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Optimization Model for Group Decisions Based on Consensus in Social Network

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Abstract**—**In today's organizations, team and group decision-making has become a part of everyday organizational life. Many people spent a good part of their workdays attending meetings for all types of purposes. Some are simply for communication purposes, but many involve making some type of joint decision. Decision making problems basically consist of finding the best option from a feasible option set. This paper develops a consensus model for group decision making based on social network concept. We develop a multi-criteria model for the problem. The consensus degree can be found to indicate how far a group of individuals is from maximum consensus.

Keywords*-Consensus, multi-criteria, group decision, Social Network.*

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I. Introduction

Frequently, in an organization an unstructured decision problem turns up and needs to be solved, which means that the authorized should make a decision regarding to solving the problem. The process involved is called decision making which can be defined as a set of activities whose aim is to find satisfactory solutions for the problem. A finite set of alternatives should be defined and analyzed . From this analysis, decision makers, judges or expert(s) should decide which of the alternatives is the best one to be chosen solve an issue [1].

Sometimes, although the best alternative has been chosen and recommended it still needs to be processed by a team member, called group decision making (GDM). The GDM is a process whereby an opportunity is provided for team members to influence a particular decision. The impact of this process would be the commitment of team members to the decision has been made [2]; such a decision-making process needs an equilibrium agreement, which can be called as consensus. In a real situation case study presented by [3], there is a group of production engineers that defines consensus as the situation where there are no further conflicts, or destructive action or interference in the chosen activity. In order to have a full insight of the definition of consensus, we could say that it is a process that is reliant upon the openness and frankness of each individual's recommendations [4]. Other methods of GDM involve conflict and debate. For example, devil's advocacy or dialectical inquiry where the ideas and assumptions of participants are presented and then are systematically evaluated and challenged [5]; [6]. One can also look at other alternatives to reaching a decision by consensus, namely majority rules or the voting process. While widely used, reaching a decision with these processes often leads to a decrease in commitment in the implementation of the decision [7]. In fact, a GDM process requires discussion and ultimately a solution that leaves no disagreement among participants.

Group decision making has been widely studied since group decision making processes are very common in many fields. Formal representation of the experts' opinions, aggregation of assessments or selection of the best alternatives have been some of main areas addressed by scientists and researchers.

In GDM problems there are two processes which is necessarily to be carried out to reach a final solution ([8]; [9]): the consensus process and the selection process. The former process refers to how to obtain the maximum degree of consensus or agreement between the set of experts on the solution set of alternatives, while the latter process consists of how to obtain the solution set of alternatives from the opinions on the alternatives given by the experts. Clearly, it is preferable that the set of experts reach a high degree of consensus on the solution set of alternatives.

Consensus has become a major area of research in GDM ([10]; [11]; [12]; [9]; [13]; [14]) Naturally, at the beginning of every GDM problem, experts' opinions may differ substantially. Therefore, it is necessary to develop a consensus process in an attempt to achieve group decision. Classically, consensus is defined as the full and unanimous agreement of all the experts regarding all the possible alternatives. However this definition is not appropriate for our purposes for two reasons. First, it only allows us to differentiate between two states, namely, the existence and absence of consensus. Second, the chances for reaching such a full agreement are rather low. It should be noted that, complete agreement is not necessary in real life.

In this paper, we focus on another promising area, the study of group decision making processes from the concept of influence and social networks. In order to do so, we present a novel model that gathers the experts' initial opinions and provides a framework to represent the influence of a given expert over the other(s). With this proposal it is feasible to estimate both the evolution of the group decision making process and the final solution before carrying out the group discussion process and consequently foreseeing possible actions.

A social network can be defined as a set of expert or groups of experts which has some pattern of interactions or ties between them ([15]; [16]). These patterns could be friendship among a group of scientists, in industry there are business relationships , and for families we have intermarriages. These are all examples of networks that have been studied in the past. From these examples we can say that social network effects can be used for understanding human behavior. People interact with different numbers of individuals and with some individuals more than others and this affects behavior in fundamental ways.

In general, a network is used to grasp the insight of information on social interactions. Each individual could be represented by a node in the network, and there is an arc between two nodes if a social interaction has occurred at any point in time between the two individuals represented by these nodes. The conceptualization of social systems as graphs and networks describe visually the opportunity for systematic investigation and conceptualizing the structure of ties among social experts beyond the pair. Whereas classical sociology tended to make a quantum leap from the individual and the pair to the triple, group, or society, graph theory offered the tools to formally visualize social structure and relatioship consisting of three and more experts.

Let $N = \{1, 2,\dots, n\}$ be a set of network nodes, with each node representing a social expert. The experts are often persons, but may also be groups, organizations or other social entities. A graph can be used to represent social network in a way of specifying relationships among each node of a network. The relationship is represented by links called edges.

Using graph, this network model of social interactions has a clear understanding mathematically.. Unfortunately, from the structure point of view, this graph model has a major drawback is that it is essentially static in that all information about the dynamic relationship among experts is unobserved. The static nature of the model would be unable to present accurate or exact information about patterns in the social activities of actors.

From optimization point of view, it would be appropriate to use the concept of centrality to characterize the measurement whether an expert's position is the most important (or popular). The concept of centrality as applied to social communication was introduced already by [17], since then many different measures of centrality have been addressed (see, for example, [18]; [19]; [20]; [15]; [21]; [22]; [23])

II. SOCIAL NETWORK MODEL

We build a model for social network based on graph formulation described in previous Section.

Firstly, the main target is to appoint an expert in the decision making process. Therefore the objective in the social network domain is to maximize the centrality of the expert. The constraints of the model consist of Density of a network's connectivity (D), Betweenness centrality (B), and Closeness centrality (Cl).

The model can be formulated as a 0-1 integer programming problem.

x_{ij} is a binary variable to describe whether expert i has an agreement with expert j.

Max

$$\sum_{i\in\delta^{-1}(i);(i,j)\in E;i,j\notin x} C_j x_{ij} \tag{1}$$

Subject to

$$\sum_{(i,j)\in\delta^{+}(i)} x_{ij} \le D_i \quad \forall i,j \in N; j \notin X \tag{2}$$

$$\sum_{(i,j)\in\delta^{-}(i)} x_{ij} = \sum_{(v,j)\in\delta^{+}(i)} x_{ij} \quad i \notin X;\ \forall i \in N \tag{3}$$

$$\sum_{(i,j)\in\delta^{+}(i)} (\tau_{ji} x_{ij}) \ge B_i \quad i,j \notin X;\ \forall i \in E \tag{4}$$

$$\sum_{(i,j)\in\delta^{-}(i)} (\tau_{ij} x_{ij}) \le Cl_i \ i,j \notin X, \forall i \in E \tag{5}$$

$$x_{ij} \in \{0,1\} \quad i,j \notin X, \forall (i,j) \in E \tag{6}$$

τ_{ij} consumption or prevalence factor.

III. CONSENSUS OPTIMIZATION MODEL.

There are several important points are necessarily to be satisfied in order we can say that a person (expert) has a dynamic interactions in the social network. These points are:

a) The outdegree ties,
b) Relationship among experts is reciprocal
c) Transitivity, and
d) Equilibrium.

The optimization model can be expressed with the objective is to maximize degree of centrality, the number of outdegree ties, and reciprocity relationship. This is a binary integer programming problem, which can be written mathematically as follows.

$$\max \sum_{i\in\delta^{-1}(i);(i,j)\in E;i,j\notin x} C_j x_{ij} + \sum_{(i,j)\in E} \delta_i^{+} x_{ij} + \sum_{(i,j)\in E} \rho x_{ij} \tag{7}$$

Subject to

$$\sum_{(i,j)\in\delta^{+}(i)} x_{ij} \le D_i \qquad \forall i,j \in N; i,j \notin X \tag{8}$$

$$\sum_{(i,j)\in\delta^{-}(i)} x_{ij} = \sum_{(v,j)\in\delta^{+}(i)} x_{ij} \qquad i \notin X; \forall i \in N \tag{9}$$

$$\sum_{(i,j)\in\delta^{+}(i)} \left(\tau_{ji} x_{ij}\right) \ge B_i \qquad i,j \notin X; \forall i \in E \tag{10}$$

$$\sum_{(i,j)\in\delta^{-}(i)} \left(\tau_{ij} x_{ij}\right) \le Cl_i \qquad i,j \notin X; \forall i \in E \tag{11}$$

$$x_{ij} \in \{0,1\} \quad i,j \notin X, \forall (i,j) \in E \tag{12}$$

Consensus measurement can be defined as follows.

$$CL(V^1,V^2,\dots,V^n) = \frac{1}{nml} \sum_{k=l}^{n} d(V^k,V^C) \tag{13}$$

where $d(V^k,V^C)$ is Manhattan distance between V^k and V^C

$$d(V^k, V^C) = \sum_{i=1}^{m} \sum_{j=1}^{l} \left| v_{ij}^k - v_{ij}^c \right|, k = 1, 2, \ldots, n \quad (14)$$

If $CL(V^1, V^2, \ldots, V^n) = 0$, then all experts should have full and unanimous consensus with the collective opinion.

Generally, the optimization model of consensus rule based on distance can be formulated as follows.

$$\begin{cases} \min \frac{1}{ml} \sum_{k=1}^{n} d(V^k, \overline{V^k}) \\ s.t \begin{cases} \min \frac{1}{nml} \sum_{k=1}^{n} d(V^k, \overline{V^k}) \leq \varepsilon \\ \overline{v_{ij}^c} = F_w^{OWA}(\overline{v_{ij}^1}, \overline{v_{ij}^2}, \ldots, \overline{v_{ij}^n}) \quad i = 1, 2, \ldots, m; j = 1, 2, \ldots, l \end{cases} \end{cases} \quad (15)$$

Where $\overline{V^k}$, $(k = 1, 2, \ldots, n)$ and $\overline{V^c}$ are the decision variables.

The consensus optimization model related to social network can be expressed as in the Eq. (15), in which the expression of Eq. (11) can be written as

$$\sum_{(i,j)\in\delta^-(i)} \tau_{ij} x_{ij} \leq CL(V^1, V^2, \ldots, V^N), \; i, j \notin X; \quad \forall i \in E \quad (16)$$

Provided that the value of $\overline{V^k}$ can be obtained from the optimal result of linear program Eq. (15).

IV. THE ALGORITHM

To solve the 0-1 integer programming model, we adopt the approach of examining a reduced problem in which most of the integer variables are held constant and only a small subset allowed varying in discrete steps.

The steps of the procedure can be summarized as follows.

Step 1. Solve the problem ignoring integrality requirements.

Step 2. Obtain a (sub-optimal) integer-feasible solution, using heuristic rounding of the continuous solution.

Step 3. Divide the set I of integer variables into the set I_1, at their bounds that were nonbasic at the continuous solution, and the set I_2, $I = I_1 + I_2$.

Step 4. Perform a search on the objective function, maintaining the variables in I_1 nonbasic and allowing only discrete changes in the values of the variables in I_2.

Step 5. At the solution in step 4, examine the reduced costs of the variables in I_1. If any should be released from their bounds, add them to the set I_2 and repeat from step 4, otherwise terminate.

It should be noted that the above procedure provides a framework for the development of specific strategies for particular classes of problems.

The integer results are kept in superbasic variables set. Then we conduct an integer line search to improve the integer feasible solution [24].

V. CONCLUSIONS

This paper presents a consensus optimization model based on social network. The assumption of the model is that the experts who are involved in the consensus process should have transitivity, reciprocal and equilibrium interaction among them. The closeness of the consensus is measured using Manhattan distance.

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