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An LMI based Robust H_∞ SOF Controller for AVR in an SMIB System

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Abstract— This paper presents the design of an H_∞ (H-infinity) controller to stabilize an uncertain power system using mixed sensitivity approach through an iterative LMI (Linear Matrix Inequality) algorithm. Here a robust control methodology is suggested to improve the voltage regulation of a synchronous generator. H∞ control method is used in this control theory to synthesize controller to obtain robust performance and stabilization. This technique has the advantage over classical control techniques that it is readily applicable to the problems including multivariable systems. The proposed robust controller enhances the performance as well as minimizes the disturbances' effect more effectively. In this paper the controller is designed and simulated under MATLAB/Simulink for electric generator stabilization studies for an SMIB system.

Keywords- *static output feedback, linear matrix Inequality, H_∞ robust controller, loop shaping.*

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I. INTRODUCTION

Power system experiences continuous deviations in operating conditions due to varying generation/load and other factors. The regulation of voltage and stability of system have been considered as an important problem for perfect system operation over the time. The AVR is installed to attain the required voltage regulation and performance.

During the past years, many studies have been done on the designing of an advanced AVR using domain partitioning, robust pole placement, adaptive control etc. Recently many methods have been developed coordinating the voltage regulation and stabilization requirements within a single controller. A desensitized controller designed using LQG approach is given in [3]. Internal Mode Control (IMC) [4]-[5] is also used to attain a trade-off in voltage regulation and stabilization. Despite all the given approaches uses linear control methods, because of control structure complexity, various unknown design aspects and neglecting real constraints, these approaches are not meant to meet the objectives of multi-machine power system. The performances of these methods depend on the time of switching. However, using multiple approaches in such highly non-linear structures increases the complexity of the designed controllers.

In this paper, stabilization and voltage regulation considering practical constraints for feasibility are formulated via an H∞ static output feedback (H∞-SOF) control problem which can be easily solved using an iterative liner matrix inequality (LMI) algorithm. The designed controller ensures effective and direct trade-off between regulation and performance. The controller uses the measured signals and has small proportional gains; giving adequate surety for implementation, mainly in an SMIB system. The developed control methodology bridges the gap between the simplicity and robustness of the system, thus satisfying the constraints. In order to explain the effective performance of the controller, it is applied to an SMIB (Single Machine Infinite Bus) system.

II. GENERALIZED H_∞ MIXED SENSITIVITY FORMULATION

The mixed-sensitivity formulation for the optimization of output control effort and disturbance rejection and is explained in figure 3, where, *K(s)* is the controller to be designed and *G(s)* is the open-loop system model. The equation $S = (I-GK)^{-1}$ gives the sensitivity transfer function from measured output *y(s)* to disturbance input *d(s)*. For reducing the effects of disturbance on the measured output, the minimization of *S* is necessary, i.e. $||S||_\infty$. For optimizing the control effort of the controller in a limited bandwidth, the minimization of the H_∞ norm of the transfer function from the disturbance input *d(s)* to the control input *u(s) is* needed. This is equivalent to minimizing $||KS||_\infty$, where *KS* is the complementary sensitivity transfer function.

$$\min_{K \in S} \left\| \begin{bmatrix} W_1 S \\ W_2 KS \end{bmatrix} \right\|_\infty \tag{1}$$

where S is set of all internally stabilizing controllers *K.*

Both these functions together represent the system. But, it is impossible to minimize both *S* and *KS* together over all the full frequency spectrum. The disturbance rejection is normally needed at low frequencies, so the *S* needs to be minimized at low frequencies, whereas *KS* should be minimized at greater frequencies where 'controlled' control action is required. This is done by using individual weighting functions $W_1(s)$ and $W_2(s)$ for each transfer function. The minimization problem is now transformed according to the transfer function *S* which is less than $\frac{1}{W_1}$ and the transfer function *KS* is less than $\frac{1}{W_2}$. With the inclusion of weights, the problem is reformed as :

To obtain a stabilizing controller, *K*, such that

$$\min_{K \in S} \left\| \begin{bmatrix} W_1 S \\ W_2 KS \end{bmatrix} \right\|_\infty < \gamma \tag{2}$$

III. PROPOSED CONTROL STRATEGY

A. Modelling

The generator is represented by the classical model with all the resistances neglected as shown in figure 2 and the excitation system used for the generator is shown in figure 1. Here E is the value voltage behind reactance, X'_d. Its magnitude is assumed to remain to remain constant at the pre-disturbance value. Let δ be angle by which E' leads the infinite voltage V_∞. As the rotor oscillates during a disturbance, δ changes. Only one winding on the rotor is considered, ie, the field winding as given in [2].

From figure 2, $\hat{I} = \dfrac{E'\angle\delta - V\infty\angle 0^0}{jX_T}$

$$= \frac{E'(\cos\delta + j\sin\delta) - V\infty}{jX_T}$$

where $$X_T = X'_d + X_e \cdot$$

(3)

The acceleration equation is given by

$$2H\frac{d(\Delta w)}{dt} + K_D(\Delta w) = \overline{T}_m - \overline{T}_e \quad (4)$$

Linearising equation we get,

$$\frac{d(\Delta w)}{dt} = \frac{1}{2H}(\Delta T_m - K_S\Delta\delta - K_D\Delta w) \cdot \quad (5)$$

The electrical torque equation is

$$\Delta T_e = K_1\Delta\delta + K_2\Delta E'_q \quad (6)$$

where K_1 and K_2 are constants.

The swing equation is given by

$$\frac{d(\Delta\delta)}{dt} = \Delta w.w_s \cdot \quad (7)$$

The voltage equations for this model are given by

$$pE'_q = -\frac{1}{T'_{d0}}\left[E'_q - (X_d - X'_d)I_d - E_{FD}\right] \cdot \quad (8)$$

Taking the Laplace transform of Eq. 3.13 and rearranging, we get

$$\Delta E_{FD} = (1 + sT'_{d0})\Delta E'_q - (X_d - X'_d)\Delta I_d \cdot$$

Taking the Laplace transform of the expression for ΔI_d and substituting it in the above equation, we get after simplification,

$$\Delta E'_q = \frac{K_3}{1 + K_3T'_{d0}s}\Delta E_{FD} - \frac{K_3K_4}{1 + K_3T'_{d0}s}\Delta\delta \cdot \quad (9)$$

where K_5 and K_6 are constants.

The constants K_1 to K_6 given in the block diagram in figure 3 represents parameters such as impedance, demagnetizing effect etc. The constants K_3 and K_4 are usually positive. Dimensionally K_3 is an impedance. It takes into account the loading effect of external impedance. K_4 is a measure of the demagnetizing effect due to a change in the rotor angle.

From (6), we get

$$\frac{d.(\Delta E'_q)}{dt} = -\frac{K_4}{T'_{d0}}\Delta\delta - \frac{1}{K_3T'_{d0}}\Delta E'_q + \frac{1}{T'_{d0}}\Delta E_{FD} \cdot \quad (10)$$

Substituting for ΔV_T in the differential equation for the voltage transducer block becomes

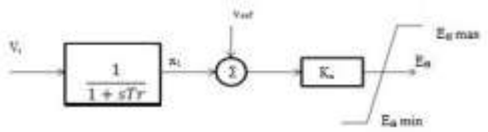

Figure 1: Block diagram of the excitation system

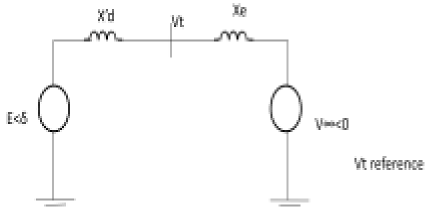

Figure 2: SMIB system

$$\frac{d(\Delta X_1)}{dt} = \frac{K_5}{T_R}\Delta\delta + \frac{K_6}{T_R}\Delta E'_q - \frac{1}{T_R}\Delta X_1 \cdot \quad (11)$$

The above state variable equations are represented in state canonical form as

$$\begin{pmatrix} \Delta\dot{w} \\ \Delta\dot{\delta} \\ \Delta\dot{E}_q \\ \Delta\dot{X}1 \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{pmatrix} \begin{pmatrix} \Delta w \\ \Delta\delta \\ \Delta E'_q \\ \Delta X_1 \end{pmatrix} + \begin{pmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{pmatrix}$$

IV. OVERALL CONTROL FRAMEWORK

The mixed-sensitivity methodology, explained before, is solved here as a generalized H_∞ problem. The first step is to form a generalized regulator *P* as per the the mixed sensitivity formulation taking the assumption D = 0. The state-space formation of the generalized regulator P is given by:

$$\begin{bmatrix} \dot{x} \\ z_1 \\ z_2 \\ y \end{bmatrix} = \begin{bmatrix} A & 0 & B \\ C & I & 0 \\ 0 & 0 & I \\ C & I & 0 \end{bmatrix} \begin{bmatrix} x \\ w \\ u \end{bmatrix} \quad (12)$$

where *x*: state variable vector of the system,
y: measured output,
z_1,z_2: regulated output.
w: disturbance input,
u: control input,
A, *B*, *C*, *D*: state space matrices.

The developed controller is found out from LTI control law $u = K(s)*y$ for an H_∞ performance index $\gamma > 0$, such that: $\|T_{wz}\|_\infty < \gamma$ where, $T_{wz}(s)$ gives the closed-loop transfer function from w to *z*. The state-space equation of controller is given by:

$$\begin{aligned} \dot{x}_k &= A_k x_k + B_k y \\ u &= C_k x_k + D_k y \end{aligned} \quad (13)$$

where A_k, B_k, C_k, D_k are the state space matrices of the controller.

The transfer function $T_{wz}(s)$ between w and *z* is derived as

$$T_{wz}(s) = D_{cl} + C_{cl}\,(sI - A_{cl})\text{- } B_{cl}$$

where,

$$A_{cl} = \begin{bmatrix} A + B_2 D_k C_2 & B_2 C_k \\ B_k C_2 & A_k \end{bmatrix}$$

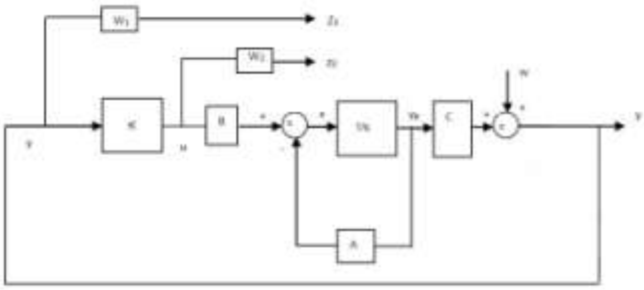

Figure 3: Simplified block diagram of the augmented plant including controller

$$B_{cl} = \begin{bmatrix} B_1 + B_2 D_k D_{21} \\ B_k D_{21} \end{bmatrix}$$

$$C_{cl} = \begin{bmatrix} C_1 + D_{12} D_k C_2 & D_{12} C_k \end{bmatrix}$$

$$D_{cl} = \begin{bmatrix} D_{11} + D_{12} D_k D_{21} \end{bmatrix} \qquad (14)$$

V. LINEAR MATRIX INEQUALITY FORMULATION

The bounded real lemma and the Schur's formula for the determinant of a partitioned matrix is equivalent to the existence of a solution $X_\infty = X_\infty^T > 0$ to the following matrix inequality as in [15].

$$\begin{pmatrix} X_\infty A_{cl} + A_{cl}^T X_\infty & B_{cl} & X_\infty C_{cl}^T \\ B_{cl}^T & -\gamma_i & D_{cl}^T \\ C_{cl} & X_\infty & -\gamma_i \end{pmatrix} < 0 \qquad (15)$$

The design specifications hold for positive semi-definite matrices X_∞ for better feasibility. The controller parameters in the matrix make closed loop matrices non-linear. The controller variables are linearized in terms of an unknown matrix

$$X = \begin{pmatrix} R & M \\ M^T & U \end{pmatrix} \qquad (16)$$

Pre- and post-multiplying the inequality by the linearized matrices $\prod_2^T$ and $\prod_2$, where $\prod_1 = \begin{pmatrix} R & I \\ M^T & 0 \end{pmatrix}$ and $\prod_2 = \begin{pmatrix} I & S \\ 0 & N^T \end{pmatrix}$, respectively and carrying out appropriate change of variables, the following LMI is obtained

$$S\begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{bmatrix} < 0$$

where,

$$\psi_{11} = \begin{bmatrix} AR + RA^T + B_2\hat{C} + \hat{C}^T B_2^T & B_1 + B_2\hat{D}D_{21} \\ \left(B_1 + B_2\hat{D}_{21}\right)^T & -\gamma I \end{bmatrix}$$

$$\psi_{21} = \begin{bmatrix} \hat{A} + \left(A + B_2\hat{D}C_2\right)^T & SB_1 + \hat{B}D_{21} \\ C_1 R + D_{12}\hat{C} & D_{11} + D_{12}\hat{D}D_{21} \end{bmatrix}$$

$$\psi_{22} = \begin{bmatrix} A^T S + SA + \hat{B}C_2 + C_2^T \hat{B}^T & \left(C_1 + D_{12}\hat{D}C_2\right)^T \\ C_1 + D_{12}\hat{D}C_2 & -\gamma I \end{bmatrix}$$

The new controller variables are defined as:

$$\hat{A} = NA_k M^T + NB_k C_2 R + SB_2 C_k M^T + S(A + B_2 D_k C_2)R$$
$$\hat{B} = Nk + SB_2 D_k$$
$$\hat{C} = C_k M^T + D_k C_2 R$$
$$\hat{D} = D_k \qquad (17)$$

VI. APPLICATION IN SMIB SYSTEM

To demonstrate the performance of the developed controller, an SMIB system is taken as the test study system which is given in figure 2. The generator is having excitation system as shown in figure 1. For simulation 100 MVA is considered as the system base MVA.

A synchronous generator is connected to bus 1 and bus 2 is taken as the infinite bus. The objective is to design a robust controller for the synchronous generator. First of all, a full order robust dynamic controller with the structure given in equation (2.2) is designed.

Then applying the proposed H_∞-SOF control method, optimal gain of the controller is obtained through an iterative LMI approach. The standard practice in H-infinity mixed-sensitivity design is choosing the weight $W_1(s)$ for rejection of output disturbance and $W_2(s)$ for reducing the control effort in the high frequency ranges. In view of that, the weights were chosen as constant weights with a value 2.

VII. SIMULATION RESULTS

To explain the performance of the developed method, simulations were carried out. The robustness of the system with H_∞ SOF controller is tested for voltage deviation and system disturbance. The analysis of the damping ratios of corresponding eigen values is given as plots in table 1.Eigen values represent the state of stability of a system. Table 1 gives the details of eigen values of plant before and after applying the controller. The eigen values were more stabilized with the implementation of the developed controller by making them more closer towards the negative infinity portion of the complex s-plane. The damping ratios were minimized due to the application of the controller thereby enhancing the robustness of the system as shown in table 2.

A state variable is the one that describes the mathematical state of a system. The state of a system explains more about the system and to determine its future characteristics in the case of absence of any external forces affecting the modelled system. Models which consist of first-order differential coupled equations are in state-variable form.

Table 1: Eigen values and damping ratios

No	Plant with conventional AVR		Plant with robust AVR	
	Eigen value	***Damping ratio***	***Eigen value***	***Damping ratio***
1	-0.0+0.0007i	0.4793	-0.093+6.54i	0.4789
2	-0.0+0.0007i	0.4793	-0.093+6.54i	0.4789
3	-0.003+0.001i	0.2051	-0.51 +17.43i	0.2011
4	-0.003+0.001i	0.2051	-0.51 +17.43i	0.2011

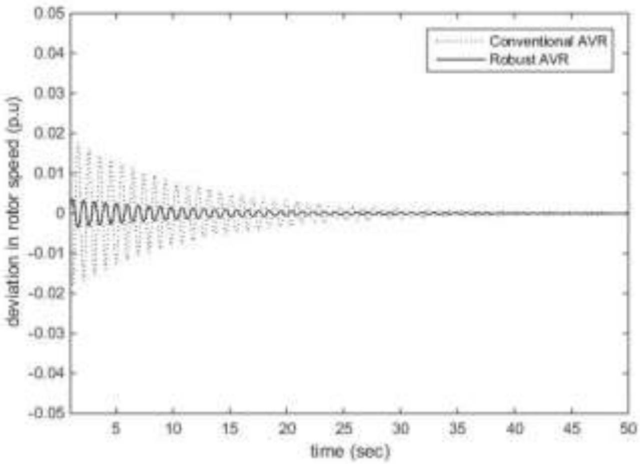

Figure 4: Rotor speed deviation response

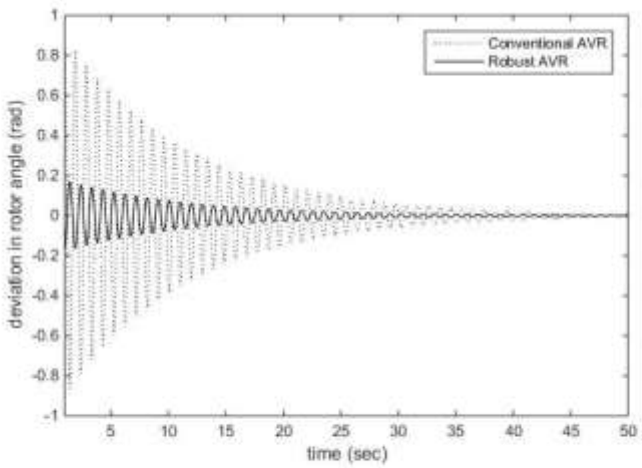

Figure 5: Rotor angle deviation response

A perturbation of 0.2 percent was included in the designing of the state variables (w, δ E_{fd}, V_t). The unforced response of the state-space model G, i.e nominal plant and the system including the conventional AVR with the initial condition of the states given in vector form X_0 was calculated from the constants of the machines. The initial state vector X_0

is a [4*1] sized matrix. The response was within the specified limits. The system exhibited more robustness with the presence of the developed controller. The settling time of the disturbance was reduced. The amplitude of the oscillations was reduced by 5-6%. Also the deviation in terminal voltage was reduced to a near zero value compared with the case of the system with the conventional controller. The system exhibited more robustness with the inclusion of the developed robust H_∞ controller. The initial response of the rotor speed and angle deviation is given in figures 4 and 5 respectively. The settling time of the oscillations is reduced as well by 5-6 sec. Figures 6 and 7 gives the initial responses of deviations in excitation voltage and terminal voltage respectively which also exhibits the reduction as explained previously.

Once the design is complete, it is required to verify the objective constraint. If the designed controller does not satisfy

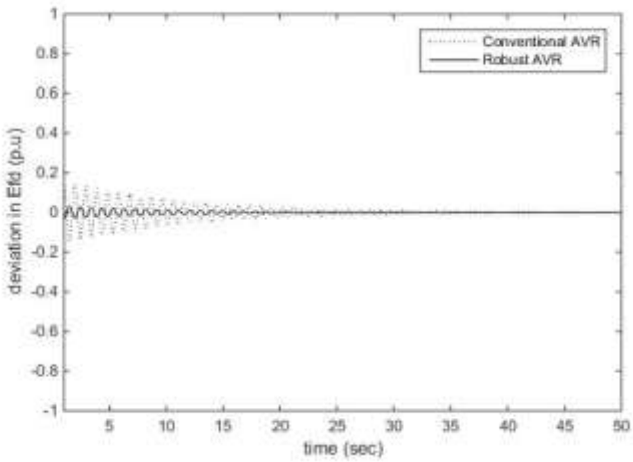

Figure 6 :Excitation voltage deviation response

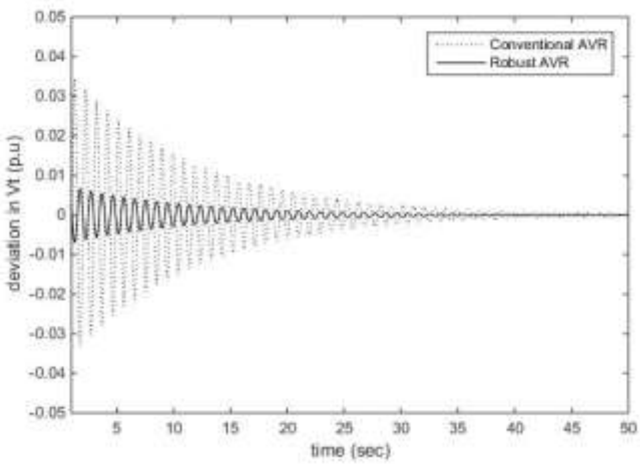

Figure 7: Terminal voltage deviation response

the constraint, then the controller should be redesigned by choosing a different set of weighting functions. It is shown that improvement in the high speed response of the controller was achieved.

The mathematical model of a control system gives an approximate true physical reality of system dynamics. The typical generation of discrepancy includes the unmodelled (high frequency) dynamics, neglected non-linearities in modelling, effect of deliberate reduced order models. And system parameter variations due to changes in environmental, torn-and-worn factors.

An uncertainty was inserted in the system model to ascertain the robustness of the controller. The system withstood the effect of disturbance effectively. The disturbance introduced was a parametric uncertainty in the state space representation of the system. The given uncertainty was a 50% change in the value of the state matrix elements {i.e.ureal('p1',a1(2,1),'pe',50)}.The system with conventional AVR proved to be unstable. The response of system variables with uncertainty including robust AVR was plotted and is given in figures 8 –11 which explains the initial responses of rotor speed and angle, excitation and terminal voltages respectively. The system performance was improved with the robust AVR.

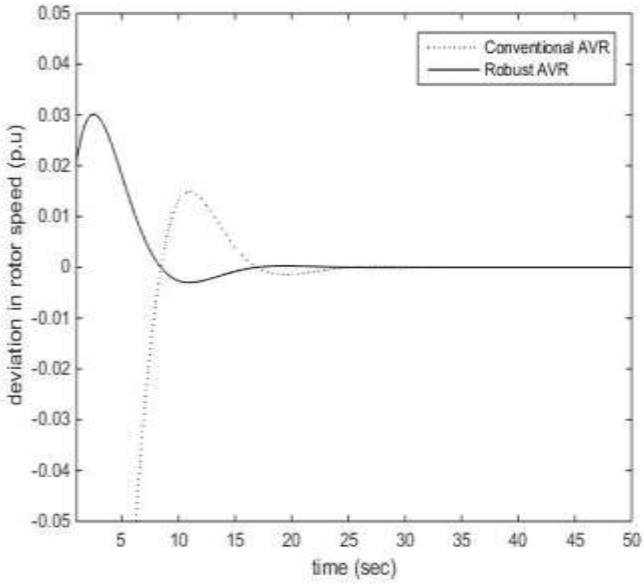

Figure 8: Rotor speed deviation response

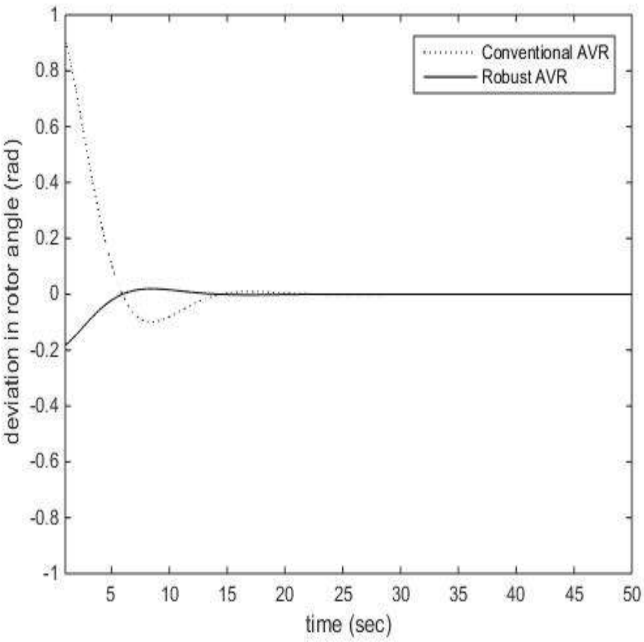

Figure 9: Rotor angle deviation response

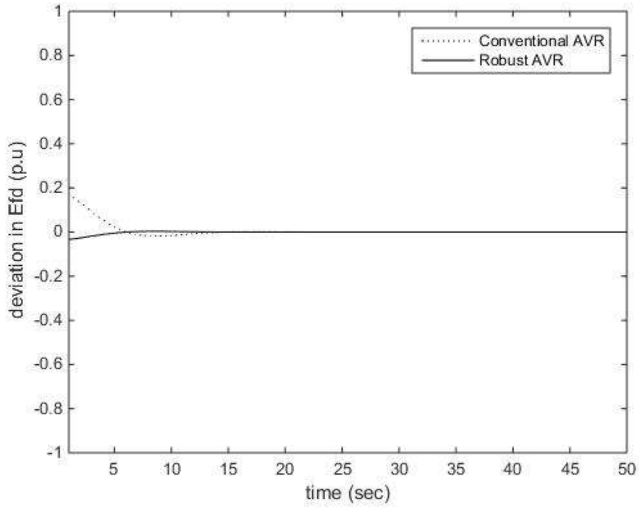

Figure 10 : Excitation voltage deviation response

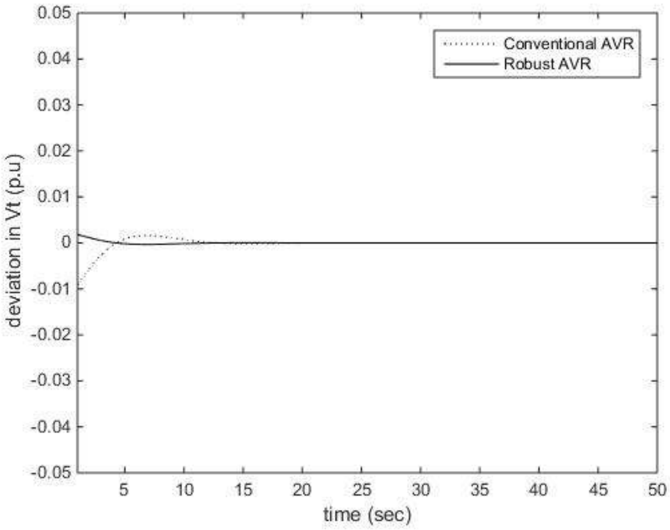

Figure 11: Terminal voltage deviation response

The step response of a control system in its initial states includes the time evolution of its outputs when its inputs are step functions. Step response of a system which is dynamic gives information about its stability, and on its ability to attain one stationary state after starting from another. Figure 12 shows the step response of system including the robust AVR along with a disturbance. A disturbance of 20% was inserted in the reference voltage at t = 80 sec and the disturbed ssignal was fed to controller. The controller damped the disturbance oscillations effectively. The iteration of the performance was done step by step and the best closed loop gain calculated was 0.200909 which is within the standard limit.

The gamma value was minimized to a value as possible and thus it can be concluded that the developed controller gives maximum performance and enhances stability. It is proved that the developed robust AVR improved the control over terminal voltage than the conventional AVR controller.

VIII. CONCLUSION

This paper presents an excitation control method using H∞ SOF method using an iterative LMI approach for enhancing the robust performance of the system. The developed method was applied to an SMIB system, and also the results were obtained

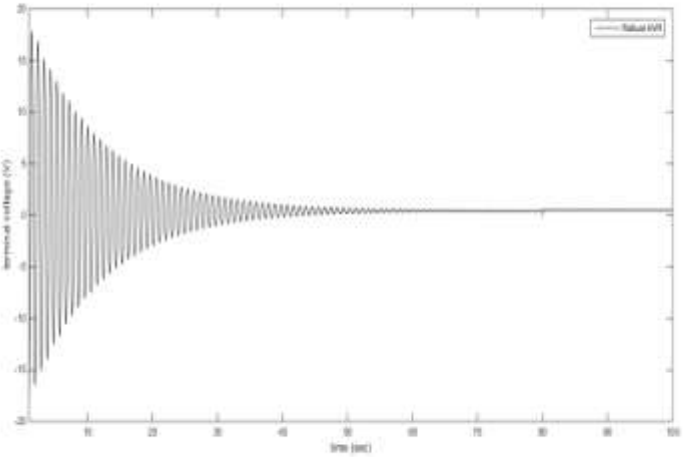

Figure 12: Step response

to be feasible. The performance of the system with robust AVR was found to be satisfactory over a wide range of operating conditions making an appropriate and effective trade-off between performance and voltage regulation by not changing the fundamental AVR concepts. The Ease and flexibility of design to give a good feasible solution, are the main pros of the deigned method used in this paper. The results also proved better performance and robustness in the case of uncertainty (i.e. test of robustness). Therefore, it can be concluded that the developed method improves efficiency, enhance dynamic performance of the power system and provides more robustness thus increasing its stability limit.

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APPENDIX

parameters	**Value**
basemva	588 MVA
Vinf	1.0 pu
freq	50 Hz
H	3.07 pu
X_q	2.15 pu
X_d	2.35 pu
T_{do}	6.0 pu
X'_d	0.253 pu
R_a	0.0023 pu
T_r	0.02 pu
ka	200
k_d	4
r_e	0.1 pu
X_e	0.65 pu