Oblong Mean Prime Labeling of Some Snake Graphs

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Abstract—A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. The graph for which every edge(uv),the labels assigned to u and v are oblong numbers and for each vertex of degree at least 2, the g c d of the labels of the incident edges is 1. Here we characterize some snake graphs for oblong mean prime labeling.

Keywords- *Graph labeling, prime graphs, prime labeling, oblong numbers, snake graphs.*

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I. INTRODUCTION

All graphs in this paper are simple, connected, finite and undirected. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [1],[2] and [3]. Some basic concepts are taken from [1]. In [4], we introduced the concept oblong mean prime labeling and proved that some path related graphs admit oblong mean prime labeling. In this paper we investigated oblong mean prime labeling of some snake graphs.

Definition: 1.1 Let G be a graph with p vertices and q edges. The greatest common divisor of a vertex of degree greater than or equal to 2, is the g c d of the labels of the incident edges.

Definition: 1.2 An oblong number is the product of a number with its successor, algebraically it has the form n(n+1).The oblong numbers are 2, 6, 12, 20, --------------.

II. MAIN RESULTS

Definition 2.1 Let G be a graph with p vertices and q edges . Define a bijection f : V(G) → {2,6,12,20 ,--------------,p(p+1)} by f(v_i) = $i(i+1)$, for every i from 1 to p and define a 1-1 mapping f^*_{ompl} : E(G) → set of natural numbers N by $f^*_{ompl}(uv) = \frac{f(u)+f(v)}{2}$. The induced function f^*_{ompl} is said to be an oblong mean prime labeling, if the g c d of each vertex of degree at least 2, is one.

Definition 2.2 A graph which admits oblong mean prime labeling is called an oblong mean prime graph.

Theorem 2.1 Triangular snake T_n admits oblong mean prime labeling.

Proof: Let G = T_n and let v_1,v_2,----------------,v_{2n-1} are the vertices of G

Here |V(G)| = 2n-1 and |E(G)| = 3n-3

Define a function f : V → {2,6,12,--------------,$(2n-1)(2n)$ } by

$$f(v_i) = i(i+1) , i = 1,2,\text{------},2n-1.$$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ompl} is defined as follows

$f^*_{ompl}(v_i\ v_{i+1}) = (i+1)^2$, i = 1,2,------,2n-2

$f^*_{ompl}(v_{2i-1}\ v_{2i+1}) = 4i^2+2i+1$, i = 1,2,------,n-1

Clearly f^*_{ompl} is an injection.

g c d of (v_{i+1}) = g c d of {$f^*_{ompl}(v_i\ v_{i+1})$, $f^*_{ompl}(v_{i+1}\ v_{i+2})$ }
= g c d of { $(i+1)^2$, $(i+2)^2$}
= g c d of {i+1 , i+2} = 1, i = 1,2,------,2n-3

g c d of (v_1) = g c d of {$f^*_{ompl}(v_1\ v_2)$, $f^*_{ompl}(v_1\ v_3)$ }
= g c d of {4,7} = 1.

g c d of (v_{2n-1}) = g c d of {$f^*_{ompl}(v_{2n-2}\ v_{2n-1})$, $f^*_{ompl}(v_{2n-3}\ v_{2n-1})$ }
= g c d of { $4n^2$-4n+1, $4n^2$-6n+3}
= g c d of { (2n-2), ($4n^2$-6n+3)} = 1.

So, g c d of each vertex of degree greater than one is 1.

Hence T_n, admits oblong mean prime labeling. ∎

Theorem 2.2 Double triangular snake D(T_n) admits oblong mean prime labeling.

Proof: Let G = D(T_n) and let v_1,v_2,----------------,v_{3n-2} are the vertices of G

Here |V(G)| = 3n-2 and |E(G)| = 5n-5

Define a function f : V → {2,6,12,--------,$(3n-2)(3n-1)$ } by

$$f(v_i) = i(i+1) , i = 1,2,\text{------},3n-2.$$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ompl} is defined as follows

$f^*_{ompl}(v_{3i-2}\ v_{3i}) = 9i^2-3i+1$, i = 1,2,------------,n-1.

$f^*_{ompl}\,(v_{3i}\; v_{3i+1})$ = 9i²+6i+1, i = 1,2,------------,n-1.
$f^*_{ompl}\,(v_{3i-2}\; v_{3i-1})$= 9i²-6i+1, i = 1,2,------------,n-1.
$f^*_{ompl}\,(v_{3i-1}\; v_{3i+1})$= 9i²+3i+1, i = 1,2,------------,n-1.
$f^*_{ompl}\,(v_{3i-2}\; v_{3i+1})$= 9i²+2, i = 1,2,------------,n-1.

Clearly f^*_{ompl} is an injection.

g c d of (v_{3i}) = g c d of {$f^*_{ompl}\,(v_{3i}\; v_{3i-2})$, $f^*_{ompl}\,(v_{3i}\; v_{3i+1})$ }
= g c d of {$9i^2$-3i+1 , $9i^2$+6i+1 }
= g c d of {$9i^2$-3i+1 , 9i }
= g c d of {6i+1 , 9i}
= g c d of {3i-1 , 6i+1}
= g c d of {3, 3i-1} = 1, i = 1,2,---------,n-1.

g c d of (v_{3i-1}) = g c d of {$f^*_{ompl}\,(v_{3i-2}\; v_{3i-1})$, $f^*_{ompl}\,(v_{3i-1}\; v_{3i+1})$ }
= g c d of {$9i^2$-6i+1 , $9i^2$+3i+1 }
= g c d of {$9i^2$-6i+1 , 9i }
= g c d of {3i+1 , 9i}
= g c d of {3i-2 , 3i+1}
= g c d of {3, 3i-2} = 1, i = 1,2,-------,n-1.

g c d of (v_{3i+1}) = g c d of {$f^*_{ompl}\,(v_{3i+1}\; v_{3i+2})$, $f^*_{ompl}\,(v_{3i}\; v_{3i+1})$ }
= g c d of {$(3i+1)^2$, $(3i+2)^2$}
= g c d of {3i+1 , 3i+2 }
= g c d of {6i+1 , 9i}
= g c d of {3i-1 , 6i+1}= 1, i = 1,2,------,n-2.

g c d of (v_1) = g c d of {$f^*_{ompl}\,(v_1\; v_2)$, $f^*_{ompl}\,(v_1\; v_3)$ }
= g c d of {4,7} = 1.

g c d of (v_{3n-2}) = g c d of {$f^*_{ompl}\,(v_{3n-2}\; v_{3n-3})$, $f^*_{ompl}\,(v_{3n-4}\; v_{3n-2})$ }
= g c d of {$(3n-2)^2$, $9n^2$-15n+7}
= g c d of {3n-2 , $9n^2$-15n+7 }= 1.

So, g c d of each vertex of degree greater than one is 1.

Hence D(T_n), admits oblong mean prime labeling. ■

Theorem 2.3 Quadrilateral snake Q_n admits oblong mean prime labeling.

Proof: Let G = Q_n and let v_1,v_2,----------------,v_{3n-2} are the vertices of G

Here |V(G)| = 3n-2 and |E(G)| = 4n-4

Define a function f : V → {2,6,12,--------,$(3n-2)(3n-1)$ } by

f(v_i) = i(i+1) , i = 1,2,------,3n-2.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ompl} is defined as follows

$f^*_{ompl}\,(v_{3i-2}\; v_{3i+1})$ = 9i²+2, i = 1,2,-------,n-1.
$f^*_{ompl}\,(v_i\; v_{i+1})$ = i²+2i+1, i =1,2,------,3n-3.

Clearly f is an injection.

g c d of (v_{i+1}) = 1, i = 1,2,------------,3n-4
g c d of (v_1) = g c d of {4,11}.
g c d of (v_{3n-2}) = g c d of {$f^*_{ompl}\,(v_{3n-2}\; v_{3n-3})$, $f^*_{ompl}\,(v_{3n-5}\; v_{3n-2})$ }
= g c d of {$(3n-2)^2$, $9n^2$-18n+11}
= g c d of {3n-2 , 3 }= 1.

So, g c d of each vertex of degree greater than one is 1.

Hence Q_n, admits oblong mean prime labeling. ■

Theorem 2.4 Alternate triangular snake A(T_n) admits oblong mean prime labeling, when n is odd and triangle starts from the first vertex.

Proof: Let G = A(T_n) and let v_1,v_2,----------------,$v_{(3n-1)/2}$ are the vertices of G

Here |V(G)| = $\frac{3n-1}{2}$ and |E(G)| = 2n-2

Define a function f : V → {2,6,12,--------------,$(\frac{3n-1}{2})(\frac{3n+1}{2})$ } by

f(v_i) = i(i+1) , i = 1,2,------,$\frac{3n-1}{2}$.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ompl} is defined as follows

$f^*_{ompl}\,(v_{3i-2}\; v_{3i})$ = 9i²-3i+1, i = 1,2,------------,$\frac{n-1}{2}$.
$f^*_{ompl}\,(v_i\; v_{i+1})$ = i²+2i+1, i = 1,2,------------,$\frac{3n-3}{2}$.

Clearly f is an injection.

g c d of (v_{i+1}) = 1, i = 1,2,------------,$\frac{3n-5}{2}$.
g c d of (v_1) = g c d of {4,7}.

So, g c d of each vertex of degree greater than one is 1.

Hence A(T_n), admits oblong mean prime labeling. ■

Theorem 2.4 Alternate triangular snake A(T_n) admits oblong mean prime labeling, when n is odd and triangle starts from the first vertex.

Proof: Let G = A(T_n) and let v_1,v_2,----------------,$v_{(3n-1)/2}$ are the vertices of G

Here |V(G)| = $\frac{3n-1}{2}$ and |E(G)| = 2n-2

Define a function f : V → {2,6,12,--------------,$(\frac{3n-1}{2})(\frac{3n+1}{2})$ } by

f(v_i) = i(i+1) , i = 1,2,------,$\frac{3n-1}{2}$.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ompl} is defined as follows

$f^*_{ompl}\,(v_{3i-2}\; v_{3i})$ = 9i²-3i+1, i = 1,2,------------,$\frac{n-1}{2}$.
$f^*_{ompl}\,(v_i\; v_{i+1})$ = i²+2i+1, i = 1,2,------------,$\frac{3n-3}{2}$.

Clearly f is an injection.

g c d of (v_{i+1}) = 1, i = 1,2,------------,$\frac{3n-5}{2}$.
g c d of (v_1) = g c d of {4,7}.

So, g c d of each vertex of degree greater than one is 1.

Hence A(T_n), admits oblong mean prime labeling. ■

Theorem 2.5 Alternate triangular snake A(T_n) admits oblong mean prime labeling, when n is odd and triangle starts from the second vertex.

Proof: Let G = A(T_n) and let v_1,v_2,----------------,$v_{(3n-1)/2}$ are the vertices of G

Here |V(G)| = $\frac{3n-1}{2}$ and |E(G)| = 2n-2

Define a function f : V → {2,6,12,---------------,($\frac{3n-1}{2}$)($\frac{3n+1}{2}$) } by

$$f(v_i) = i(i+1) \ , i = 1,2,\text{------},\frac{3n-1}{2}.$$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ompl} is defined as follows

$f^*_{ompl}\,(v_{3i-1}\ v_{3i+1})$ = $9i^2+3i+1$, i = 1,2,------------,$\frac{n-1}{2}$.

$f^*_{ompl}\,(v_i\ v_{i+1})$ = i^2+2i+1, i = 1,2,------------,$\frac{3n-3}{2}$.

Clearly f is an injection.

g c d of (v_{i+1}) = 1, i = 1,2,------------,$\frac{3n-5}{2}$.

g c d of ($v_{(3n-1)/2}$) = g c d of{$f^*_{ompl}\,(v_{(3n-1)/2}\ v_{(3n-3)/2})$, $f^*_{ompl}\,(v_{(3n-5)/2}\ v_{(3n-1)/2})$}

= g c d of {$\frac{9n^2-6n+1}{4}$, $\frac{9n^2-12n+7}{4}$ }

= g c d of {$\frac{3n-3}{2}$, $\frac{9n^2-12n+7}{4}$ }= 1.

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So, g c d of each vertex of degree greater than one is 1.

Hence A(T_n), admits oblong mean prime labeling. ■

Theorem 2.6 Alternate triangular snake A(T_n) admits oblong mean prime labeling, when n is even and triangle starts from the first vertex.

Proof: Let G = A(T_n) and let v_1,v_2,----------------,$v_{(3n)/2}$ are the vertices of G

Here |V(G)| = $\frac{3n}{2}$ and |E(G)| = 2n-1

Define a function f : V → {2,6,12,-----------------,($\frac{3n}{2}$)($\frac{3n+2}{2}$) } by

$$f(v_i) = i(i+1) \ , i = 1,2,\text{------},\frac{3n}{2}.$$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ompl} is defined as follows

$f^*_{ompl}\,(v_{3i-2}\ v_{3i})$ = $9i^2-3i+1$, i = 1,2,------------,$\frac{3n-2}{2}$.

$f^*_{ompl}\,(v_i\ v_{i+1})$ = i^2+2i+1, i = 1,2,------------,$\frac{n}{2}$.

Clearly f is an injection.

g c d of (v_{i+1}) = 1, i = 1,2,------------,$\frac{3n-4}{2}$.

g c d of (v_1) = g c d of {4,7}.

g c d of ($v_{(3n)/2}$) = g c d of{$f^*_{ompl}\,(v_{(3n)/2}\ v_{(3n-2)/2})$, $f^*_{ompl}\,(v_{(3n-4)/2}\ v_{(3n)/2})$}

= g c d of {$\frac{9n^2-6n+4}{4}$, $\frac{9n^2}{4}$ }

= g c d of {$\frac{3n-2}{2}$, $\frac{9n^2-6n+4}{4}$ }

= 1.

So, g c d of each vertex of degree greater than one is 1.

Hence A(T_n), admits oblong mean prime labeling. ■

Theorem 2.7 Alternate triangular snake A(T_n) admits oblong mean prime labeling, when n is even and triangle starts from the second vertex.

Proof: Let G = A(T_n) and let v_1,v_2,----------------,$v_{(3n)/2}$ are the vertices of G

Here |V(G)| = $\frac{3n-2}{2}$ and |E(G)| = 2n-3

Define a function f : V → {2,6,12,-----------------,($\frac{3n-2}{2}$)($\frac{3n}{2}$) } by

$$f(v_i) = i(i+1) \ , i = 1,2,\text{------},\frac{3n-2}{2}.$$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ompl} is defined as follows

$f^*_{ompl}\,(v_{3i-1}\ v_{3i+1})$ = $9i^2+3i+1$, i = 1,2,------------,$\frac{n-2}{2}$.

$f^*_{ompl}\,(v_i\ v_{i+1})$ = i^2+2i+1, i = 1,2,------------,$\frac{3n-4}{2}$.

Clearly f is an injection.

g c d of (v_{i+1}) = 1, i = 1,2,------------,$\frac{3n-6}{2}$.

So, g c d of each vertex of degree greater than one is 1.

Hence A(T_n), admits oblong mean prime labeling. ■

REFERENCES

[1] F Harary, Graph Theory, Addison-Wesley, Reading, Mass.(1972)

[2] Joseph A Gallian, A Dynamic Survey of Graph Labeling, The Electronic Journal of Combinatorics, #DS6,(2016) pp 1 – 408.

[3] T K Mathew Varkey, Some Graph Theoretic Generations Associated with Graph Labeling, PhD Thesis, University of Kerala (2000).

[4] T K Mathew Varkey, Sunoj B S, Oblong Mean Prime Labeling of Some Path Graphs, Aryabhatta Journal of Mathematics and Informatics, Volume 9, Issue 1,(2017),pp 320-326.