Unsteady Free Convective Flow past an Infinite Vertical Plate through a Porous Medium in Sleep Flow Regime with Variable Suction and Periodic Plate Temperature in Presence of a Heat Source

Deepjyoti Kalita
Department of Mathematics, Hojai College
Assam, India, PIN - 782435
E mail ID: deepjyoti_kalita2007@rediffmail.com

Abstract: The study of a two dimensional unsteady free convective and heat transfer flow of an incompressible viscous fluid through a porous medium bounded by an infinite vertical porous plate in presence of a heat source has been presented in sleep flow regime. The plate temperature varies periodically with time. The governing equations are solved by regular perturbation technique. The expressions for non-dimensional skin friction in the direction of flow and rate of heat transfer in terms of Nusselt number at the plate are obtained. The effect of various parameters on the transient velocity, transient temperature, the skin friction and the rate of heat transfer are discussed with the help of graphs.

2000 Mathematics subject classification: 76D.

Key words and phrases: *Free convective, viscous incompressible, slip flow regime, rarefaction.*

*****

I. INTRODUCTION

In recent year the oscillatory free convective flow and heat transfer problems have attracted the attention of a number of scholars due to its important in technological applications. Unsteady oscillatory free convective flows play an important role in chemical engineering, turbo machinery and aerospace technology. Such flows arise due to either unsteady motion of a boundary or boundary temperature. The unsteadiness may also be due to oscillatory free stream velocity or temperature. The free convection heat transfer on a vertical semi-infinite plate has been investigated by many researchers under different physical situation. In most of the investigations the plates were assumed to be maintained at a constant temperature, which is also the temperature of the surrounding fluid.

The transient free convection flow in the unsteady flow phenomena in the cooling towers is useful from practical point of view. However, in some cases free convection flow is enhanced by superimposing oscillating temperature on the mean plate-temperature. In the initial stage of melting adjacent to a heated surface on transient heating of insulating air gapes by heat input at the start-up of furnaces, transient natural convection is of much interest.

In last few decades much attention has been given to the theory of flow through porous medium. The studies concerning porous medium has got importance also because of their occurrence of movements of water. Investigation of such problems is also importance in purification process in petroleum technology and the field of agricultural engineering. Moreover porous medium is widely used in high temperature heat exchange, turbine, blades, jet, nozzles etc.

Soundalgekar and Waves (1977) investigated the unsteady free convection flow past an infinite vertical plate with constant suction and mass transfer. They assumed that the plate temperature oscillates in such a way that its amplitude is small. The effect of fluctuating surface temperature and concentration on natural convection flow from a vertical flat plate was studied by Hussain *et. al*. (2001). The effect of viscous heating on flow past a vertical plate in slip flow regime with periodic temperature variation was studied by Jain and Sharma (2006). Free convective effects on flow past a vertical surface studied by Vedhanayagam *et al*. (1980), Kolar and Sastri (1988), Camargo *et al*. (1996), Ahmed and Kalita (2008) with different boundary conditions. Kim and Vafai (1989) and Harris *et al*. (1997) solved the problem of natural convection flow through porous medium past a vertical plate.

The aim of the present work is to investigate the effect of porosity of the porous medium and heat penetration parameter on an unsteady free convective flow past a vertical plate in slip-flow regime with periodic temperature variation. This work is an extension of the work done by Jain and Sharma (2006) when the medium is porous.

II. MATHEMATICAL FORMULATION

We consider an unsteady two-dimensional free convective flow of a viscous incompressible fluid past an infinite vertical flat porous plate in a slip-flow regime, with variable suction in the form $\bar{v} = -v_0\left(1+\varepsilon A e^{i\bar{\omega}\bar{t}}\right)$. We introduce a coordinate system $\left(\bar{x},\bar{y},\bar{z}\right)$ with x-axis vertically upwards along the stationary plate and y-axis perpendicular to it directed to the fluid region and Z-axis along the width of the stationary plate. Since the plate is considered infinite in $\bar{x}$-direction, all the physical quantities except possibly the pressure p will be independent of $\bar{x}$. Under these assumption, the physical variables are functions of $\bar{y}$ and $\bar{t}$ only. We neglecting viscous dissipation and assuming variation of density in the body force term (Boussinesq's approximation) the problem is governed by the following equations.

<u>Equation of continuity</u>

$$\frac{\partial \bar{v}}{\partial \bar{y}} = 0 \Rightarrow \bar{v} = -v_0\left(1+\varepsilon A e^{i\bar{\omega}\bar{t}}\right) \qquad (2.1)$$

where v_0 and A being constants.

<u>Momentum equation</u>

$$\frac{\partial \bar{u}}{\partial \bar{t}} - v_0\left(1+\varepsilon A e^{i\bar{\omega}\bar{t}}\right)\frac{\partial \bar{u}}{\partial \bar{y}} = g\beta(\bar{T}-\bar{T}_\infty) + \upsilon\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\upsilon \bar{u}}{\bar{k}} \qquad (2.2)$$

<u>Energy equation</u>

$$\frac{\partial \bar{T}}{\partial \bar{t}} - v_0\left(1+\varepsilon A e^{i\bar{\omega}\bar{t}}\right)\frac{\partial \bar{T}}{\partial \bar{y}} = \frac{\kappa}{\rho c_p}\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \upsilon\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 + \bar{S}(\bar{T}_\infty - \bar{T}) \quad (2.3)$$

(Neglecting the higher powers of $\bar{u}$)

The boundary conditions are

at $\bar{y} = 0$; $\bar{u} = \bar{L}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)$, $\bar{T} = \bar{T}_w + \varepsilon\left(\bar{T}_w - \bar{T}_\infty\right)e^{i\bar{\omega}\bar{t}}$

at $\bar{y} \to \infty$; $\bar{u} \to 0, \bar{T} \to \bar{T}_\infty$ (2.4)

Where, $\bar{L}$ is the characteristic dimension in the flow field, κ is the thermal conductivity, A is the suction parameter, $\bar{\omega}$ is the frequency, v_0 is the suction velocity, c_p is the specific heat at constant pressure, ε is the small reference parameter, g is the gravitational acceleration, $\bar{T}_w$ is the temperature at the wall, $\bar{T}_\infty$ is the temperature at infinity, $\bar{k}$ is the porous medium permeability, $\bar{S}$ is the volumetric rate of heat generation, υ is the kinematic viscosity, β is the coefficient of volume expansion for heat transfer and the other symbols have their usual meanings.

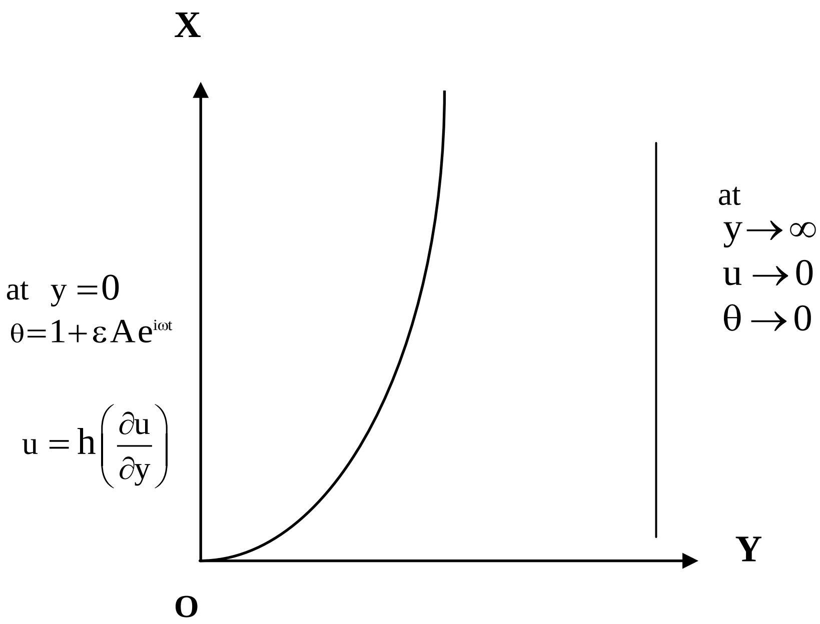

Figure 1: Flow configuration of the problem

We introduce the following non-dimensional quantities

$y = \frac{\bar{y}}{h}$ (distance), $t = \frac{\bar{t}v_0^2}{\upsilon}$ (time), $u = \frac{\bar{u}}{v_0}$ (fluid velocity), $\theta = \frac{\bar{T} - \bar{T}_\infty}{\bar{T}_w - \bar{T}_\infty}$ (fluid temperature), $\omega = \frac{\upsilon\bar{\omega}}{v_0^2}$ (frequency), $G_r = \frac{g\beta\upsilon(\bar{T}_w - \bar{T}_\infty)}{v_0^3}$ (Grashoff number for heat transfer), $P_r = \frac{\mu c_p}{\lambda}$ (Prandtl number),

 ISSN: 2321-8169
149 – 157

$E_c = \dfrac{v_0^2}{c_p\left(\bar{T}_w - \bar{T}_\infty\right)}$ (Eckert number), $h = \dfrac{v_0 \bar{L}}{\upsilon}$ (Rarefaction parameter), $k = \dfrac{v_0^2 \bar{k}}{\upsilon^2}$ (permeability of porous medium), $\alpha = \dfrac{\bar{S}\upsilon}{v_0^2}$ (heat source strength).

The non-dimensional equations with boundary conditions are

$$\frac{\partial u}{\partial t} - \left(1 + \varepsilon A e^{i\omega t}\right)\frac{\partial u}{\partial y} = G_r \theta + \frac{\partial^2 u}{\partial y^2} - \frac{u}{k} \tag{2.5}$$

$$P_r \frac{\partial \theta}{\partial t} - P_r\left(1 + \varepsilon A e^{i\omega t}\right)\frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2} + E_c P_r \left(\frac{\partial u}{\partial y}\right)^2 - P_r \alpha\theta \tag{2.6}$$

Subjected to boundary conditions

$$\left.\begin{aligned} &y = 0; u = h\frac{\partial u}{\partial y}, \theta = 1 + \varepsilon A e^{i\omega t} \\ &y \to \infty; u \to 0, \theta \to 0 \end{aligned}\right\} \tag{2.7}$$

III. SOLUTION OF THE PROBLEM

Assuming the small amplitude oscillation ($\varepsilon \ll 1$), we represent the velocity u and temperature θ, near the plate as

$$\left.\begin{aligned} u &= u_0(y) + \varepsilon e^{i\omega t} u_1(y) + O(\varepsilon^2) \\ \theta &= \theta_0(y) + \varepsilon e^{i\omega t} \theta_1(y) + O(\varepsilon^2) \end{aligned}\right\} \tag{3.1}$$

Substituting (3.1) in (2.5) and (2.6) and by equating the harmonic terms and neglecting the higher powers of ε, the following equations are obtained

$$u_0'' + u_0' - \frac{1}{k}u_0 = -G_r\theta_0 \tag{3.2}$$

$$u_1'' + u_1' - \left(i\omega + \frac{1}{k}\right)u_1 = -G_r\theta_1 - Au_0' \tag{3.3}$$

$$\theta_0'' + P_r\theta_0' - P_r\alpha\theta_0 = -E_c P_r u_0'^2 \tag{3.4}$$

$$\theta_1'' + P_r\theta_1' - P_r\left(\alpha + i\omega\right)\theta_1 = -2E_c P_r u_0' u_1' - P_r A\theta_0' \tag{3.5}$$

The corresponding boundary conditions are

$$\left.\begin{aligned} &y = 0; u_0 = h\frac{\partial u_0}{\partial y}, \theta_0 = 1, u_1 = h\frac{\partial u_1}{\partial y}, \theta_1 = A \\ &y \to \infty; u_0 \to 0, \theta_0 \to 0, u_1 \to 0, \theta_1 \to 0 \end{aligned}\right\} \tag{3.6}$$

 ISSN: 2321-8169
149 – 157

Where, dashes denote differentiation with respect to y.

The equations (3.2) to (3.5) are still coupled for the variables u_0, u_1, θ_0 and θ_1. To solve them we note that E_c <1 for all incompressible fluids and assume that

$$\left.\begin{aligned} u_0 = u_{00} + E_c u_{01} + O\left(E_c^{\,2}\right); u_1 = u_{10} + E_c u_{11} + O\left(E_c^{\,2}\right) \\ \theta_0 = \theta_{00} + E_c \theta_{01} + O\left(E_c^{\,2}\right); \theta_1 = \theta_{10} + E_c \theta_{11} + O\left(E_c^{\,2}\right) \end{aligned}\right\} \quad (3.7)$$

Substituting from (3.7) in the equations (3.2) to (3.5) and equating the terms independent of E_c and coefficient of E_c in each equation and neglecting higher powers of E_c the following equations are obtained

$$u_{00}'' + u_{00}' - \frac{1}{k} u_{00} = -G_r \theta_{00} \quad (3.8)$$

$$u_{01}'' + u_{01}' - \frac{1}{k} M u_{01} = -G_r \theta_{01} \quad (3.9)$$

$$u_{10}'' + u_{10}' - \left(i\omega + \frac{1}{k}\right) u_{10} = -A u_{00}' - G_r \theta_{10} \quad (3.10)$$

$$u_{11}'' + u_{11}' - \left(i\omega + \frac{1}{k}\right) u_{11} = -G_r \theta_{11} - A u_{01}' \quad (3.11)$$

$$\theta_{00}'' + P_r \theta_{00}' - P_r \alpha \theta_{00} - = 0 \quad (3.12)$$

$$\theta_{01}'' + P_r \theta_{01}' - P_r \alpha \theta_{01} = -P_r u_{00}'^{2} \quad (3.13)$$

$$\theta_{10}'' + P_r \theta_{10}' - P_r (\alpha + i\omega) \theta_{10} = -A P_r \theta_{00}' \quad (3.14)$$

$$\theta_{11}'' + P_r \theta_{11}' - P_r (\alpha + i\omega) \theta_{11} = -A P_r \theta_{01}' - 2 P_r u_{00}' u_{10}' \quad (3.15)$$

subject to boundary conditions

$$\text{at } y = 0, \left\{\begin{aligned} u_{00} = h\frac{\partial u_{00}}{\partial y}, u_{01} = h\frac{\partial u_{01}}{\partial y}, u_{10} = h\frac{\partial u_{10}}{\partial y}, u_{11} = h\frac{\partial u_{11}}{\partial y} \\ \theta_{00} = 1, \theta_{01} = 0, \theta_{10} = 1, \theta_{11} = 0 \end{aligned}\right\} \quad (3.16)$$

$$\text{at } y \to \infty, \quad \left\{\begin{aligned} u_{00} \to 0, u_{01} \to 0, u_{10} \to 0, u_{11} \to 0 \\ \theta_{00} \to 0, \theta_{01} \to 0, \theta_{10} \to 0, \theta_{11} \to 0 \end{aligned}\right\} \quad (3.17)$$

Solving the equations from (3.8) to (3.15) with the help of boundary conditions (3.16) and (3.17) we get

$$\theta_{00}(y) = e^{-\gamma_1 y} \quad (3.18)$$

$$u_{00}(y) = A_1 e^{-\gamma_1 y} + A_2 e^{-\gamma_2 y} \quad (3.19)$$

$$\theta_{01}(y) = A_6 e^{-\gamma_3 y} + A_3 e^{-2\gamma_1 y} + A_4 e^{-2\gamma_2 y} + A_5 e^{-(\gamma_1+\gamma_2)y} \quad (3.20)$$

$$u_{01}(y) = A_{11} e^{-\gamma_2 y} + A_7 e^{-\gamma_3 y} + A_8 e^{-2\gamma_1 y} + A_9 e^{-2\gamma_2 y} + A_{10} e^{-(\gamma_1+\gamma_2)y} \quad (3.21)$$

$$\theta_{10}(y) = B_1 e^{-\gamma_1 y} + B_2 e^{-\gamma_2 y} \quad (3.22)$$

$$u_{10}(y) = B_3 e^{-\gamma_1 y} + B_4 e^{-\gamma_2 y} + B_5 e^{-\gamma_4 y} + B_6 e^{-\gamma_5 y} \quad (3.23)$$

$$\theta_{11}(y) = B_{26} e^{-\gamma_4 y} + B_{18} e^{-\gamma_3 y} + B_{19} e^{-2\gamma_1 y} + B_{20} e^{-2\gamma_2 y} + B_{21} e^{-(\gamma_1+\gamma_2)y} + B_{22} e^{-(\gamma_1+\gamma_4)y} + B_{23} e^{-(\gamma_1+\gamma_5)y} + B_{24} e^{-(\gamma_2+\gamma_4)y} + B_{25} e^{-(\gamma_5+\gamma_2)y} \quad (3.24)$$

$$u_{11}(y) = B_{46} e^{-\gamma_5 y} + B_{36} e^{-2\gamma_1 y} + B_{37} e^{-\gamma_2 y} + B_{38} e^{-2\gamma_2 y} + B_{39} e^{-(\gamma_1+\gamma_2)y} + B_{40} e^{-\gamma_3 y} + B_{41} e^{-\gamma_4 y} B_{42} e^{-(\gamma_1+\gamma_4)y} + B_{43} e^{-(\gamma_1+\gamma_5)y} + B_{44} e^{-(\gamma_4+\gamma_2)y} + B_{45} e^{-(\gamma_5+\gamma_2)y} \quad (3.25)$$

Now substituting equations (3.18) to (3.25) in equation (3.1) and splitting into real and imaginary parts we get the expressions for the velocity, temperature and species concentration profiles in the following form:

$$u(y,t) = u_{00}(y) + E_c u_{01}(y) + \varepsilon\left\{\left(u_{10}{}^R + E_c u_{11}{}^R\right)\cos\omega t - \left(u_{10}{}^I + E_c u_{11}{}^I\right)\sin\omega t\right\}$$

$$\theta(y,t) = \theta_{00}(y) + E_c \theta_{01}(y) + \varepsilon\left\{\left(\theta_{10}{}^R + E_c \theta_{11}{}^R\right)\cos\omega t - \left(\theta_{10}{}^I + E_c \theta_{11}{}^I\right)\sin\omega t\right\}$$

where, superscripts R and I respectively represent the real and imaginary parts and

$$\gamma_1 = \frac{P_r + \sqrt{P_r{}^2 + 4P_r\alpha}}{2} = \gamma_3,\ \gamma_2 = \frac{k + \sqrt{k^2 + 4k}}{2k},\ \gamma_4 = \frac{P_r + \sqrt{P_r{}^2 + 4P_r(\alpha + i\omega)}}{2},\ \gamma_5 = \frac{k + \sqrt{(k^2 + 4k) + (4i\omega)}}{2k}.$$

The other constants $A_1, A_2, A_3, \ldots\ldots\ldots\ldots\ldots\ldots A_{11}$ (real) and $B_1, B_2, B_3, \ldots\ \ldots\ldots B_{46}$ (complex) are obtained but not presented here for the sake of brevity.

IV. COEFFICIENT OF SKIN-FRICTION

The shear stress distribution is given by $\bar{\tau} = \mu \frac{\partial \bar{u}}{\partial \bar{y}}$, where μ is the viscosity.

The skin-friction in the non-dimensional form on the plate y = 0 in the direction of free stream is given by

$$\tau = \left.\frac{\mu \frac{\partial \bar{u}}{\partial \bar{y}}}{\rho v_0{}^2}\right]_{\bar{y}=0} = \left.\frac{\partial u}{\partial y}\right]_{y=0} = \left\{\frac{\partial u_0}{\partial y} + \varepsilon e^{i\omega t}\frac{\partial u_1}{\partial y}\right\}_{y=0}$$

ISSN: 2321-8169
149 – 157

V. RATE OF HEAT-TRANSFER

The (Nusselt number) rate of heat transfer between the fluid and the plate is given by

$$N_u = -\frac{\upsilon\left(\dfrac{\partial \bar{T}}{\partial \bar{y}}\right)_{\bar{y}=0}}{v_0\left(\bar{T}_w - \bar{T}_\infty\right)} = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} = -\left\{\frac{\partial \theta_0}{\partial y} + \varepsilon e^{i\omega t}\frac{\partial \theta_1}{\partial y}\right\}_{y=0}$$

VI. RESULTS AND DISCUSSION

in order to get physical insight into the problem the numerical values of velocity distribution, temperature distribution, skin friction, rate of heat transfer in terms of Nusselt number have been obtained and they are demonstrated graphically.

For the purpose of discussing the effects of various parameters on the flow behaviour near the plate, numerical calculations have been carried out for different values of k, α, h and for fixed values of Pr, Gr, ω, Ec, ε, A and ωt. Throughout our investigation the Prandtl number Pr is taken to be equal to 0.71 which corresponds to the air at 20^0 C. The values of Grashof number for heat transfer Gr is taken to be 5 and the Eckert number E_c is assumed to be 0.01. Here, Grashof number for heat transfer Gr > 0 corresponds to externally cooled plate. The values of small reference parameter ε, frequency ω, suction parameter A and ωt are taken 0.001, 1, 2 and π/2 respectively.

Figures (2)-(4) depict the change of behavior of velocity profile u against y under the effects of porosity of porous medium, heat source parameter and rarefaction parameter. From these figures we observe that fluid motion is accelerated due to the porosity of porous medium. But due to the application of heat source fluid velocity is retarded. We also notice from the figure 4 that near the plate the transient velocity is decreased due to reduction of density of the fluid (increase values of rarefaction parameter) but it shows reverse effect at far away from the plate.

Figures (5)-(7) correspond to the change of behavior of temperature against y under the effects of k, α and h. It is inferred from these figures that an increase in porosity and density of the fluid or a decrease in heat strength leads the temperature to increase.

The variation of skin friction τ at the plate y=0 against porosity k for different values of α and h is demonstrated in figure 8. It is seen from the figure that skin friction at the plate y=0 is increased for k but it shows reverse trend for increasing values of h and α. In other words we can say that drag force at the plate increases due to effect of porosity of porous medium but it is decreased due to heat strength and reduction of density of the fluid.

Figures 9 exhibit how the Nusselt number N at the plate is affected by the parameters k, α and h. This figure clearly establishes the fact that N falls due to the effect of heat strength and low density of the fluid whereas it rises under the effect of porosity of porous medium.

 ISSN: 2321-8169
149 – 157

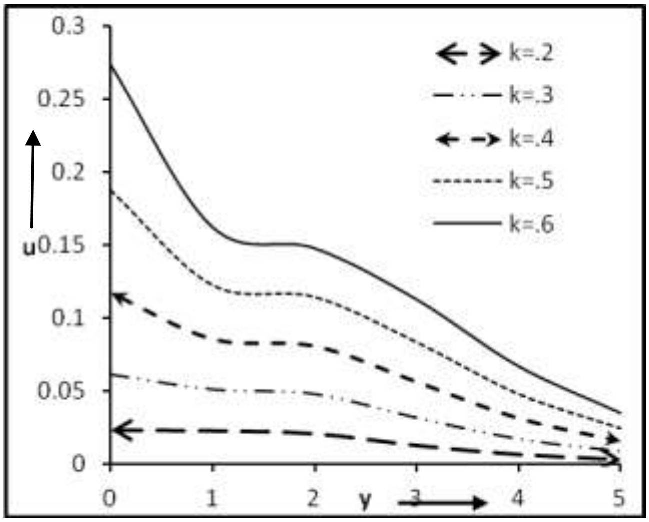

Fig 2: Velocity profile u versus y when α=0.1, h=1

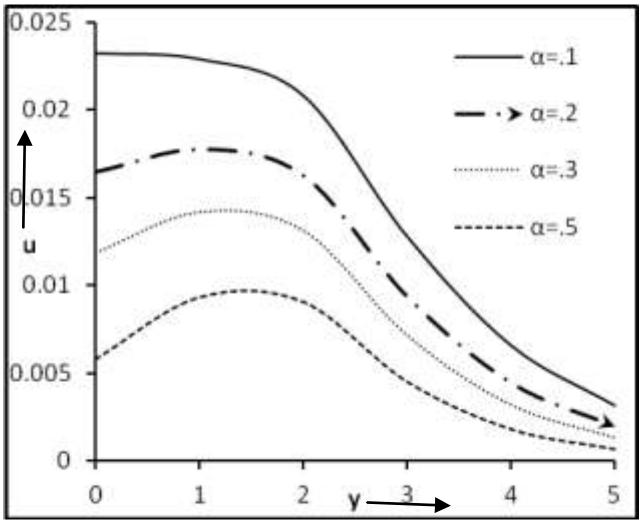

Fig 3: Velocity profile u versus y when k=0.2, h=1

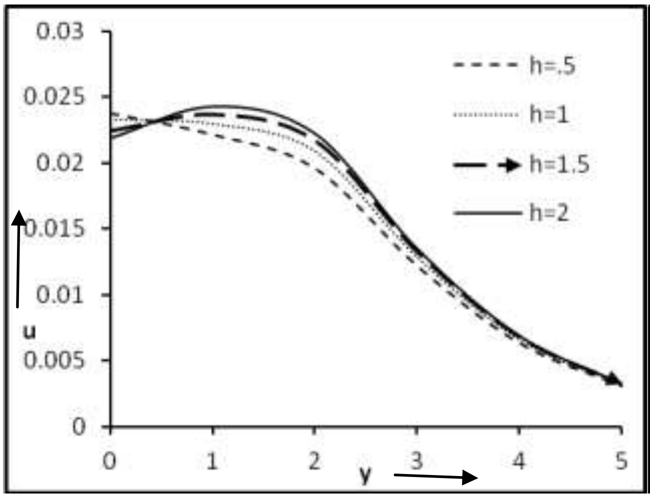

Fig 4: Velocity profile u versus y when α=0.1, k=0.2

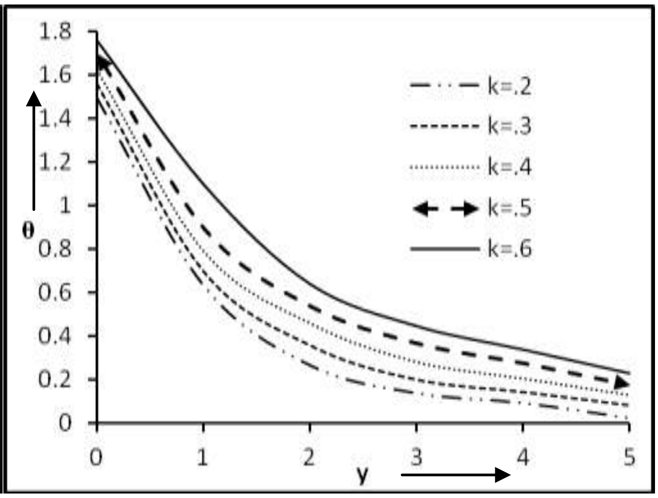

Fig 5: Temperature profile θ versus y when α=0.1, h=1

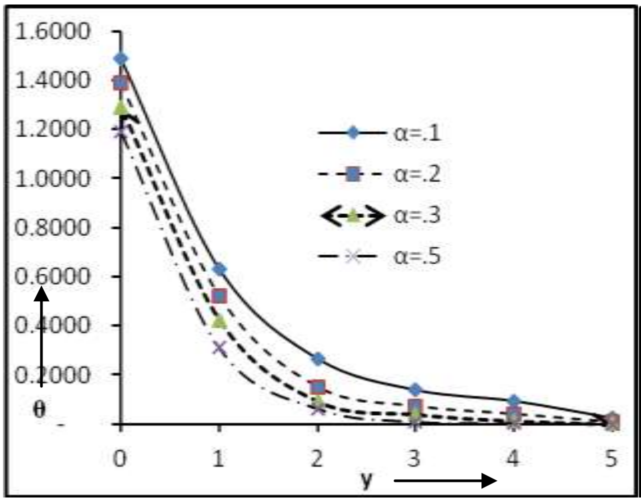

Fig 6: Temperature profile θ versus y when k=0.2, h=1

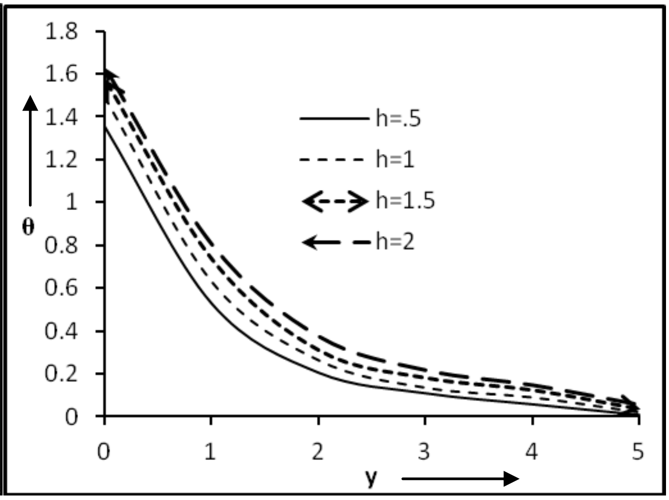

Fig 7: Temperature profile θ versus y when α=0.1, k=0.2

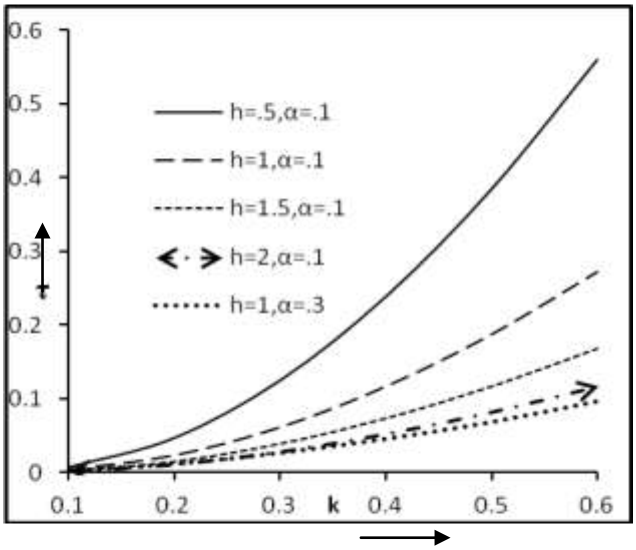

Fig 8: Skin friction τ versus k

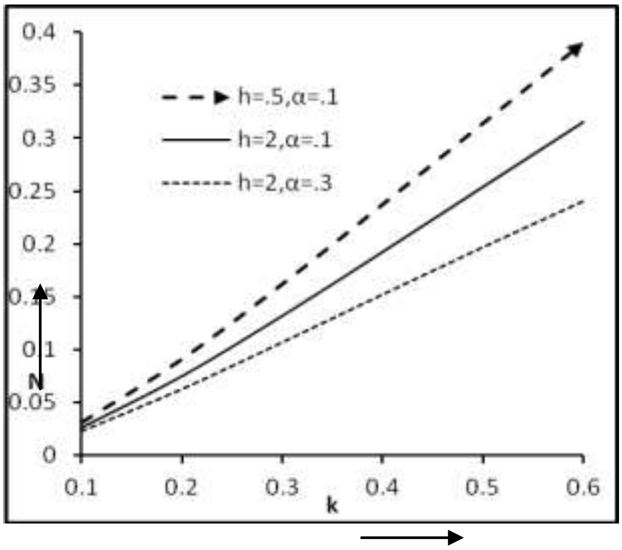

Fig 9: Nusselt number N versus k

VII. CONCLUSIONS

1. Fluid motion is accelerated due to the porosity of porous medium. But due to the application of heat source fluid velocity is retarded
2. Viscous drag increases at the plate due to effect of porosity of porous medium but it is decreased due to heat strength and reduction of density of the fluid.
3. Fluid temperature falls due to strength of heat source but it rises for porosity of the porous medium.
4. Heat flux falls due to the effect of heat strength and reduction of density of the fluid whereas it rises under the effect of porosity of porous medium

REFERENCES

[1] V.M. Soundalgekar, and P. D. Wavre (1977): Unsteady free convection flow past an infinite vertical plate with constant suction and mass transfer," *Int. J. Heat Mass Transfer*, **20**, 1363-1373.

[2] M. Vedhanayagam, R. A. Altenkrich, R. Eichhorn (1980): A transformation of the boundary layer equation for free convection flow past a vertical flate plate with orbitary blowing and wall temperature variation, Int. J. Heat and Mass Transfer, 23. 1236-1288

[3] A. K. Kolar, V. M. Sastri (1988): Free convective transpiration over a vertical plate: a numerical study. Heat and Mass Transfer, 23(6), 327-336.

[4] S. J. Kim , V. Vafai (1989): Analysis of natural convection about a vertical plate embedded in porous medium, Int. J. Heat and Mass Transfer, 32, 665-677.

[5] R. Camargo, E. Luna, C. Trevino (1996): Numerical study of natural convective cooling of a vertical plate, Heat and Mass Transfer, 32, 89-95.

[6] S. D. Harris, D. B. Ingham, I. Pop (1997): Free convection from a vertical platein a porous medium subject to sudden change in surface heat flux, Transport in porous media, 26, 207-226.

[7] M. A. Hussain, S. Hussain, D.A.S. Rees (2001): Influence on fluctuating surface temperature and concentration on natural convection flow from a vertical flat plate, ZAAM, 81, 699-709.

[8] Sunita R. Jain, Pawan K. Sharma (2006): Effect of viscous heating flow past a vertical plate in slip-flow regime with periodic temperature variations, J. Rajasthan Acs. Psy, Sci, Vol. 5, No 2, June 2006, pp 209-220.

[9] N. Ahmed and D. Kalita (2008): MHD oscillatory free convective flow past a vertical plate in slip-flow regime with variable suction and periodic plate temperature. International Journal of Heat and Technology, Italy, Vol 26, n-2.