

Ripple Algorithm to Evaluate the Importance of Network Nodes

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Abstract—In this paper, the ripple algorithm to evaluate the importance of network nodes was proposed, its principle is based on the direct influence of adjacent nodes, and affect farther nodes indirectly by closer ones just like the ripples on the water. Then we defined two judgments, the discrimination of node importance and the accuracy of key node selecting, to verify its efficiency. The greater degree of discrimination and higher accuracy means better efficiency of algorithm. At last we performed experiment on ARPA network, to compare the efficiency of different algorithms, closeness centrality, node deletion, node contraction method, algorithm raised by Zhou Xuan etc. and ripple method. Results show that ripple algorithm is better than the other measures in the discrimination of node importance and the accuracy of key node selecting.

Keywords—network Nodes, Ripple Algorithm, Discrimination of Node Importance, Accuracy of Key Node Selecting

I. INTRODUCTION

With the further research on network reliability and anti-destructive, it is found that under random attack, scale-free networks are more reliable than random networks. And under the selective attack, scale-free networks are more vulnerable than random networks [1]. Therefore, to evaluate the importance of nodes, and protect the principle nodes in network protection is particularly important.

There are many methods to evaluate the importance of nodes in networks, most of which are based on graph theory. However, each algorithm has its limitations and may not be applied to all kinds of networks.

The traditional methods for judging the importance nodes in networks are a lot. Such as, Degree Centrality, Closeness Centrality [2], Betweenness Centrality, etc.

Kitsak et al. [3-4] raised K - Shell Decomposition. Stripping nodes of peripheral layers until the center nodes of inner layers, and judge nodes of inner layers have got more influence. The effect of this method is remarkable for the analysis of spreading. But shortcomings are obvious when this method are used on the analysis of nodes evaluations in communication networks. Firstly, the analysis result is too coarse graining. Secondly, it is the default that nodes in the same layer link the same number of neighbors in the outer layer, which leading to the error.

Node Deletion method analyze network after target node is deleted, observing the change of network topology and some properties, and judge node important degree on the basis of the parameters changing. For instance, the Spanning Tree is widely used in this method. Node Deletion method is limited by the topology of network. If there are multiple nodes, any one of which are deleted may lead to the network separating, the importance of these nodes will not be distinguished. Aiming at the shortcomings of the Node Deletion method, Tan Yuejin et al. raised the Node Contraction method [1], After a node and its neighbor nodes shrink into a new node, if the condensation of the network become better, the node is considered to be more important.

In addition, there are many methods to determine the importance of nodes, but most of them are improvements or integration of existing methods. Such as Chen Jing, Sun Linfu [5] consider Closeness Centrality as the global importance of nodes, and Betweenness of a node within its neighborhood as the local importance, then multiply the two results to get the node important degree. Wu Guo, Fang Liguang and Li Zhong put forward a method based on D - S evidence theory, which evaluate the importance of nodes of complex network by comprehend multi-index [6]. Comprehend degree, Closeness, Betweenness, tenacity, and Condensation degree by using the D-S evidence theory, so as to get the node important degree. Shasha Wang et al. raised Efficiency Centrality method, evaluate the importance of a node by comparing the efficiency changing after the node is deleted [7]. Junyi Wang et al. put forward a new algorithm that take the weight of neighbor nodes into account to evaluate the important degree of the node. This method is more accurate in the importance of nodes in local nodes calculation [8], Zhong-Kui Bao et al. comprehend the length and number of the shortest path between nodes of a network, and transmission rates, then put forward a new algorithm for network node evaluation [9]. Zhou Xuan etc. defined node efficiency and node importance evaluation matrix [10], proposed an evaluation method for finding out the key node in the complex network by importance evaluation matrix. This method comprehend node efficiency, degree, and the importance of neighbor nodes. The contribution of a node for the importance of its neighbor is calculated with its efficiency and degree. However, the calculation of its node efficiency is similar to that of the Closeness Centrality, the distance of every pair of nodes in the network are need to be calculated, which lead to higher computational complexity.

II. RIPPLE ALGORITHM

The changing of the importance of a node in the network will cause a series of changing of other nodes in the network which leading to a chain reaction. According to this characteristic, this paper proposes a method for judging node importance names ripple algorithm. The effect of a node to others is transported with iterative calculation like ripple

layers spreading to the edge of the network. Finally select out the important nodes in the network. We can represent the importance of a node with the weight of its neighbors by a function in a communication network.

There are several characteristics of the backbone routing network of communication network has the following comparing with other networks:

1) There is information exchange between adjacent nodes, and the amount of information exchanged is not necessarily equal.

2) The information transmitted in the network must be sent from a source node to a destination node;

3) The information in the network is from the outside;

4) The information will be sent back to the outside after reaching the destination node.

Based on the above characteristics, we should do the following provisions while evaluating the importance of node in network:

1) The probability that a node v_i sends information to any other node in the network is equal to $\frac{1}{n-1}$;

2) During a unit time, the amount of information received from the outside through any node v_i in the network is equal to the amount of information sent to the outside world through node v_i , all of which are u_i .

On the basis of the provisions mentioned above, we can use the weight of the node to represent the importance of the node, and make the following definition for the node weight:

Definition: the weight of node v_i is the sum of the amount of information received from other nodes and the input information from outside during the unit time, which is represented by w_i .

$$w_i = \sum_{j=1}^n p_{j,i} (w_j - u'_j) + u_i \quad (1)$$

Among Eq.(1), $p_{i,j}$ represents the probability that node v_i sends information to node v_j ; u_i is the amount of information input to the network through node v_i in the unit time. u'_j is the amount of information that v_j send to the outside world during the unit time.

P in Eq.(1) could be a fixed value or variable. When p is constant value, the value of w can be calculated immediately. When p is variable, the equations could be calculated by iterative algorithm, and iterative equations are as follows:

$$w_i^{(k+1)} = \sum_{j=1}^n p_{j,i}^{(k)} (w_j^{(k)} - u'_j) + u_i, \quad k = 1, 2, \dots, m \quad (2)$$

Eq.(2) can be represented as matrix form:

$$\begin{bmatrix} w_1^{(k+1)} \\ w_2^{(k+1)} \\ \vdots \\ w_n^{(k+1)} \end{bmatrix} = \begin{bmatrix} p_{1,1}^{(k)} & p_{2,1}^{(k)} & \dots & p_{n,1}^{(k)} \\ p_{1,2}^{(k)} & p_{2,2}^{(k)} & \dots & p_{n,2}^{(k)} \\ \vdots & \vdots & \ddots & \vdots \\ p_{1,n}^{(k)} & p_{2,n}^{(k)} & \dots & p_{n,n}^{(k)} \end{bmatrix} \begin{bmatrix} (w_1^{(k)} - u'_1) \\ (w_2^{(k)} - u'_2) \\ \vdots \\ (w_n^{(k)} - u'_n) \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}, \quad k = 1, 2, \dots, m \quad (3)$$

In the upper formula, the initial value of the weight vector can be set in advance. The step transfer probability of node v_i to node v_j can be estimated by the importance degree of node v_j relative to node v_i , and two errors should be taken into account, as shown in Figure 1:

1) In computing the relative importance of node v_2 to node v_1 , we should get rid of the influence of the node v_1 to v_2 . Or the

information may spread back and forth between the two nodes, leading to the values of the two nodes be over evaluated;

2) Node v_8 is a common neighbor of v_1 and v_2 . The three nodes make up a triangle. We should get rid of the influence of the node v_8 when evaluating the relative importance of v_2 to v_1 , because the connection of v_1 and v_8 makes the connection of v_2 and v_8 no sense to v_1 .

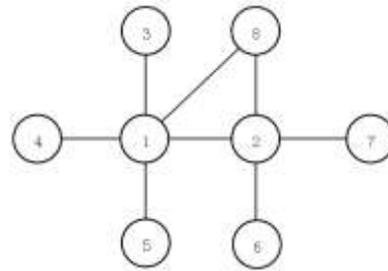


Figure 1. Simple Network

Then we give the following calculation formula:

$$p_{i,j}^{(k+1)} = \frac{a_{i,j} (w_j^{(k+1)} - e_{j,i}^{(k)})}{a_i (W^{(k+1)} - e_i^{(k)})} \quad (4)$$

$i = 1, 2, \dots, n$
 $j = 1, 2, \dots, n$
 $k = 1, 2, \dots, m$

Where, $a_{i,j}$ is an element in the adjacency matrix A ; a_i is a row vector of the i 'th row of the adjacency matrix A ; W is the column vector of weights of all nodes in the network; $e_{i,j}$ is the error generated when calculating the importance of node v_j relative to node v_i ; e_i is the i 'th column vector of the matrix $e_{i,j}$. Let's discuss how to get $e_{i,j}$, for every node of every network, we can list the following formula:

$$e_{j,i}^{(k)} = p_{i,j}^{(k)} (w_i^{(k)} - u'_i) + \sum_{t=1}^n a_{i,t} p_{t,j}^{(k)} (w_t^{(k)} - u'_t) \quad (5)$$

Plug $e_{i,j}$ from Eq.(5) into Eq. (4), calculate the value of $p_{i,j}^{(k+1)}$, and plug it into Eq.(3) to get a new weight of the node.

III. EXPERIMENTAL RESULTS

In order to facilitate the analysis of the performance of different network node importance evaluation methods, this paper analyzes two aspects. The first one is the ability for distinguishing the importance degree of each node in different types of network. The second is the accuracy in searching the specific node with the maximum degree of importance. Therefore, the definition of the importance of network node is given as the following words:

Definition: the specific value of the number of nodes with different importance degree and the number of nodes in the whole network, which is called the discrimination degree of the algorithm in calculating the network.

$$I = \frac{N_d}{N} \quad (6)$$

N is the total number of nodes in the network; N_d is the number of nodes with different importance evaluation.

For complex networks, if not for a particular purpose, it is strictly to prove which one or several nodes are the most important ones in the network. So this paper suggests the

method of "vote" to determine the most important node in the network:

Definition: evaluate the node in the network with several methods. Each method will select one or more nodes to be the most important nodes in the network. And the node which is selected by the most methods should be voted to be the most valuable one. Then according to the outcome of the vote, in turn, calculate the voting accuracy of each algorithm. Voting accuracy calculation formula:

$$C = \frac{n_c}{n_v} \quad (7)$$

In Eq.(7), C is the voting accuracy of algorithm; n_c represents the number of nodes that the algorithm "vote" for and is elected as most important ones in the network; N_c is the number of the most important nodes in the network; n_v is the total number of this algorithm votes for.

Figure 2 shows the network of ARPA (Advanced Research Project Agency), which is a commonly used network topology in testing the evaluation algorithms. The network is experimentally studied by Closeness Centrality, Contraction method, Deletion method, algorithm of literature [10] and Ripple algorithm.

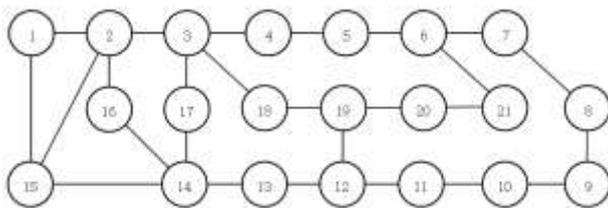


Figure 2. Network Topology of ARPA

TABLE I. THE NODE IMPORTANCE EVALUATION RESULTS OF ARPANETWORK TOPOLOGY

node	Closeness	Contraction	Deletion	literature[10]	Ripple
v ₁	0.0127	0.1270	0.6262	0.1528	2.2295
v ₂	0.0149	0.2514	0.9721	0.2987	10.1247
v ₃	0.0179	0.3080	0.9930	0.2984	10.5049
v ₄	0.0159	0.1911	0.8387	0.1562	2.4741
v ₅	0.0147	<u>0.1911</u>	<u>0.8387</u>	0.1090	1.2307
v ₆	<u>0.0147</u>	0.2550	0.9836	0.1261	1.2167
v ₇	<u>0.0127</u>	0.1835	0.8797	0.0935	1.1356
v ₈	0.0115	<u>0.1835</u>	<u>0.8797</u>	0.0634	1.1304
v ₉	0.0116	<u>0.1835</u>	<u>0.8797</u>	0.0624	1.1303
v ₁₀	<u>0.0127</u>	<u>0.1835</u>	<u>0.8797</u>	0.0680	1.1308
v ₁₁	0.0143	<u>0.1835</u>	<u>0.8797</u>	0.1062	1.1370
v ₁₂	0.0169	0.2615	0.9780	0.1815	1.2300
v ₁₃	<u>0.0159</u>	0.1911	0.8051	0.1839	1.1929
v ₁₄	<u>0.0159</u>	0.2754	0.9864	0.2369	1.9184
v ₁₅	0.0139	0.1855	0.8787	0.2522	2.7991
v ₁₆	0.0137	0.1255	0.6639	0.1978	2.5803
v ₁₇	0.0154	0.1484	0.6977	0.2214	2.7185
v ₁₈	0.0167	0.1667	0.7701	0.1970	2.5061
v ₁₉	0.0172	0.2308	0.9671	0.1845	1.3303
v ₂₀	0.0149	0.1499	0.8279	0.1115	1.1435
v ₂₁	0.0135	<u>0.1499</u>	<u>0.8279</u>	0.1023	1.1364

the number shown in bold is the maximum value; the underlined value is nodes has one or more similar node with the same value. the number in brackets is the ranking of the node's importance in the network.

Have a look at the experimental results, literature [10] and the ripple algorithm have the maximum discrimination degree. Closeness, Contraction and Deletion method discriminate the nodes into fewer parts, with 16, 15 and 15 different important values respectively. In terms of voting accuracy, four of the five methods consider node v₃ as the most important node. Only the literature[10] select node v₂ as

the most important node, turning out that the voting accuracy of it is low.

TABLE II shows the discrimination of node importance evaluation of the five node importance evaluation method. TABLE III shows the node with the max value of node importance evaluation and accuracy of voting respectively of the five node importance evaluation method.

TABLE II. THE DISCRIMINATION OF NODE IMPORTANCE EVALUATION

Network Topology	Clo.	Con.	Del.	Lit. [10]	Rip.
ARPA	0.7619	0.7143	0.7143	1	1

TABLE III. THE NODE WITH THE MAX VALUE OF NODE IMPORTANCE EVALUATION AND ACCURACY OF VOTING

Network Topology	Clo.	Con.	Del.	Lit. [10]	Rip.
ARPA	v ₃	v ₃	v ₃	v ₂	v ₃
	1	1	1	0	1

TABLE II and TABLE III show the compares of the discrimination degree and voting accuracy of the algorithms experimented on ARPA network. The literature [10] and ripples have a good perform at discrimination, but the literature [10] performs worse at voting accuracy. Therefore, considering algorithm at discrimination degree and voting accuracy, the ripple algorithm has the best performance in this experiment.

IV. ALGORITHM PERFORMANCE ANALYSIS

We conduct experiments on simple network, mesh network, fat tree network, ARPA network and random network of different sizes, and obtained the following data:

TABLE IV. THE AVERAGE DISCRIMINATION OF NODE IMPORTANCE EVALUATION

Algorithm	Clo.	Con.	Del.	Lit. [10]	Rip.
Average Discrimination	0.629	0.634	0.467	0.667	0.771

TABLE V. THE NODE WITH THE MAX VALUE OF NODE IMPORTANCE EVALUATION AND AVERAGE ACCURACY OF VOTING

Algorithm	Clo.	Con.	Del.	Lit. [10]	Rip.
Average Voting Accuracy	0.667	0.667	0.417	0.833	0.936

TABLE IV shows that, the average discrimination degree of the Ripple algorithm is the highest in these 5 methods, and the Betweenness method is the lower, and the Deletion method is the lowest. As can be seen from TABLE V, the Ripple algorithm has the highest accuracy in searching the most important nodes in the network, the Betweenness method is the lower, and the Deletion is the least accurate. So the conclusion of the two forms is, Ripple algorithm perform the best, followed by the method of Betweenness, Closeness and Contraction method, Deletion method and literature [10] algorithm integrated performance is poorer. The calculation speed of the Ripple algorithm and the Betweenness method is compared in the following figure.

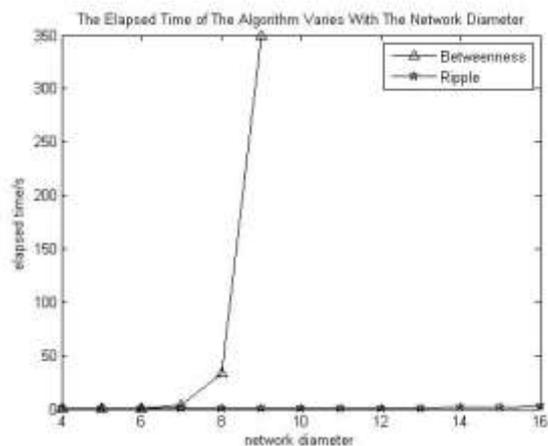


Figure 3. the Elapsed Time of the Algorithm Varies With the Network Diameter

Figure 3 shows the elapsed time of the Ripple and the Betweenness algorithm varies with the network diameter. The elapsed time of the Betweenness method increases exponentially with the increase of network diameter. The elapsed time of the ripple algorithm increases linearly with the increase of network diameter.

Based on the experimental data above, the Ripples algorithm can balance the location of the node in the network, local importance, bridge characteristics and the contribution of the network robustness index to evaluate network node importance degree. So the Ripple algorithms has several advantages such as high degree of discrimination, excellent selection accuracy of importance node, short elapsed time and wide applicable scope.

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