Survey on wavelet based image fusion techniques

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***Abstract*—** Image fusion is the process of combining multiple images into a single image without distortion or loss of information. The techniques related to image fusion are broadly classified as spatial and transform domain methods. In which, the transform domain based wavelet fusion techniques are widely used in different domains like medical, space and military for the fusion of multimodality or multi-focus images. In this paper, an overview of different wavelet transform based methods and its applications for image fusion are discussed and analysed.

Keywords-*image fusion;transform domain; multi-scale decomposition; wavelet based fusion techniques*

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I. INTRODUCTION

Image fusion is the process of combining information from multiple images of different modality, focus, view, sensors and time into a single image with complementary information and without redundant information. The fused image gives a better description than the source images and also it has better quality in the aspects of contrast, edge, texture and information. A good fusion process should include the redundant information but at the same time should not overload the fused image with artifacts or noise.

In the domain of fusion research, many techniques for fusion have been proposed and implemented [1]. Image fusion techniques are broadly classified as pixel level, region level and feature level techniques or spatial (pixel) and transformation techniques. Pixel level techniques are related to spatial information with appropriate fusion rules. In region level technique, the image was decomposed into regions and then pixel level fusion is applied. Feature level techniques are widely used in the data fusion process rather than the image fusion process [2, 3].

The spatial domain methods are: averaging, Brovey method, Principal Component Analysis (PCA), High pass filtering and Intensity Hue Saturation (IHS), Neural Networks (NN), Fuzzy approach etc. The resultant fused image of these methods has low contrast with spatial and spectral distortion. The recent spatial fusion methods related to machine learning techniques are neural networks, fuzzy approach and neuro-fuzzy approach. Neural network is adapted as Pulse Coupled Neural Network (PCNN) algorithm [4] and its variants multichannel PCNN, dual channel PCNN, simplified PCNN are used in fusion for different domain images such as satellite, infrared and medical images, which were taken from multiple sensors [5]. While analyzing the review of fuzzy logic, it has been proposed and used for image fusion from two decades early onwards. Fuzzy logic still plays a vital role in fusion research, since it is practiced for processing the uncertain and ambiguous data derived in real time applications [6]. For the image uncertainty, the membership functions are used to describe the distribution and clustering of the pixel values; to derive fusion operators and decision rules for image fusion. Therefore, image fusion rules (decision rules) based on fuzzy logic uses fuzzy inference to solve the issue of uncertainty in the images. The Mamdani type MIN-SUM-MOM technique, proposed by [6], has been validated for two types of modality (CT and MRI) images only. This method basically utilizes the fuzzy implication operation for MIN algorithm, then the membership degree of the derived output fuzzy sets are calculated using SUM algorithm and the defuzzification of the output set is processed with MOM algorithm. Like the fuzzy approach, neuro-fuzzy approach is developed in three steps: Fuzzy sets are utilized to describe the gray levels of the source images and Neuro-Fuzzy Inference System (NFIS) is established, Hybrid algorithm is applied to train the parameters of membership functions and neuro-fuzzy inference is carried out according to fuzzy rules. Then, the output gray value is calculated and the fused image is obtained [7, 8]. From the analysis, we identified that PCNN method is better than other two methods, since the contrast and information are retained with less spectral degradation. These methods provide a better spatial orientation in the fused image but they suffer from spectral degradation. To overcome that, transform based techniques are emerged. The transform domain based techniques can also be called as Multi-Scale Decomposition (MSD) techniques.

MSD techniques involve transformation of source images into different scales before applying the fusion rules. The commonly used MSD techniques are wavelet, pyramid, contourlet and curvelet transforms. These methods show a better performance in spatial and spectral quality of the fused image compared to spatial methods of fusion.

A few of the pyramid-based methods of fusion are: Filter Subtraction Decimate pyramid (FSD), gradient pyramid, laplacian pyramid, morphological pyramid, ratio pyramid, contrast pyramid. In all these methods each approach has its own limitation in fusion process. For example, contrast pyramid method loses too much information from the source images; ratio pyramid method produces false information that does not exist in the source images and morphological pyramid method creates many false edges.

The curvelet transform has evolved as a tool for the representation of curved shapes in graphical applications. Then, it was extended to the fields of edge detection and image denoising. Recently, some authors have proposed the application of curvelet transform in image fusion. Curvelet based image fusion is carried out in 4 steps as sub-band decomposition, smooth partitioning, renormalization and ridgelet analysis. This approach will retain the edge information, however it will lost the spatial information. To retain both a combined fusion techniques can be performed [9].

Another widely used transformation method is region based contourlet transform. This method retains localization, directionality and anisotropy on the fused image. It is generally implemented in two stages: transformation and decomposition, followed by fusion rule [10, 11]. In the first stage double filter bank scheme is applied for transformation and in the next stage decomposition is done with fusion rules. Finally the fused image is retrieved using reconstruction procedure. However, the computational cost is high in this method.

The majority of fusion techniques are based on wavelet transformation. The two dimensional Discrete Wavelet Transform (DWT) is becoming one of the standard tools for image fusion. The DWT is computed by successive lowpass and highpass filtering of the images. This is called the Mallat algorithm or Mallat-tree decomposition. Its significance is in the manner it connects the continuous time multi-resolution to discrete-time filters. The principle of image fusion using wavelets is to merge the wavelet decompositions of the two original images using fusion methods applied to approximations coefficients and details coefficients.

The further sections, discusses the wavelet based image fusion and its variants in detail with its applications.

II. WAVELET TRANSFORM

Wavelet transforms, provides a framework in which a signal or image is decomposed into levels. Each level corresponds to a coarser resolution or lower frequency band and higher frequency bands. The image fusion process using wavelet transform has three steps in sequence: Decomposition, Applying fusion rule and Reconstruction.

A. Decomposition

In this step, the images are decomposed into approximation information or low-frequency coefficients and detailed information or high-frequency coefficients using the scaling and wavelet functions.

B. Fusion rule

The process of applying fusion rule involves two steps. They are (i) Activity level measurement, (ii) Coefficient combining methods.

1) Activity level measurement: The process of measuring an activity level (quality of a pixel in an image) can be categorized into three methods as Coefficient based, Window based and Region based measures.

Coefficient Based Activity (CBA)- In CBA, the activity level is measured as given in Equation (1).

$$AI(P) = |DI(p)| \;\; or \; AI(P) = |DI(p)|^2 \qquad (1)$$

$AI(p)$ - the activity level of a pixel at position p.

Window Based Activity (WBA)-The WBA employ a small (typically 3 × 3 or 5 × 5) window centred at the current coefficient position. Thus, the activity level $AI(p)$ is determined by the coefficients surrounding p using a small window.

Region Based Activity (RBA)-The regions used in RBA measurement is similar to window based activity but windows with odd shapes only used.

2) Coefficient combining methods: Coefficient combining is the process of combining low-frequency and high-frequency coefficients from the source images.

Selection- The simplest selection method is choose-max (CM) as given in Equation (2).

$$D_z(p) = \begin{Bmatrix} D_x(p), if \; A_x(p) \geq A_y(p) \\ D_y(p), if \; A_x(p) \leq A_y(p) \end{Bmatrix} \qquad (2)$$

In the high-frequency bands, the larger DWT coefficient corresponds to sharpness, brightness changes and thus leads to the salient features in the image such as edges, lines, and region boundaries. Therefore, the CM method is useful in the collection of detailed information.

Weighted average (WA)- For each p, the composite D_Z is obtained using Equation (3).

$$D_z(p) = W_x(p) D_x(p) + W_y(p)\, D_y(p) \qquad (3)$$

The weighting factors $W_X(p)$ and $W_y(p)$ can be deterministic or dependent on the activity levels of X and Y.

Adaptive weighted average (AWA)-The AWA scheme is a special WA scheme that the weight $W_X(p)$ is not deterministic or dependent on the cross-correlation but only relevant to the neighborhood around p, as given in Equation (4).

$$W_x(p) = \left| D_x(p) - \overline{D_x(p)} \right|^2 \qquad (4)$$

where, $D_{x(}p)$ is average value over the neighborhood (say $N \times M$) centered at p. Simply speaking, the weight represents the degree of interest of pixel p.

C. Reconsruction

In this step, the inverse process is applied to display the image format from coefficient level.

In general, wavelet transform is classified into two types. They are: Continuous Wavelet Transform and Discrete Wavelet Transform. The continuous wavelet transform doesn't support multi resolution analysis and scaling functions. Therefore, they are not used in image fusion.

III. DISCRETE WAVELET TRANSFORM (DWT)

A. Model

The DWT is a spatial-frequency decomposition that provides flexible multi-resolution analysis of an image. DWT involves separate filtering and down sampling process in the horizontal and vertical directions. This gives four sub-bands at each scale of the transformation with sensitivity to vertical, horizontal and diagonal frequencies. Of these one subband contains approximation or low-frequency coefficients and other three are related to detailed or high-frequency coefficients as shown in Figure 1. In Figure 1, three level of decomposition is shown, where LL subband has approximation coefficients and HL, LH, HH are subbands of detailed coefficients. In DWT, downsampling is done during decomposition and upsampling is done in reconstruction step. To fuse the subbands general fusion rules like selection, weighted average and adaptive weighted average has been used. However, the DWT in medical image fusion results with shift variant and additive noise in the fused image [12].

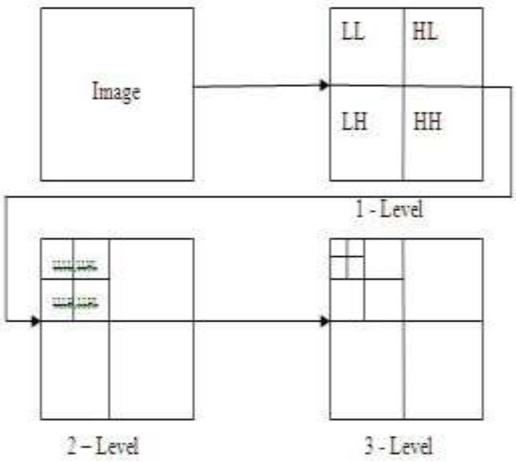

Figure 1. Three level decomposition of DWT.

Decimation of the wavelet coefficients is an intrinsic property of the Discrete Wavelet Transform (DWT). The decimation step removes every other of the coefficients of the current level. Thus the computation of the wavelet transform is faster and more compact in terms of storage space. More importantly, the transformed signal can be perfectly reconstructed from the remaining coefficients. Unfortunately, the decimation is causing shift variance of the wavelet transform. Shift variance means that the DWTs of a signal and its shifted version are not the same. In order to achieve shift invariance, research from different fields with various goals have invented several wavelet transform algorithms. This type of transforms is known under the common name Undecimated Discrete Wavelet Transform (UDWT). In addition to the shift invariance the UDWT gives increased amount of information about the transformed signal compared to the DWT. The modification of DWT i.e. Redundant DWT and UDWT, overcomes the problem of shift variance and preserves the exact edge and spectral information without much of spatial distortion [13]. However, in [14, 15, 16, 17] Yang et al proposed different fusion rules to fuse the low-frequency sub-bands and high-frequency sub-bands separately. In [18], Pei et al proposed a separate fusion rules for the three high-frequency sub-bands. In [19], sub-bands are denoised and different fusion rules are applied to fuse the sub-bands. In [20], Zheng et al analysed optimal wavelet and decomposition levels. In [21], Solanki et al compared DWT with pixel-based techniques and proved its effectiveness over them.

In [22], Li et al studies the feasibility of compressive sensing (CS) principle as an alternate to DWT and in [23], Kim et al proposes an improved additive wavelet. Several variants of DWT are: Shift Invariant Wavelet, Redundant Discrete Wavelet, Complex Wavelet and Stationery Wavelet Transforms.

B. *Shift Invariant Wavelet Transform (SIWT)*

To overcome the shift dependency of the wavelet fusion method, the input images must be decomposed into a shift invariant wavelet representation. This is done by computing the wavelet transform for all possible (circular) shifts of the input images. This results in over-complete wavelet representation.

To overcome the above stated issue, Beylkin developed an efficient computation scheme as given in [24]. Another approach, related to the concept of wavelet frames, was proposed by Unser as in [25]. Each stage of the SIDWT, splits the input image using scaling and wavelet functions. The zeroth level scale coefficients are set equal to the input image, thus defining the complete SIDWT decomposition scheme. In contrast to the standard DWT decomposition scheme, the subsampling is dropped, which resulted a high redundant or over complete wavelet representation.

In [26], Xin et al used SIDWT to fuse the video sequences. In [27], Xu et al used SIWT in exposure fusion method by decomposing luminance information and applying fusion rules. In [28], Nirmala et al decomposed source images using SIDWT and then applied SVM for the fusion of sub-bands.

C. *Redundant Discrete Wavelet Transform*

In DWT, the redundant information is eliminated during the decomposition stage of process. Redundant Discrete Wavelet Transform (RDWT) has been developed and studied from Sun et al, Roux et al, Rajkumar et al and Prasad et al in [29, 30, 31, 32, 33].

D. *Complex Wavelet Transform*

The complex wavelet transform (CWT) is a complex-valued extension to the standard discrete wavelet transform (DWT). It is a two-dimensional wavelet transform which provides multi resolution, sparse representation, and useful characterization of the structure of an image. Further, it has a high degree of shift-invariance in its magnitude. However, a drawback to this transform is that it exhibits 2^d (where d is the dimension of the image being transformed) redundancy compared to a separable (DWT). Variants of CWT are discussed in [34, 35].

C. *Dual-tree Complex Wavelet Transform*

The dual-tree complex wavelet transform (DT-CWT) is an over-complete wavelet transform that provides both good shift invariance and directional selectivity over the DWT. But, there is an increased memory and computational cost. Two fully decimated trees are computed, one for the odd samples and one for the even samples generated at the first level. The DT-CWT has reduced over completeness compared with the SIDWT and is able to distinguish between positive and negative orientations giving six distinct sub-bands at each level, the orientations of which are $\pm 15^0$, $\pm 45^0$, $\pm 75^0$.

Different fusion strategies have been used to fuse DT-CWT sub-bands. Chabi et al proposed min, max, and energy schemes for fusing DT-CWT sub-bands in [36]. In [37], Cai and Hu proposed DT-CWT for image fusion of palm print and palm vein. In [38], Guomundsson and Sveinsson proposed DT-CWT for the fusion of TOF and CCD camera images. In [39], Ioannidou et al applied DT-CWT and SIDWT on quickbird images and proved its effectiveness over DWT and IHS. In [40], Chen and Gao implemented variation of DT-CWT called Double Density DT-CWT for image fusion. Canagarajah et al proposed a region level DT-CWT based image fusion and proved its advantages over pixel level image fusion schemes in [41].

E. *Daubechies Wavelet Transform*

The Daubechies wavelet transform is named after its inventor, the mathematician Ingrid Daubechies. It has been successfully implemented by Singh et al and discussed in [34, 35].

F. *Stationary Wavelet Transform*

The Stationary Wavelet Transform (SWT) is a wavelet transform algorithm designed to overcome the lack of translation-invariance of the discrete wavelet transform (DWT). Translation-invariance is achieved by removing the downsampling and upsampling the filter coefficients by a factor of $2^{(j-1)}$ in the jth level of the algorithm. It was introduced by Holschneider et al in [43]. It is also known as Undecimated wavelet transform. It has been studied from Zhou in [44], Kannan et al in [45] and Guo in [46].

G. LiftingWavelet Transform

Lifting Wavelet Transform (LWT) proposed by Sweldens in [47], is a new wavelet construction method using the lifting scheme of time domain. Its reconstruction can be implemented by adjusting the computation order or the signs during the process of decomposition. Therefore, the computational complexity of the lifting wavelet transform can be reduced to half, making the wavelet transform simple and fast. Therefore, the amount of data can be reduced to three quarters of the original. It was successfully implemented by Yancchun et al, Ramesh et al, Khor, Ganeshan, Ling and Chai et al in [48, 49, 50, 51, 52, 53].

Discrete wavelet transform method and its variants are summarized in the Tables from I to VII.

IV. Conclusion

In [54], James and Dasarathy have performed a survey on medical image fusion techniques. In that paper they have listed all the papers based on medical image fusion. They classified the papers based on methods, image modality and application domains. In addition, the metrics related to fusion are grouped under different categories and explained in [55] can be considered for analyzing the performance of fusion algorithms. In this paper, a complete overview for wavelet transform based image fusion with its types and variants are discussed. Also, the discussed papers are based on multi-focus, multi-sensor, and multi-view image fusion for ease of analysis and understanding.

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TABLE I. DISCRETE WAVELET TRANSFORM

Author	Year	Title	Fusion rule	Image set	Remarks
Yang Yong, et al	2010	Medical image fusion via an effective wavelet-based approach	Maximum selection	Multi-modal medical images (Brain)	Window-based activity measurement. low-frequency- Visibility high-frequency- Variance.
Yang Yong	2010	Multimodal medical image fusion through a new DWT based technique	Maximum selection	Multi-modal medical images (Brain)	Window-based activity measurement. low-frequency- Visibility high-frequency- Variance.

Hong Zheng et al	2010	Study on the optimal parameters of image fusion based on wavelet transform	Nine types of fusion rule (max-max, max-mean, max-min, mean-max, mean-mean, mean-min, min-max, min-mean, min-min)	Multi-focus images	Analyses and compares 54 wavelet bases, 6 kinds of decomposition levels and 9 kinds of fusion operator
Yijian Pei et al	2010	The improved wavelet transform based image fusion algorithm and the quality assessment	Weighted average.	Multi-focus images	Two window-based activity measurement for high-frequency coefficients namely energy and matching degree.
SM Mahbubur Rahman et al	2010	Contrast-based fusion of noisy images using discrete wavelet transform	Fusion using contrast measure	Multi-focus images (out of focus toy)	The source images are decomposed by DWT and the coefficients are denoised.
Yang Yong	2011	A novel DWT based multi-focus image fusion method	Maximum selection	Multi-focus images	Window-based activity measurement. low-frequency- Visibility high-frequency- Variance.
Yang Yong	2011	Performing wavelet based image fusion through different integration schemes	Maximum selection	Multi-focus, Multi-modal medical images	Window-based activity measurement. low-frequency- variance high-frequency- Energy of image gradient.
Chetan Solanki et al	2011	Pixel based and Wavelet based Image fusion methods with their comparative study	Mean-max	Multi-focus images	Compares DWT based image fusion with pixel-based image fusion.
X. Li and S-Y. Qin	2011	Efficient fusion for infrared and visible images based on compressive sensing principle	Self-adaptive weighted average	Multi-spectral images (visible and infrared)	Compressive sensing is used. Activity measurement using standard deviation.
Yonghyun Kim et al	2011	Improved additive-wavelet image fusion	Injects high-frequency values into MS images	Multi-spectral images (PAN and MS)	Transformation of MS images into IHS space. Decomposition of the intensity component using DWT.

TABLE II. SHIFT INVARIANT WAVELET TRANSFORM

Author	Year	Title	Fusion rule	Image set	Remarks
Naoki Saito and Gregory Beylkin	1993	Multi-resolution representations using the autocorrelation functions of compactly supported wavelets	Not specified	Not specified	Shift-invariant wavelet transform is introduced.
Michael Unser et al	1998	Shift-orthogonal wavelet bases	Not specified	Not specified	Shift-invariant wavelet transform is introduced.
Jinhua Wang et al	2011	Exposure fusion based on shift-invariant discrete wavelet transform	Averaging and max selection	Multi-focus images	Enhancement is applied for the fused image.
V.Vaidehi et al	2011	A novel multi-modal image fusion method using shift invariant discrete wavelet transform and support vector machines	Using SVM	Multi-spectral images (visible and infrared)	Activity measurement by energy, entropy and standard deviation.
Wang Xin et al	2013	A new multi-source image sequence fusion algorithm based on SIDWT	Maximum selection for low-frequency and weighted average for high-frequency	Video sequence fusion (visible and infrared)	Window-based activity measurement. low-frequency- contrast high-frequency- edge gradient energy.

TABLE III. REDUNDANT DISCRETE WAVELET TRANSFORM

Author	Year	Title	Fusion rule	Image set	Remarks
M. Roux et al	2010	Multi-focus image fusion based on redundant wavelet transform.	Decision map	Multi-focus images	Activity measurement using edge enhanced details.

Author	Year	Title	Fusion rule	Image set	Remarks
Ling Jiang et al	2010	A new multi-focus image fusion algorithm based on redundant wavelet transform	Max selection	Multi-focus images	Low-frequency: variance high-frequency: absolute value
Rajkumar et al	2010	Redundancy Discrete Wavelet Transform and Contourlet Transform for multi-modality medical image fusion with quantitative analysis	Mean-max	Multi-modal brain images	Activity measurement of high-frequency bands are computed using entropy.
Saurabh Prasad et al	2012	Information fusion in the redundant-wavelet-transform domain for noise-robust hyperspectral classification	Decision-fusion mechanism	Multi-spectral images	Not specified
Chandra Prakash et al	2012	Medical image fusion based on redundancy DWT and Mamdani type min-sum mean-of-max techniques with quantitative analysis	Mean-max	Multi-modal brain images	Activity measurement of high-frequency bands are computed using entropy. RDWT is compared with min-sum-mom algorithm.

TABLE IV. DUAL TREE COMPLEX WAVELET TRANSFORM

Author	Year	Title	Fusion rule	Image set	Remarks
Lewis, J. J. et al	2004	Region-based image fusion using complex wavelets	Weighted averaging	Fusion of infrared and visible images	Variance and entropy can be used as activity measurement.
Runbin Cai and Dewen Hu	2010	Image fusion of palm print and palm vein: multi-spectral palm image fusion	Weighted average and max selection	Palm print and palm vein	Not specified
Guomundsson, S. A., and Johannes R. Sveinsson	2011	TOF-CCD image fusion using complex wavelets	Denoising	Time Of Flight (TOF) and CCD images	Edge and energy measurement
Huimin Lu et al	2013	Multi-frame medical images enhancement on dual tree complex wavelet transform domain.	Weighted fusion	Multi-frame medical images	The decomposed coefficients are denoised.
Guangqiu Chen and Yinhan Gao	2012	Multisource image fusion based on double density dual-tree complex wavelet transform	Weighted average and max selection	Multi-focus images	Activity measurement in high-frequency sub-band by variance.
Negar Chabi et al	2013	An efficient image fusion method based on dual tree complex wavelet transform	Max selection and decision map	Multi-modal, multi-focus and multi-sensor images	Activity measurement in high-frequency sub-band by region energy.

TABLE V. DAUBECHIES COMPLEX WAVELET TRANSFORM

Author	Year	Title	Fusion rule	Image set	Remarks
Rajiv Singh and Ashish Khare	2012	Fusion of multimodal medical images using Daubechies complex wavelet transform–A multiresolution approach	Max selection	Multi-modal brain images	Not specified
Rajiv Singh et al.	2012	Mixed scheme based multimodal medical image fusion using DCWT	Max selection	Multi-modal brain images	Activity measurement in high-frequency by energy.

TABLE VI. STATIONARY WAVELET TRANSFORM

Author	Year	Title	Fusion rule	Image set	Remarks
M. Holschneider	2007	Nonstationary Gaussian processes in wavelet domain: synthesis, estimation, and significance testing	Not specified	Not specified	Stationary wavelet transform is introduced.
Qing Guo and Shutian Liu	2011	Performance analysis of multi-spectral and panchromatic image fusion techniques based on two wavelet discrete approaches	Not specified	Fusion of PAN and MS images	Compares mallet and a trous algorithms.
K. Arulmozhi et al	2010	Area level fusion of multi-focused images using multi-stationary wavelet packet transform	Max selection, max absolute value, selection and averaging	Multi-focus images	Activity measurements like energy and match measure.

Author	Year	Title	Fusion rule	Image set	Remarks
Houkui Zhou	2012	A stationary wavelet transform & curvelet transform based infrared and visible images fusion algorithm	Not specified	Infrared and visible images	Decomposing source images using SWT and again decomposing the low-frequency sub-bands using curvlet transform

TABLE VII. LIFTING WAVELET TRANSFORM

Author	Year	Title	Fusion rule	Image set	Remarks
Chaveli Ramesh and T. Ranjith	2001	Fusion performance measures and a lifting wavelet transform based algorithm for image fusion	Averaging and max selection	Multi-sensor	Fusion symmetry and fusion factor are introduced.
Sudipta Kor and Umashanker Tiwary.	2004	Feature level fusion of multimodal medical images in lifting wavelet transform domain	Decision map	Multi-modal medical images	Modulus of gradient
Yangping Wang, et al	2014	Medical image fusion method based on lifting wavelet transform and dual-channel PCNN	Max selection	Multi-modal Medical image	Low-frequency: spatial frequency high-frequency: PCNN