

# Performance Analysis and Optimal Detection of Spatial Modulation

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**Abstract**—In this paper, we propose the optimal detector for spatial modulation. The new detector performs significantly better than the original (~4 dB gain), and we derive the closed form expression for the average bit error probability. The optimal detector of SM shows performance gain (~1.5 –3 dB) over popular multiple antenna system, making it an excellent prospect for future wireless communication.

**Keywords**—Antenna modulation, spatial modulation, maximum likelihood detection, MIMO.

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## I. INTRODUCTION

Multiple Input Multiple output (MIMO) scheme have been proposed for wireless communication system to significantly increase capacity, range and reliability when comparing with conventional single antenna system. It provides increase in data throughput and minimum probability of error without additional frequency spectrum and transmission power. MIMO systems can be categorized into Beam forming, spatial multiplexing (SM) and diversity. Several MIMO techniques, among which the Space Time Block Code (STBC) and spatial multiplexing achieving diversity and multiplexing gain. The spatial diversity gain can be exploited by STBC because of its implementation simplicity and low decoding complexity. The maximum likelihood (ML) decoder with linear complexity is the main attraction of orthogonal STBC (OSTBC). Full-rate full-diversity code for more than two transmit antennas with linear complexity is proven impossible to be constructed. Maximum multiplexing gain by simultaneous transmission over all antennas can be achieved using V-BLAST scheme. The joint ML decoding provides high capacity for the data stream, but the complexity increases with the number of streams. The error performance of the system can be significantly reduced using linear sub-optimal decoders for V-BLAST, such as linear minimum mean square error (MMSE), successive cancellation, but the inter channel interference (ICI) and Inter antenna interference (IAI) increases. Spatial Modulation (SM) is a recently developed low-complexity Multiple-Input Multiple-Output scheme that uses antenna indices and a conventional signal set to convey information. It has been shown that the Maximum-Likelihood (ML) detection in an SM system involves joint detection of the transmit antenna index and the transmitted symbol, and hence, the ML search complexity grows linearly with the number of transmit antennas and the size of the signal set. In this paper, we show that the ML search complexity in an SM system becomes independent of the constellation size when the signal set

employed is a square- or a rectangular-QAM. Further, we show that Sphere Decoding (SD) algorithms become essential in SM systems only when the number of transmit antennas is large and not necessarily when the employed signal set is large. We propose a novel sphere decoding detector whose complexity is lesser than that of the existing detector and a generalized detection scheme for SM systems with number of transmit antennas. We support our claims with simulation results that the proposed detectors are ML-optimal and offer a significantly reduced complexity.

*Organization:* This paper is organized as follows. Section II introduces the existing system and disadvantages of this system. Section III introduces the basic system model. In section IV we introduces the proposed system and its advantages, In section V we derive the optimal detector and provide a performance analysis for the SM system. Section VI presents some simulation results and we conclude the paper in section VII.

## II. EXISTING SYSTEM AND ITS DISADVANTAGES

### A. EXISTING SYSTEM

Transmission techniques designed for multiple input multiple output (MIMO) systems, such as the Bell Laboratories layered space-time (BLAST) architecture. Due to inter-channel interference (ICI) caused by coupling multiple symbols in time and space, maximum likelihood (ML) detection increases exponentially in complexity with the number of transmit antennas. Consequently, avoiding ICI greatly reduces receiver complexity, and contributes in attaining performance gains. The so-called spatial modulation (SM), is an effective means to remove ICI and the need for precise time synchronization amongst antennas. SM is a pragmatic approach for transmitting information, where the modulator uses well known amplitude/phase modulation (APM) techniques such as phase shift keying (PSK) and quadrature amplitude modulation, but also employs the

antenna index to convey information. Only one antenna remains active during transmission so that ICI is avoided. As well, inter-antenna synchronization (IAS) during transmission is no longer needed as in the case of Vertical-BLAST (V-BLAST) [4], in which all antennas transmit symbols at the same time. A sub-optimal detection method is presented and only valid under some constrained assumptions about the channel. For conventional channels, their detector fails and even with their assumption, detection is not optimal. We present the optimal detector for SM and show that the detection is a joint optimization problem that cannot be separated. We analyze the performance of the SM system and derive a closed form expression for the bit error probability when real constellations are used. As well, prior to this work, SM's advantages lied in removing ICI and IAS from the communication systems, where gains in performance over other schemes in the literature was not present. With optimal SM however, we show that performance gains over maximum ratio combining (MRC) and V-BLAST is observed, making the use of SM in practical systems more attractive.

### B. DISADVANTAGES

Inter-channel interference (ICI) caused by coupling multiple symbols in time and space, maximum likelihood (ML) detection increases exponentially in complexity with the number of transmit antennas.

## III. SPATIAL MODULATION

### A. SYSTEM MODEL

The general system model consists of a MIMO wireless link with  $N_t$  transmit antennas and  $N_r$  receive antennas. The general system is shown in fig 1. The random sequence of independent bits  $b$  enters the SM mapper, which groups  $B$  bits and maps them to a constellation vector  $x = [x_1 \ x_2 \ \dots \ x_{N_t}]^T$  where we assume power constraint of unity (i.e.  $E_x[x^H x] = 1$ ).

Where,  $[\cdot]^T$  represents transpose and  $E_x[\cdot]$  represents statistical expectation with respect to  $x$ .

In SM only one antenna is active at a time any other antenna is deactivated and hence only one of the  $x_i$  in  $x$  is non-zero. The signal is transmitted over an  $N_r \times N_t$  wireless channel  $H$  and experiences an  $N_r$ -dim additive white Gaussian noise  $\eta = [\eta_1 \ \eta_2 \ \dots \ \eta_{N_r}]^T$ . The received can be expressed as

$$y = \sqrt{\rho} Hx + \eta \quad (1)$$

Where,  $\rho$  is the average signal to noise ratio (SNR) at each receive antenna, and  $H$  and  $\eta$  have independent and identically distributed (iid) entries according to  $\mathcal{CN}(0,1)$ .

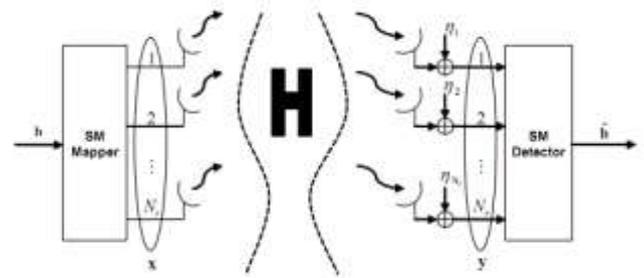


Fig. 1 General system model.

### B. SM MODULATION

As mentioned earlier, SM utilizes the antenna index as another means to transmit information. The antenna combined with symbol index make up the SM mapper. The mapper collects  $B = \log_2(MN_t)$  bits and maps them to a constellation vector.

$$x_{jq} \triangleq [0 \ 0 \ \dots \ x_q \ 0 \ \dots \ 0]^T$$

↑  
 $j^{th}$  position

Where,  $j$  is the activated antenna and  $x_q$  is the  $q^{th}$  symbol from the constellation  $\chi_M$ . Hence  $j^{th}$  antenna remains active during symbol transmission. For example in 3 bits/s/Hz transmission with  $N_t = 4$  antennas, the information bits are mapped to a  $\pm 1$  binary PSK (BPSK) symbol, and transmitted on one of the four available antennas. When  $x_a$  is transmitted from the  $j^{th}$  antenna the output of the channel is expressed as,

$$y = \sqrt{\rho} h_j x_a + \eta$$

Where,  $h_j$  is the  $j^{th}$  column of  $H$ .

### C. SM DETECTION

In [1], assuming constant modulus signaling such as PSK a sub-optimal detection rule is given by,

$$\hat{j} = \arg_j \max_{j \in \{1, \dots, N_t\}} |h_j^H y|$$

$$\hat{q} = \arg_q \max_{q \in \{1, \dots, M\}} \text{Re}\{(h_j x_q)^H y\}$$

Where,  $\hat{j}$  and  $\hat{q}$  is the estimated antenna and symbol index, respectively. Since the mapping is one to one, the demapper obtain an estimate of the transmitted bits by taking  $j$  and  $q$  as inputs. However, this detector only works for transmission over normalized channel.

## IV. PROPOSED SYSTEM AND ITS ADVANTAGES

### A. PROPOSED SYSTEM.

We propose the optimal detector for SM and show that the detection is a joint optimization problem that cannot be separated. We analyze the performance of the SM system and derive a closed form expression for the bit error probability when real constellations are used. SM's advantages lied in removing ICI and IAS from the communication systems, where gains in performance over other schemes in the

literature was not present. We show that performance gains over maximum ratio combining (MRC) and V-BLAST is observed, making the use of SM in practical systems more attractive. A MIMO wireless link with  $N_t$  transmit and  $N_r$  receive antennas. A random sequence of independent bits  $b$  enters the SM mapper. SM exploits the antenna index as an additional means to transmit information. The antenna combined with the symbol index make up the SM mapper. The MRC scheme is essentially a single input multiple output (SIMO) communication system using APM and employing an ML receiver, where we use 8-QAM to achieve the spectral efficiency requirement. V-BLAST using BPSK with  $N_t = 4$  antennas and ordered successive interference cancellation (OSIC) using the minimum mean squared error (MMSE) receiver is also compared.

### B. ADVANTAGE

SM was inferior in terms of performance over V-BLAST and MRC, and its advantages mainly lied in enabling simple detection as well as removing the need for ICI and IAS.

## V. OPTIMAL DETECTION AND PERFORMANCE ANALYSIS

### A. OPTIMAL DETECTION

Since the channel inputs are assumed equally likely, the optimal detector is ML, which is given by,

$$[\hat{J}_{ML}, \hat{q}_{ML}] = \underset{j,q}{\operatorname{argmax}} p_Y(y|x_{jq}, H) \\ = \underset{j,q}{\operatorname{argmin}} \sqrt{\rho} \|g_{jq}\|_F^2 - 2\operatorname{Re}\{y^H g_{jq}\} \quad (2)$$

Where,  $g_{jq} = h_j x_q$ ,  $1 \leq j \leq N_t$ ,  $1 \leq q \leq M$ , and  $p_Y(y|x_{jq}, H) = \pi^{-N_r} \exp\{-\|y - \sqrt{\rho} H x_{jq}\|_F^2\}$  is PDF of  $y$ , conditioned on  $x_{jq}$  and  $H$ . It can be seen that detection is a joint optimization problem which cannot easily be separated. Even with normalized channels and constant modulus signaling (i.e.  $\|g_{jq}\|_F^2 = 1$ ), the detector reduces to,

$$[\hat{J}_{ML}, \hat{q}_{ML}]_{PSK} = \underset{j,q}{\operatorname{argmax}} \operatorname{Re}\{y^H g_{jq}\}$$

### B. PERFORMANCE ANALYSIS

The performance of SM system will be derived using well known union bounding technique [5, P. 261-262]. The average bit error rate (BER) in SM is union bounded as

$$P_{e,bit} \leq E_x \left[ \sum_{\hat{j}\hat{q}} N(q, \hat{q}) P(x_{jq} \rightarrow x_{\hat{j}\hat{q}}) \right]$$

$$= \sum_{j=1}^{N_t} \sum_{q=1}^M \sum_{\hat{j}=1}^{N_t} \sum_{\hat{q}=1}^M \frac{N(q, \hat{q}) P(x_{jq} \rightarrow x_{\hat{j}\hat{q}})}{N_t M} \quad (3)$$

Where,  $N(q, \hat{q})$  is the number of bits in error between the symbol  $x_q$  and  $x_{\hat{q}}$  and  $P(x_{jq} \rightarrow x_{\hat{j}\hat{q}})$  denotes the pairwise error probability (PEP) of deciding on the constellation vector  $x_{\hat{j}\hat{q}}$  given that  $x_{jq}$  is transmitted. By simplifying (2) the PEP conditioned on  $H$  is given by,

$$P(x_{jq} \rightarrow x_{\hat{j}\hat{q}}|H) = P(d_{jq} > d_{\hat{j}\hat{q}}|H) = Q(\sqrt{\kappa})$$

Where,  $d_{jq} = (\sqrt{\rho} \|g_{jq}\|_F^2 - 2\operatorname{Re}\{y^H g_{jq}\})$  and  $Q(x) = \int_x^\infty \frac{1}{2\pi} e^{-t^2/2} dt$ . We define  $\kappa$  as

$$\kappa \triangleq \frac{\rho}{2N_r} \|g_{jq} - g_{\hat{j}\hat{q}}\|_F^2 = \sum_{n=1}^{N_r} |A(n) + iB(n)|^2 \quad (4)$$

Where,  $i = \sqrt{-1}$  and

$$A(n) = \sqrt{\frac{\rho}{2N_r}} (h_{nj}^R x_q^R - h_{nj}^I x_q^I - h_{nj}^R x_{\hat{q}}^R + h_{nj}^I x_{\hat{q}}^I)$$

$$B(n) = \sqrt{\frac{\rho}{2N_r}} (h_{nj}^R x_q^I + h_{nj}^I x_q^R - h_{nj}^R x_{\hat{q}}^I - h_{nj}^I x_{\hat{q}}^R)$$

The superscript R and I denote the real and imaginary part, respectively, and  $h_{nm}$  is the element of  $H$  in the  $n^{th}$  row, and  $m^{th}$  column. In this case, the performance can be evaluated numerically. However, for symbols  $x$  drawn from a real constellation  $X_M$ , this independence is satisfied and (4) reduces to  $\kappa = \sum_{n=1}^{2N_r} \alpha_n^2$  where,  $\alpha_n \sim \mathcal{N}(0, \sigma_\alpha^2)$  with  $\sigma_\alpha^2 = \frac{\rho(|x_q|^2 + |x_{\hat{q}}|^2)}{2N_r}$

Hence,  $\kappa$  is a chi-squared random variable with  $s = 2N_r$  degrees of freedom and PDF  $p_\kappa(\vartheta)$  given in [5, p.41]. The PEP can then be formulated as

$$P(x_{jq} \rightarrow x_{\hat{j}\hat{q}}) = E_\kappa [P(x_{jq} \rightarrow x_{\hat{j}\hat{q}}|H)] \\ = \int_{\vartheta=0}^\infty Q(\sqrt{\vartheta}) p_\kappa(\vartheta) d\vartheta \\ = \int_{t=0}^\infty \frac{\exp\{-\frac{t^2}{2}\} F_\kappa(t^2) dt}{\sqrt{2\pi}} \quad (5)$$

where the last line follows from a simple change of integration order and  $F_\kappa(y) = \int_{\vartheta=0}^y f_\kappa(\vartheta) d\vartheta$  is the chi-squared cumulative distribution function (CDF). We use the expression for  $F_\kappa(y)$  given in [5, p.42 Eq. (2.1-114)] and closed form integral expression from [6, p.337, Eq. (3.326-2)] to simplify equation (5) as

$$P(x_{jq} \rightarrow x_{\hat{j}\hat{q}}) = \frac{1 - \sum_{k=0}^{m-1} \frac{\Gamma(k')(2\sigma_\alpha^2)^{-k} \left[\frac{\mu_\alpha}{\sqrt{2}}\right]^{-2k'}}{\sqrt{2\pi} k!}$$

Where,  $\mu_\alpha = \sqrt{\frac{\sigma_\alpha^2 + 1}{\sigma_\alpha^2}}$ ,  $m = \frac{s}{2} = N_r$  and  $k' = k + \frac{1}{2}$  using [6, p.897 Eq. (8.339-2)], with some straightforward algebra, we get the PEP expression as

$$P(x_{jq} \rightarrow x_{\hat{j}\hat{q}}) = \frac{\mu_\alpha - \sum_{k=0}^{N_r-1} \binom{2k}{k} [2\mu_\alpha \sigma_\alpha]^{-2k}}{2\mu_\alpha} \quad (6)$$

Plugging in (6) into (3), we obtain

$$P_{e,bit} \leq \sum_{q=1}^M \sum_{\hat{q}=1}^M \frac{N(q, \hat{q}) \left( \mu_\alpha - \sum_{k=0}^{N_r-1} \binom{2k}{k} [2\mu_\alpha \sigma_\alpha]^{-2k} \right)}{4M\mu_\alpha} \quad (7)$$

### VI. SIMULATION RESULTS

We perform montecarlo simulation. We perform this simulation for at least  $10^5$  channel realization and plot the average BER performance versus  $\rho$ , the average SNR per receive antenna. In all schemes 3bits/s/Hz transmission with  $N_t = 4$  antennas are assumed. In fig.2 dotted lines represents constraint channel and solid line represents conventional channel. We use 8-QAM with MRC scheme using APM and employing ML-receiver. This scheme increases spectral efficiency. V-BLAST using BPSK with  $N_t = 3$  antennas and ordered successive interference constellation (OSCI) using the minimum mean square error(MMSE) receiver is also compared. SM with BPSK and  $N_t = 4$  antenna is shown for sub-optimal [1] and optimal receivers along with the SM BER bound of [7].

When simulation over conventional channels (solid line) are performed. Higher gains are achieved. Mesleh’s detector fails in achieving higher gain.

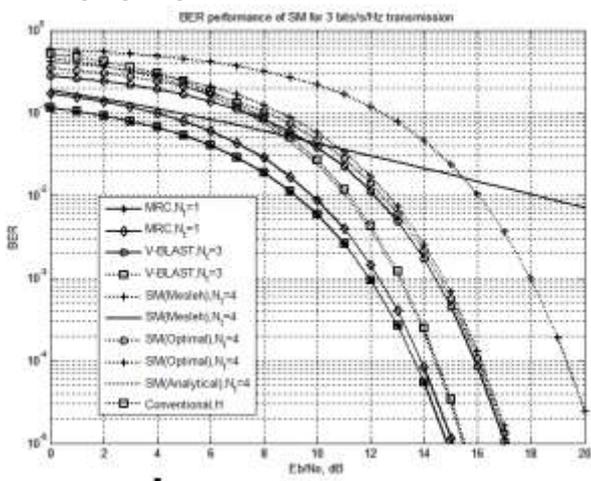


Fig. 2 BER performance of SM for 3 bits/s/Hz transmission.

### VII. CONCLUSION

In this paper, optimal detector for SM is derived performance gain of SM is observed over the detector in [1]. Closed form expression for the average BER of SM is derived. Shown in fig.2 optimal SM is better than V-BLAST and MRC. SM is an excellent candidate future communication system.

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