Saw-Dmss Model For Intuitionistic Fuzzy Multi Attribute Decision Making Problems

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Abstract—This work introduces a new SAW-DMSS (Simple Additive Weighting-Decision Making Support System) technique for decision-makers to choose the most ideal alternative that has been provided. This also deals with the problem based on SAW algorithm which is a multiple criteria decision making approach with weight determining methods which gives the weights to indicators which is partially or completely unknown or not presented by the decision makers. The SAW algorithm deals with the conflicts between indicators based on certain way to sort the scheme and choose the best scheme. A numerical example is proposed to illustrate the effectiveness of this algorithm. However, comparison of two weight determining methods based on Gaussian distribution and Linguistic quantifier guided aggregation is performed to make the result of evaluations more objective and accurate.

Keywords—SAW; DMSS; Decision Making; Multi Criteria Decision Making.

I. INTRODUCTION

Decision-making support systems (DMSS) are computer based information systems designed to support some or all phases of the decision-making process. Decision-making support systems utilize creative, behavioral, and analytic foundations that draw on various disciplines. DMSS evolution has presented unique challenges and opportunities for information system professionals. These foundations give rise to various architectures that deliver support to individual and group DMSS users. Once created, DMSS must be evaluated and managed. Economic-theory-based methodologies, quantitative and qualitative process and outcome measures, and the dashboard approach have been used to measure DMSS effectiveness. This work deals with the DMSS problems based on SAW algorithm (Simple Additive Weighting) which is a multiple criteria decision making approach with intuitionistic fuzzy sets. The SAW algorithm deals with the conflicts between indicators based on certain way to sort the scheme and choose the best scheme. Some values of the multi attribute decision models are often subjective. The weights of the criteria and the scoring values of the alternatives against the subjective (judgmental) criteria contain always some uncertainties. It is therefore an important question how the final ranking or the ranking values of the alternatives is sensitive to the changes of some input parameters of the decision model.

In multiple attribute decision making (MADM) problem, a decision maker (DM) has to choose the best alternative that satisfies the evaluation criteria among a set of candidate solutions. It is generally hard to find an alternative that meets all the criteria simultaneously, so a better solution is preferred. The SAW method was developed for multi-criteria optimization of complex systems. This method focuses on ranking and selecting from a set of alternatives in the presence of conflicting criteria. Multi-criteria optimization is the process of determining the best feasible solution according to the established criteria (representing different effects). Practical problems are often characterized by several non-commensurable and conflicting criteria and there may be no solution satisfying all criteria simultaneously. Thus, the solution is a set of non-inferior solutions, or a compromise solution according to the decision maker’s preferences. The compromise solution was established by Zeleny, (1982) for a problem with conflicting criteria and it can help the decision makers to reach a final solution. In classical MADM methods, the ratings and the weights of the criteria are known precisely, whereas in the real world, in an imprecise and uncertain environment, it is an unrealistic assumption that the knowledge and representation of a decision maker or expert are so precise. For example, human judgment including preferences is often vague and decision maker (DM) cannot estimate his preference with exact numerical values. In these situations, determining the exact value of the attributes is difficult or impossible. So, to describe and treat imprecise and uncertain elements present in a decision problem, fuzzy approaches and linguistic terms are frequently used. In the works of linguistic terms decision making, linguistic terms are assumed to be with known by fuzzy linguistic membership function. However, in reality to a decision maker it is not always easy to specify the membership function in an inexact environment. At least in some of the cases, the use of interval numbers may serve the purpose better. An interval number can be thought as an extension of the concept of a real number, however, in decision problems its use is not much attended as it merits (Hwang & Yoon, 1981). Thiagarasu & Thinaharan, (2015) and Thiagarasu & Rengaraj, (2015) have contributed to the field of DMSS using SAW and VIKOR methods. Chen, (2012) presented a comparative model based
on SAW and TOPSIS. Zavadskas et al., (2007) presented a sensitivity analysis for SAW method.

Vagueness and uncertainty are the two important aspects of imprecision. IFS is an intuitively straightforward extension of Zadeh’s, (1965) fuzzy sets. IFS theory basically defines the claim that from the fact that an element x “belongs” to a given degree (say $\mu$) to a fuzzy set $A$, it naturally follows that $x$ should “not belong” to $A$ to the extent $1-\mu$, an assertion implicit in the concept of a fuzzy set. On the contrary, IFSs assign to each element of the universe both a degree of membership $\mu$ and one of non-membership $\gamma$ such that $\mu + \gamma \leq 1$, thereby relaxing enforced duality $\gamma = 1-\mu$ from fuzzy set theory. Obviously, when $\mu + \gamma = 1$ for all elements of the universe, the traditional fuzzy set concept is recovered. In IFS this identity is weakened into an inequality, or in other words: a denial of the law of the excluded middle occurs, one of the main ideas of intuitionism. Let $X$ be the universe of discourse defined by $X = \{x_1, x_2, ..., x_n\}$. The grade of membership of an element $x \in X$ in a fuzzy set is represented by real values between 0 and 1. It indicates the evidence for $x \in X$, but does not indicate the evidence against $x \notin X$. Atanassov, (1986; 1989 ) pointed out that this single value combines the evidence for $x \in X$ and the evidence against $x \notin X$. An IFS $A$ in $X$ is characterised by a membership function $\mu_A (x_i)$ and a non-membership function $\gamma_A (x_i)$. Here, $\mu_A (x_i)$ and $\gamma_A (x_i)$ are associated with each point in $X$, a real number in [0,1] with the values of $\mu_A (x_i)$ and $\gamma_A (x_i)$ at $X$ representing the grade of membership and non-membership of $x_i$ in $A$. Thus closeness of the value of $\mu_A (x_i)$ to unity and the value of $\gamma_A (x_i)$ to zero, raise high the grade of membership and lower the grade of non-membership of $x_i$. An IFS becomes a fuzzy set when $\gamma_A (x_i) = 0$.

Consensus processes imply that experts achieve an agreement about a problem before taking a decision, thus yielding a solution accepted by the organization, society or themselves. In these approaches, it is crucial to establish a consensus measure to calculate the level of agreement. Consensus measures are indicators to evaluate how far a group of experts’ opinions is from unanimity. Mohanty & Bhasker, (2005), Mohanty & Zahir, (2007) and Mohanty, (2008) have applied the concepts of Linguistic Quantifiers in the product classifications based on customer preference in Internet-Business. In this paper, The RIM-Linguistic Quantifiers (Yager, 1988) based on the Ordered Weighted Averaging (OWA) operators are used to derive the weights of the experts and Gaussian Distribution based method proposed by Xu, (2005) is also used for the same purpose. An algorithm for the proposed model of MADM for intuitionistic fuzzy sets is presented in this chapter. A numerical illustration is presented when the weights are unknown. A comparison of the proposed methods is also presented.

II. APPLICATION OF SAW AS A DECISION SUPPORT SYSTEM TECHNIQUE

A DMSS is intended to support, rather than replace, decision maker’s role in solving problems. Decision makers’ capabilities are extended through using DSS, particularly in ill-structured decision situations. In this case, a satisfied solution, instead of the optimal one, may be the goal of decision making. Solving ill-structured problems often relies on repeated interactions between the decision maker and the DSS. Decision support systems are built upon various decision support techniques, including models, methods, algorithms and tools. A cognition-based taxonomy for decision support techniques, including six basic classes as follows: Process models, Choice models , Information control techniques, Analysis and reasoning techniques, Representation aids and Human judgment amplifying/refining techniques. The Multi-criteria decision making and Multi-attribute decision making comes under the category of Choice models. Multiple Attribute decision support systems are provided to assist decision makers with an explicit and comprehensive tool and techniques in order to evaluate alternatives in terms of different factors and importance of their weights. Some of the common Multi-Attribute Decision-Making (MADM) techniques are:

- Simple Additive Weighted (SAW)
- Weighted Product Method (WPM)
- Cooperative Game Theory (CGT)
- Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)
- Elimination et Choice Translating Reality with complementary analysis(ELECTRE)
- Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE)
- Analytical Hierarchy Process (AHP)

The merit of the SAW method is that it can deal with both quantitative and qualitative assessment in the process evaluation with little computation load. It basises upon the concept that the chosen alternative is derived from the weighted decision matrix. In the process of SAW, the performance ratings and the weights of the criteria are given as crisp values. In fuzzy SAW, attribute values are represented by fuzzy numbers.

A. SAW method

Decision-making problem is the process of finding the best option from all of the feasible alternatives. In almost all such problems, the multicriteria of criteria for judging the alternatives is pervasive. For many such problems, the DM wants to solve a multiple attribute decision making (MADM) problem (Hwang & Yoon, 1981). A MADM problem can be concisely expressed in matrix format as:

$$ C_j \begin{bmatrix} A_1 & A_2 & \ldots & A_m \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \ldots \\ x_m \end{bmatrix} $$

where $A_1, A_2, \ldots, A_m$ are possible alternatives among which decision makers have to choose, $C_1, C_2, \ldots, C_n$ are criteria with which alternative performance are measured, $x_q$ is the rating of alternative $A_i$ with respect to criterion $C_j$. 

IJRITCC | May 2017, Available @ http://www.iiritcc.org

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SAW Technique is one of the most used MADM techniques. It is simple and is the basis of most MADM techniques such as AHP and PROMETHEE that benefits from additivities. In SAW technique, final score of each alternatives is calculated as follows and they are ranked.

\[ P_i = \sum_{j=1}^{k} w_j r_{ij}; \quad i = 1, 2, \ldots, m. \]

Where \( r_{ij} \) are normalized values of decision matrix elements and calculated as follow: For profit, attributes, we have, \( r_{ij} = d_{ij}; \quad d_{ij}^{\max} = Max \; d_{ij}; \quad j = 1, 2, \ldots, k \) And for cost attributes, \( r_{ij} = d_{ij}; \quad d_{ij}^{\min} = Min \; d_{ij}; \quad j = 1, 2, \ldots, k \)

If the there is any qualitative attributive, then we can use some methods for transforming qualitative one’s.

### B. INTUITIONISTIC FUZZY SETS

Let \( X \) be the universe of discourse. An intuitionistic fuzzy set \( A \) in \( X \) is an object having the form \( A = \{(x, \mu_A(x), \gamma_A(x)) | x \in X\} \) where \( \mu_A(x), \gamma_A(x) : x \rightarrow [0,1] \) denote membership function and non-membership function, respectively, of \( A \) and satisfy \( 0 \leq \mu_A(x) + \gamma_A(x) \leq 1 \) for every \( x \in X \). \( \mu_A(x) \) is the lowest bound of membership degree derived from proofs of supporting \( x \); \( \gamma_A(x) \) is the lowest bound of non-membership degree derived from proofs of rejecting \( x \). It is clear that the membership degree of Intuitionistic Fuzzy set \( A \) has been restricted in \([\mu_A(x), 1-\gamma_A(x)]\) which is a subinterval of \([0,1]\).

For each IFS \( A \) in \( X \) we call \( \pi_A(x) = 1 - \mu_A(x) - \gamma_A(x) \) as the intuitionistic index of \( x \) in \( A \). It is hesitation degree (or degree of indeterminacy) of \( x \) to \( A \). It is obvious that \( 0 \leq \mu_A(x) \leq 1 \) for each \( x \in X \).

For example, let \( A \) be an IFS with membership function \( \mu_A(x) \) and non-membership function \( \gamma_A(x) \), respectively. If \( \mu_A(x) = 0.5 \) and \( \gamma_A(x) = 0.3 \), then we have \( \mu_A(x) = 1 - 0.5 - 0.3 = 0.2 \). It could be interpreted as the degree that the object \( x \) belongs to the IFS \( A \) is 0.5, the degree that the object \( x \) does not belong to the IFS \( A \) is 0.3 and the degree of hesitation is 0.2. Thus, the IFS \( A \) in \( X \) can be expressed as \( A = \{(x, \mu_A(x), \gamma_A(x), \pi_A(x)) | x \in X\} \)

If \( A \) is an ordinary fuzzy set, then \( \pi_A(x) = 1 - \mu_A(x) - (1 - \mu_A(x)) = 0 \) for each \( x \in X \). It means that the third parameter \( \pi_A(x) \) cannot be casually omitted if \( A \) is a general IFS, not an ordinary fuzzy set. Therefore, the representation of IFS should consider all three parameters in calculating the degree of similarity between IFSs.

For \( A, B \in IFS(X) \), the set of all IFSs, the notion of containment is defined as follows:

\[ A \subseteq B \iff \mu_A(x) \leq \mu_B(x) \text{ and } \gamma_A(x) \geq \gamma_B(x), x \in X. \]

### C. OPERATIONS DEFINED OVER INTUITIONISTIC FUZZY SETS:

First, some operations \((\cup, \cap, +, \cdot)\) are defined over intuitionistic fuzzy sets (IFSs). Here we shall discuss some of their basic properties. For every two IFSs \( A \) and \( B \) the following are valid (let \( a, \beta \in [0,1] \)):

\[ A \cup B \iff (\forall x \in E) \left( \mu_A(x) \leq \mu_B(x) \text{ and } \gamma_A(x) \geq \gamma_B(x) \right). \]

\[ A \cap B \iff B \subseteq A. \]

\[ A = B \iff (\forall x \in E) \left( \mu_A(x) = \mu_B(x) \text{ and } \gamma_A(x) = \gamma_B(x) \right). \]

### D. GAUSSIAN METHOD OF DETERMINING UNKNOWN WEIGHTS

Let us consider a situation where there is an unfair argument among the experts in fixing the weights in a decision making problem. In that case we need to relieve the influence of unfair arguments on the decision variables. Xu, (2005) introduced a procedure for generating the weights based on the use of the Gaussian distribution. They are referred as Gaussian weights which are given as follows:

Consider a Gaussian distribution \( G(\mu_n, \sigma_n) \), where \( \mu_n \) is the mean of the collection and \( \sigma_n \) is the deviation of the collection, and given by:

\[ \mu_n = \frac{1}{n} \sum_{j=1}^{n} j = \frac{n+1}{2} \] and \[ \sigma_n = \sqrt{\frac{1}{n} \sum_{j=1}^{n} (j-m_n)^2}. \]

Let \( G(j) = \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-(j-m_n)^2/2\sigma_n^2} \).

Then the associated weights are defined as:

\[ w_j = \frac{G_j}{\sum_{j \in J} G(j)} = \frac{e^{-(j-m_n)^2/2\sigma_n^2}}{\sum_{j \in J} e^{-(j-m_n)^2/2\sigma_n^2}} \]

where \( w_j \in [0,1] \) and \( \sum_{j=1}^{n} w_j = 1 \).

It can be noted that the closer \( j \) is to \( m_n = \frac{n+1}{2} \), the larger \( W_j \)

Furthermore, if \( n \) is odd, the maximal value of \( W_j \) occurs
for \( j = \frac{n+1}{2} \). If \( n \) is even, the maximal value of \( W_j \) occurs for \( j = \frac{n}{2} \) and \( j = \frac{n}{2} + 1 \). It can also be shown that the weighting vector generated using this approach is symmetric, i.e., \( W_j = W_{n+1-j} \).

E. LINGUISTIC (RIM) QUANTIFIERS FOR DETERMINING UNKNOWN WEIGHTS

The problem of determining weights for an OWA operator can be addressed in different ways, for example with the use of the so-called ‘Linguistic Quantifiers’. A relative linguistic quantifier \( Q \), such as ‘most’, ‘few’, ‘many’, and ‘all’, can be represented as a fuzzy subset of the unit interval, where for a given proportion \( r \in [0,1] \) of the total of the values to aggregate, \( Q(r) \) indicates the extent to which this proportion satisfies the semantics defined in \( Q \). For example, given \( Q \) = ‘most’, if \( Q(0.7) = 1 \) then it would mean that a proportion of 70% totally satisfies the idea conveyed by the quantifier ‘most’, whereas \( Q(0.55) = 0.25 \) indicates that the proportion 55% is barely compatible with this concept (i.e., only 25%).

Regular Increasing Monotone (RIM) quantifiers are especially interesting for their use in OWA operators. These quantifiers present the following properties:

i. \( Q(0) = 0 \)

ii. \( Q(1) = 1 \)

iii. If \( r_1 < r_2 \) then \( Q(r_1) \geq Q(r_2) \).

Yager, (1988) suggested the following method to compute weights \( w_i \), with the use of a RIM quantifier \( Q \):

\[
W_i = Q \left( \frac{i}{n} \right) - Q \left( \frac{i-1}{n} \right), \quad i = 1, 2, ..., n.
\]

Where the membership function of a linear RIM quantifier \( Q(r) \) is defined by two parameters \( a, b \in [0,1] \) as:

\[
Q(r) = \begin{cases} 
0 & \text{if } r < a \\
\frac{r-a}{b-a} & \text{if } a \leq r \leq b \\
1 & \text{if } r > b
\end{cases}
\]

An example of RIM quantifier \( Q \) = ‘most’, with \( a = 0.5 \) and \( b = 0.7 \) is given as:

\[
Q(r) = \begin{cases} 
0 & \text{if } r < 0.5 \\
5r - 2.5 & \text{if } 0.5 \leq r \leq 0.7 \\
1 & \text{if } r > 0.7
\end{cases}
\]

Since the use of OWA with RIM quantifiers captures the notion of the soft consensus correctly, they can be adopted for the purpose of studying the effect of different aggregation operators on the resolution of a consensus problem with many experts, and expressing a desired group’s attitude.

**Figure –1**

**Representation of the Linguistic Quantifier \( Q(r) \)**

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F. ALGORITHM FOR INTUITIONISTIC FUZZY SAW-DMSS

The following steps are followed for the intuitionistic fuzzy SAW-DMSS proposed in this paper:

G. Step-1: For the decision matrix \( \mathbf{R} \), find the defuzzyfied matrix and the normalized matrix.

Step-2: Calculate the weights of the attributes by Gaussian method and linguistic quantifier method.

Step-3: For the expected attribute value matrix \( \mathbf{R} \), calculate the weighted normalized matrix.

Step-4: Calculate the expected value from the weighted normalized matrix.

Step-5: Rank the alternatives and choose the best one according the ranking order.

III. NUMERICAL ILLUSTRATION

We assume an MADM problem that has three alternatives and four attributes where in attributes \( C_1, C_4 \) are cost type and attributes \( C_2, C_3 \) are of profit type. The intuitionistic fuzzy decision matrix is given as follows:

\[
\mathbf{D}_{ij} = \begin{pmatrix}
(0.29,0.19) & (0.3137,0.1568) & (0.3809,0.1904) & (0.4615,0.2307) \\
(0.45,0.35) & (0.5490,0.2745) & (0.5079,0.2539) & (0.3590,0.1795) \\
(0.48,0.24) & (0.4706,0.2353) & (0.4444,0.2222) & (0.5128,0.2564)
\end{pmatrix}
\]

The defuzzyfied matrix from the above intuitionistic fuzzy matrix is given as follows:
The normalized matrix is given as:

\[
\begin{pmatrix}
0.3846 & 1 & 1 & 0.7500 \\
0.2800 & 0.5555 & 0.7776 & 1 \\
0.2800 & 0.5555 & 0.7776 & 1
\end{pmatrix}
\]

A. Weight Determination by Gaussian Distribution Method:

The three possible alternatives of the above decision making problem are to be evaluated by three decision makers whose weighting vector is completely unknown. The mean and the deviation of the collection 1, 2, ..., n are given by the following equations as follows:

\[
\mu_k = \frac{1}{n} \sum_{j=1}^{n} x_j, \quad \sigma_k = \sqrt{\frac{1}{n} \sum_{j=1}^{n} (x_j - \mu_k)^2}
\]

Where \(\mu_1 = 1, \mu_2 = 1.5, \mu_3 = 2\), \(\sigma_1 = 0, \sigma_2 = 0.5, \sigma_3 = 0.8164\)

Then the weights are calculated using the following equation as follows:

\[
w_j = \frac{G_j}{\sum_{j=1}^{n} G(j)} = \frac{e^{-(x_j-\mu_k)^2/2\sigma_k^2}}{\sum_{j=1}^{n} e^{-(x_j-\mu_k)^2/2\sigma_k^2}}
\]

where \(w_j \in [0,1]\) and \(\sum_{j=1}^{n} w_j = 1\).

\(w_1 = 0.2429, \quad w_2 = 0.5142, \quad w_3 = 0.2429\)

Hence the weights of the experts are taken as \(V = (0.2429, 0.5142, 0.2429)^T\). Using the weight vector and proceeding with the above weighted normalized matrix, we get:

The weighted normalized matrix is given as follows:

\[
\begin{pmatrix}
0.0934 & 0.2429 & 0.2429 & 0.4821 \\
0.5142 & 0.1713 & 0.2856 & 0.2571 \\
0.1735 & 0.1349 & 0.1888 & 0.2429
\end{pmatrix}
\]

Calculating the expected value from the weighted normalized matrix, we get:

\[
P_1 = \sum_{j=1}^{k} t_{ij} \times w_j, \quad i = 1, 2, ..., m
\]

\[
P_1 = \sum_{j=1}^{k} t_{ij} \times w_j = 0.7613
\]

\[
P_2 = \sum_{j=1}^{k} t_{ij} \times w_j = 1.2282
\]

\[
P_3 = \sum_{j=1}^{k} t_{ij} \times w_j = 0.7401
\]

The ranking of the above three alternatives will give the best alternative: \(P_2 > P_1 > P_3\).

Hence the best alternative is \(P_2\) (Second alternative).

B. Weight Determination by Linguistic Quantifier Method:

The three possible alternatives of the above decision making problem are to be evaluated by three decision makers whose weighting vector is completely unknown. The membership function for the linguistic quantifier \(Q = \text{most}\) is given as follows:

\[
\mu_{\text{most}} = \begin{cases} 
0 & \text{if } x \leq 0.5 \\
0.5 & \text{if } 0.5 < x < 0.9 \\
1 & \text{if } x \geq 0.9
\end{cases}
\]

The unknown weights are computed by the RIM quantifier \(Q\) as follows:

\[
w_i = Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right), \quad i = 1, 2, ..., n
\]

Which gives the weights as \(w_j = (0.26, 0.68, 0.06)^T\). Using this weight vector derived from the RIM linguistic quantifier, and
proceeding with the above weighted normalized matrix, we get:

\[
P_i = \sum_{j=1}^{k} r_{ij} \times w_j, \quad i = 1, 2, ..., m
\]

\[
P_1 = \sum_{j=1}^{k} r_{ij} \times w_j = 0.8149,
\]

\[
P_2 = \sum_{j=1}^{k} r_{ij} \times w_j = 1.6243,
\]

\[
P_3 = \sum_{j=1}^{k} r_{ij} \times w_j = 0.1827.
\]

Calculating the expected value from the weighted normalized matrix, we get:

\[
hence the best alternative is \ P_2 (Second alternative).
\]

**Table 1**  
Proposed SAW-DMSS models with unknown weights

<table>
<thead>
<tr>
<th>SAW-DMSS Models</th>
<th>Ranking of Alternatives</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method-1: Unknown Expert Weights (Gaussian Distribution)</td>
<td>( P_2 &gt; P_1 &gt; P_3 )</td>
</tr>
</tbody>
</table>

IV. CONCLUSION:

FINDINGS AND SUGGESTIONS

The proposed research work has concentrated on applying SAW-DMSS method to real world decision making problems. The general SAW-DMSS method was proposed and new algorithm was proposed for Multiple Attribute Decision Making efficiently. A numerical illustration with the theory of selecting the best alternative is analyzed with the help of the proposed algorithm of SAW-DMSS method extended with applying the changes taking place in determining unknown expert weighting vector is presented. The numerical illustration presented utilizing the SAW-DMSS method displays the effectiveness of the proposed algorithm.

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