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Some Classes of Cubic Harmonious Graphs

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Abstract- In this paper we proved some new theorems related with Cubic Harmonious Labeling. A (n,m) graph $G=(V,E)$ is said to be **Cubic Harmonious Graph(CHG)** if there exists an injective function $f: V(G) \rightarrow \{1,2,3,\ldots\ldots m^3+1\}$ such that the induced mapping $f^*_{chg}: E(G) \rightarrow \{1^3,2^3,3^3,\ldots\ldots.m^3\}$ defined by $f^*_{chg}(uv) = (f(u)+f(v)) \bmod (m^3+1)$ is a bijection. In this paper, focus will be given on the result "cubic harmonious labeling of star, the subdivision of the edges of the star $K_{1,n}$, the subdivision of the central edge of the bistar $B_{m,n}$, $P_m \Theta nK_1$".

***Key words**: Bistar graph,Cubic harmonious graph, Cubic harmonious labeling, Path graph, Star graph,*

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I. Introduction

A particular topic of interest was on labeling of graphs- specifically, on harmoniously labeling graphs. The work of Tanna[4] involved reiterations of proofs, as well as , supplementary examples to an earlier work of Graham and Sloane (1980) concerning harmonious labeling of certain classes of graphs. Square. For standard and terminology and notation we follow Graham and Sloane [4]. Graham and Sloane[4] defined a (n,m)- graph G of order n and size m to be harmonious, if there is an injective function $f: V(G) \rightarrow Z_m$,where Z_m is the group of integers modulo m, such that the induced function $f^*: E(G) \rightarrow Z_q$, defined by $f^*(uv) = f(u) + f(v)$ for each edge $uv \in E(G)$ is a bijection. Square harmonious graphs were introduced in [10]. Cubic graceful graphs were introduced in [6]. Cubic harmonious graphs were defined in [7]. Throughout this paper we consider simple, finite, connected and undirected graph.

Definition 1.1

The ***Path graph*** P_n is the n- vertex graph with $n-1$ edges, all on a single path.

Definition1.2

A complete bipartite graph $K_{1,n}$ is called a ***star*** and it has (n+1) vertices and n edges

Definition 1.3

The ***Trivial graph*** K_1 or P_1 is the graph with one vertex and no edges.

II. Main Resuts

Theorem 2.1

The star $K_{1,n}$ is cubic harmonious for all n.

Proof:

Let G be the star graph $K_{1,n}$.

Let $V(K_{1,n}) = \{u_r; \quad 1 \le r \le n+1\}$

and

$E(K_{1,n}) = \{u_r u_{n+1};\quad 1 \le r \le n\}$

Define an injection $f : V(K_{1,n}) \to \{1,2,........n^3+1\}$

$f(u_r) = (n+1-r)^3;\quad 1 \le r \le n$

$f(u_{n+1}) = n^3+1$

The induced edge mapping are

$f^*(u_r u_{n+1}) = (n+1-r)^3;\quad 1 \le r \le n$

The vertex labels are in the set $\{1^3, 2^3........n^3+1\}$. The vertex labels are distinct and edge labels are also distinct and cubic. So the star graph $K_{1,n}$ is cubic harmonious for all n.

Theorem 2.2

The graph obtained by the subdivision of the edges of the star $K_{1,n}$ is a cubic harmonious graph for all $n \ge 2$

Proof:

Let G be a graph obtained by the subdivision of the edges of the star $K_{1,n}$ is denoted as $K_{1,n,n}$.

Let the vertex set $V(G) = v, w_r, u_r;\quad 1 \le r \le n$

and the edge set $E(G) = V_{wr}, w_r u_r;\quad 1 \le r \le n$

Define an injection $f: V(G) \to \{1,2,.....(2n)^3+1\}$

$f(v) = (2n)^3 + 1$

$f(w_r) = (n+r)^3;\quad 1 \le r \le n$

$f(u_r) = ((2n)^3+1) + r^3 - (n+r)^3;\quad 1 \le r \le n.$

The induced edge mapping are

$f^*(vw_r) = (n+r)^3;\quad 1 \le r \le n$

$f^*(u_r w_r) = r^3;\quad 1 \le r \le n$

The vertex labels are in the set $\{1,2\ldots.....(2n)^3+1\}$.Then the edge labels are arranged in the set $\{1^3,2^3.........(2n)^3\}$. The vertex labels are distinct and edge labels are also distinct and cubic. So the subdivision of the edges of the star $K_{1,n}$ is a cubic harmonious graph.

Theorem 2.3.

The graph obtained by the subdivision of the central edge of the bistar $B_{m,n}$ is cubic harmonious graph.

Proof :

Let G be the graph obtained by the subdivision of the central edge of the bistar $B_{m,n}$

Let $V(G) = \begin{cases} w\,; & \\ u_r\,; & 1 \leq r \leq m+1 \\ v_s\,; & 1 \leq s \leq n+1 \end{cases}$

Then,

$E(G) = \quad u_r\, u_{m+1}; \qquad 1 \leq r \leq m$

$\begin{cases} v_s\, v_{n+1}; & 1 \leq s \leq m \\ w\, u_{m+1}; & \\ w v_{n+1}; & \end{cases}$

$|n(G)| = m+n+3;$ and $|m(G)| = m+n+2$

Define an injection $f: V(G) \rightarrow \{1,2 \ldots\ldots\ldots\ldots [(m+n+2)]^3 + 1]\}$ by

$f(u_r) = (m+n-r+1)^3\,; \qquad 1 \leq r \leq m$

$f(v_r) = (n-r+1)^3 + 3\,(m+n)^2 + 9\,(m+n) + 7\,; \qquad 1 \leq r \leq n$

$u_{m+1} = (m+n+2)^3 + 1$

$v_{n+1} = (m+n+1)^3 + 1$

$w = (m+n+2)^3$

The induced edge labels are

$f^*(u_r u_{m+1}) = (m+n-r+1)^3\,; \qquad 1 \leq r \leq m$

$f^*(w u_{m+1}) = (m+n+2)^3;$

$f^*(w v_{n+1}) = (m+n+1)^3;$

$f^*(v_s v_{n+1}) = (n+1-s)^3; \qquad 1 \leq s \leq n$

In all the above the three cases, f induces a bijection $f^*: E(G) \rightarrow \{1^3, 2^3, 3^3 \ldots. (m+n+2)^3\}$. Hence the theorem.

Theorem 4.

The graph $P_m \,\Theta\, nK_1 (\, n \geq 2\,)$ is cubic harmonious graph.

Proof:

Let $\{u_1, u_2, u_3 \ldots \ldots u_m\}$ be the vertices of path P_m and $\{v_{1j}\, v_{2j}\, v_{3j} \ldots \ldots v_{nj}\}$ be the j^{th} copy of the graph nK_1. Then $\{v_1 v_1 \ldots . v_n\}$ are the n pendent vertices adjacent to the vertex u_j of P_m for $1 \leq j \leq m$.

Define an injection $f : V(P_m \,\Theta\, nK_1) \to \{1,2 \ldots\ldots (mn + m - 1)^3 + 1\}$ by

$f(u_1) = (mn+m-1)^3 + 1$

$f(u_{i+1}) = [(n + 1)(m - i]^3 + (mn+m-1)^3 + 1 - f(ui)$; $1 \leq i \leq m - 1$

$f(v_{ij}) = \left[mn + n - \left((j - 1)(n + 1)\right) - i\right]^3 + \left[\left(mn + m - 1\right)^3 + 1\right] - f(u_j)$;

$1 \leq i \leq n\,,\, 1 \leq j \leq m$

The induced edge mapping are

$f^*(u_i u_{i+1}) = [(n + 1)(m - i)]^3$; $1 \leq i \leq m-1$

$f^*(u_j v_{ij}) = [mn + m - ((j - 1)(n + 1)) - i]^3$; $1 \leq j \leq m$, $1 \leq i \leq n$

The vertex labels are in the set $\left\{1, 2, \ldots \left(mn + m - 1\right)^3 + 1\right\}$. Then the edge labels are $\left\{1^3, 2^3, 3^3, \ldots \ldots \ldots \ldots . \left(mn + m - 1\right)^3 + 1\right.$. Hence the theorem.

Corollary 3.1

The Hoffman tree $P_n \odot K_1$ is cubic harmonious graph.

Proof:

Let $P_n \odot K_1$ be the Hoffman tree which is the graph obtained from a path P_n by attaching pendant edge at each vertex of the path and it is also denoted by P_n^+ or comb.

Let $V(P_n \odot K_1) = \{u_i, v_i ;\ 1 \leq i \leq n\}$

and $E(P_n \odot K_1) = \begin{cases} u_i v_i ; & 1 \leq i \leq n \\ u_i u_{i+1} ; & 1 \leq i \leq n - 1 \end{cases}$

The vertex sets $|V(P_n \odot K_1)| = 2n$; and the edge sets $|E(P_n \odot K_1)| = 2n - 1$;

Define an injection $f : V(P_n \odot K_1) \to \left\{1,2 \ldots \ldots \left(2n - 1\right)^3 + 1\right\}$ by

$f(u_1) = (2n - 1)^3 + 1$

$f(u_i) = (2n+1-i)^3 + (2n-1)^3 + 1 - f(u_{i-1})$; $2 \leq i \leq n$

$f(v_i) = (2n-1)^3 + 1 + i^3 - f(u_i)$; $1 \leq i \leq n$

The induced edge mapping are

$f^*(u_iu_{i+1}) = (2n-i)^3;$ $1 \le i \le n-1$

$f^*(u_iv_i) = i^3;$ $1 \le i \le n$

The vertex labels are in the set $\{1,2 \ldots \ldots (2n-1)^3 + 1\}$. Then the edge labels are distinct and cubic,

$\{1^3, 2^3, 3^3 \ldots (2n-1)^3\}$. Hence the theorem

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