conducted. ANNOVA or Analysis of Variance is the technique used to give conclusion on the experimental data. Basically, ANNOVA is used to give which factor is significant or which factor has most significant effect.

Table *V*
Table for ANNOVA of C_D - Lower-the-better

Source	DF	SS	MS	% contribution
Length	2	13.7236	6.8618	80.96
Kinematic viscosity	2	0.04278	0.0213	0.25
Free stream velocity	2	3.21049	1.60524	18.94
Error	2	0.02766	0.0138	
Total	8	16.94931		

Basically, ANNOVA is used to give which factor is significant or which factor has most significant effect. Length has 80.96% contribution, kinematic viscosity has 0.25% contribution and free stream velocity has 18.94% contribution. From the above table it is seen that free stream velocity is the most influential factor followed by energy thickness and density for loss of energy per meter length of spillway.

C. VALIDATION AND CONCLUSION

As from table II, it is seen that there is no combination of A1B3C1, We re-perform the experiment for the above said combination A1B3C1 and get the S/N ratio. The calculated C_D for the combination is 0.02375. The S/N ratio is -32.484, which is the highest value of all the combinations considered. In order to see the accuracy of the result obtained from the Minitab software, it is necessary to perform a confirmation test. This confirmation test will be used for comparing Optimum Theoretical or predicted value with the Experimental value. To obtain the Theoretical optimum value, following formula is used.

$$\eta_{optimum} = \eta_{mean} + \Sigma(\eta_j - \eta_{mean}) \quad (4)$$

Where, $\eta_{optimum}$ is optimum S/N ratio, η_j is optimal level of factors and η_{mean} is overall mean S/N ratio. The combination here is A3B1C3, hence the formula becomes;

$$\eta_{optimum} = \eta_{mean} + (\eta_{A1} - \eta_{mean}) + (\eta_{B3} - \eta_{mean}) + (\eta_{C1} - \eta_{mean})$$

$$\boldsymbol{\eta_{optimum} = -32.4845}$$

The optimum S/N ratio is -32.4845; we see that optimum value matches the combination A1B3C1 respectively.

The energy loss per meter length of the spillway is optimized for the combination A1B3C1. The main effect plot for all the three considered parameters were plotted, the maximum value from the each plot was considered to be the optimum value for the considered parameters of study. This was then followed by ANNOVA, which gave the percentage contribution of each parameter of study. The confirmation check gave the optimum S/N ratio, which matched the value with the combination A1B3C1

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