

Where A is an $m \times n$ matrix, x is a column vector with n entries, and b is a column vector with m entries.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

A. Solution set

A solution of a linear system is an assignment of values to the variables x_1, x_2, \dots, x_n such that each of the equations is satisfied. The set of all possible solutions is called the solution set [13].

A linear system may behave in any one of three possible ways:

- 1-The system has infinitely many solutions.
- 2-The system has a single unique solution.
- 3-The system has no solution.

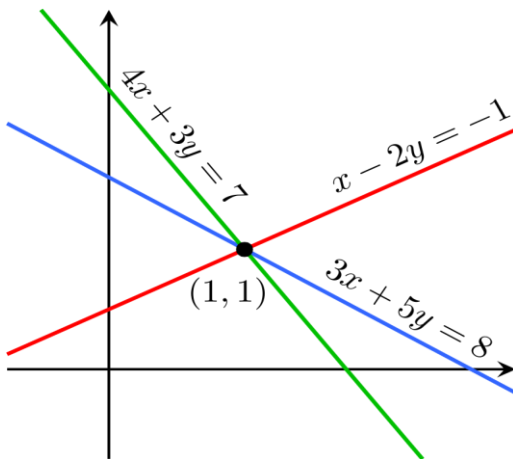


Figure 1. Graph for three independent equations [18].

B. Independence

The equations of a linear system are independent if none of the equations can be derived algebraically from the others. When the equations are independent, each equation contains new information about the variables, and removing any of the equations increases the size of the solution set [14-17]. For linear equations, logical independence is the same as linear. For example, the following equations;

$$3x + 2y = 6 \quad \text{and} \quad 6x + 4y = 12$$

are not independent, they are the same equation when scaled by a factor of two, and they would produce identical graphs. This is an example of equivalence in a system of linear equations. For a more complicated example, the equations:

$$\begin{aligned} x - 2y &= -1 \\ 3x + 5y &= 8 \\ 4x + 3y &= 7 \end{aligned}$$

are not independent, because the third equation is the sum of the other two. Indeed, any one of these equations can be

derived from the other two, and any one of the equations can be removed without affecting the solution set. The graphs of these equations “Fig. 1” are three lines that intersect at a single point.

II. SOLVING A LINEAR SYSTEM

There are several algorithms for solving a system of linear equations. When the solution set is finite, it is reduced to a single element. In this case, the unique solution is described by a sequence of equations whose left-hand sides are the names of the unknowns and right-hand sides are the corresponding values, for example $(x = 3, y = -2, z = 6)$. When an order on the unknowns has been fixed, for example the alphabetical order the solution may be described as a vector of values, like $(3, -2, 6)$ for the previous example [19].

It can be difficult to describe a set with infinite solutions. Typically, some of the variables are designated as free (or independent, or as parameters), meaning that they are allowed to take any value, while the remaining variables are dependent on the values of the free variables [20]. For example, consider the following system:

$$\begin{aligned} x + 3y - 2z &= 5 \\ 3x + 5y + 6z &= 7 \end{aligned}$$

The solution set to this system can be described by the following equations:

$$x = -7z - 1 \quad \text{and} \quad y = 3z + 2.$$

Here z is the free variable, while x and y are dependent on z . Any point in the solution set can be obtained by first choosing a value for z , and then computing the corresponding values for x and y . Each free variable gives the solution space one degree of freedom, the number of which is equal to the dimension of the solution set. For example, the solution set for the above equation is a line, since a point in the solution set can be chosen by specifying the value of the parameter z . An infinite solution of higher order may describe a plane, or higher-dimensional set [21].

Different choices for the free variables may lead to different descriptions of the same solution set. For example, the solution to the above equations can alternatively be described as follows:

$$y = -\frac{3}{7}x + \frac{11}{7} \quad \text{and} \quad z = -\frac{1}{7}x - \frac{1}{7}.$$

Here x is the free variable, and y and z are dependent.

A. Elimination of variables

The simplest method for solving a system of linear equations is to repeatedly eliminate variables. This method can be described as follows:

1. In the first equation, solve for one of the variables in terms of the others.

2. Substitute this expression into the remaining equations. This yields a system of equations with one fewer equation and one fewer unknown.
3. Continue until you have reduced the system to a single linear equation.
4. Solve this equation and then back-substitute until the entire solution is found.

For example, consider the following system:

$$\begin{aligned} x + 3y - 2z &= 5 \\ 3x + 5y + 6z &= 7 \\ 2x + 4y + 3z &= 8 \end{aligned}$$

Solving the first equation for x gives;

$x = 5 + 2z - 3y$, and plugging this into the second and third equation yields:

$$\begin{aligned} -4y + 12z &= -8 \\ -2y + 7z &= -2 \end{aligned}$$

Solving the first of these equations for y yields $y = 2 + 3z$, and plugging this into the second equation yields $z = 2$. We now have:

$$\begin{aligned} x &= 5 + 2z - 3y \\ y &= 2 + 3z \\ z &= 2 \end{aligned}$$

Substituting $z = 2$ into the second equation gives $y = 8$, and substituting $z = 2$ and $y = 8$ into the first equation yields $x = -15$. Therefore, the solution set is the single point $(x, y, z) = (-15, 8, 2)$.

B. Row reduction

In row reduction, the linear system is represented as an augmented matrix:

$$\left[\begin{array}{ccc|c} 1 & 3 & -2 & 5 \\ 3 & 5 & 6 & 7 \\ 2 & 4 & 3 & 8 \end{array} \right].$$

This matrix is then modified using elementary row operations until it reaches reduced row echelon form. There are three types of elementary row operations [22]:

- Type 1: Swap the positions of two rows.
- Type 2: Multiply a row by a nonzero scalar.
- Type 3: Add to one row a scalar multiple of another.

Because these operations are reversible, the augmented matrix produced always represents a linear system that is equivalent to the original.

There are several specific algorithms to row-reduce an augmented matrix, the simplest of which are Gaussian elimination and Gauss-Jordan elimination. The following computation shows Gauss-Jordan elimination applied to the matrix above [23]:

$$\begin{aligned} &\left[\begin{array}{ccc|c} 1 & 3 & -2 & 5 \\ 3 & 5 & 6 & 7 \\ 2 & 4 & 3 & 8 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 3 & -2 & 5 \\ 0 & -4 & 12 & -8 \\ 2 & 4 & 3 & 8 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 3 & -2 & 5 \\ 0 & -4 & 12 & -8 \\ 0 & -2 & 7 & -2 \end{array} \right] \sim \\ &\sim \left[\begin{array}{ccc|c} 1 & 3 & -2 & 5 \\ 0 & 1 & -3 & 2 \\ 0 & -2 & 7 & -2 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 3 & -2 & 5 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & 1 & 2 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 3 & -2 & 5 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 2 \end{array} \right] \sim \\ &\sim \left[\begin{array}{ccc|c} 1 & 3 & 0 & 9 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 2 \end{array} \right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & -15 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 2 \end{array} \right] \end{aligned}$$

The last matrix is in reduced row echelon form, and represents the system $x = -15, y = 8, z = 2$. A comparison with the example in the previous section on the algebraic elimination of variables shows that these two methods are in fact the same; the difference lies in how the computations are written down.

C. Cramer's rule

It is an explicit formula for the solution of a system of linear equations, with each variable given by a quotient of two determinants [24]. For example, the solution to the system:

$$\begin{aligned} x + 3y - 2z &= 5 \\ 3x + 5y + 6z &= 7 \\ 2x + 4y + 3z &= 8 \end{aligned}$$

is given by:

$$x = \frac{\begin{vmatrix} 5 & 3 & -2 \\ 7 & 5 & 6 \\ 8 & 4 & 3 \end{vmatrix}}{\begin{vmatrix} 1 & 3 & -2 \\ 3 & 5 & 6 \\ 2 & 4 & 3 \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} 1 & 5 & -2 \\ 3 & 7 & 6 \\ 2 & 8 & 3 \end{vmatrix}}{\begin{vmatrix} 1 & 3 & -2 \\ 3 & 5 & 6 \\ 2 & 4 & 3 \end{vmatrix}}, \quad z = \frac{\begin{vmatrix} 1 & 3 & 5 \\ 3 & 5 & 7 \\ 2 & 4 & 8 \end{vmatrix}}{\begin{vmatrix} 1 & 3 & -2 \\ 3 & 5 & 6 \\ 2 & 4 & 3 \end{vmatrix}}.$$

For each variable, the denominator is the determinant of the matrix of coefficients (must be not equal to zero), while the numerator is the determinant of a matrix in which one column has been replaced by the vector of constant terms. Though Cramer's rule is important theoretically, it has little practical value for large matrices, since the computation of large determinants is somewhat cumbersome. (Indeed, large determinants are most easily computed using row reduction.) Further, Cramer's rule has very poor numerical properties, making it unsuitable for solving even small systems reliably, unless the operations are performed in rational arithmetic with unbounded precision.

D. Matrix solution

If the equation system is expressed in the matrix form $Ax = b$, the entire solution set can also be expressed in matrix form. If the matrix A is square (has m rows

and $n=m$ columns) and has full rank (all m rows are independent), then the system has a unique solution given by

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$$

where \mathbf{A}^{-1} is the inverse of \mathbf{A} . More generally, regardless of whether $m=n$ or not and regardless of the rank of \mathbf{A} , all solutions (if any exist) are given using the Moore-Penrose pseudoinverse of \mathbf{A} , denoted \mathbf{A}^g , as follows:

$$\mathbf{x} = \mathbf{A}^g\mathbf{b} + (\mathbf{I} - \mathbf{A}^g\mathbf{A})\mathbf{w}$$

where \mathbf{W} is a vector of free parameters that ranges over all possible $n \times 1$ vectors. A necessary and sufficient condition for any solution(s) to exist is that the potential solution obtained using $\mathbf{w} = \mathbf{0}$ satisfy $\mathbf{A}\mathbf{x} = \mathbf{b}$ — that is, that $\mathbf{A}\mathbf{A}^g\mathbf{b} = \mathbf{b}$. If this condition does not hold, the equation system is inconsistent and has no solution. If the condition holds, the system is consistent and at least one solution exists [25]. For example, in the above-mentioned case in which \mathbf{A} is square and of full rank, \mathbf{A}^g simply equals \mathbf{A}^{-1} and the general solution equation simplifies to:

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b} + (\mathbf{I} - \mathbf{A}^{-1}\mathbf{A})\mathbf{w} = \mathbf{A}^{-1}\mathbf{b} + (\mathbf{I} - \mathbf{I})\mathbf{w} = \mathbf{A}^{-1}\mathbf{b}$$

as previously stated, where \mathbf{W} has completely dropped out of the solution, leaving only a single solution. In other cases, though, \mathbf{W} remains and hence an infinitude of potential values of the free parameter vector \mathbf{W} give an infinitude of solutions of the equation.

E. Graphical User Interface

A graphical user interface (GUI) is a pictorial interface to a program. A good GUI can make programs easier to use by providing them with a consistent appearance and with intuitive controls like pushbuttons, list boxes, sliders, menus, and so on. The GUI should behave in an understandable and predictable manner, so that a user knows what to expect when anyone performs an action. A graphical user interface provides the user with a familiar environment in which to work. This environment contains pushbuttons, toggle buttons, lists, menus, text boxes, and so on, all of which are already familiar to the user, so that anyone can concentrate on using the application rather than on the mechanics involved in doing things [28-30].

III. RESULT AND RELATED WORK

The result of this work is a Matlab Software Package designed based on MATLAB GUI. This package consists of two main modules using the MATLAB Graphical User Interface (GUI); the first one deals with applying main important methods of linear algebra system and has the following main features as shown in the following figures:

The “Fig.2” shows the main window of linear algebra package system.

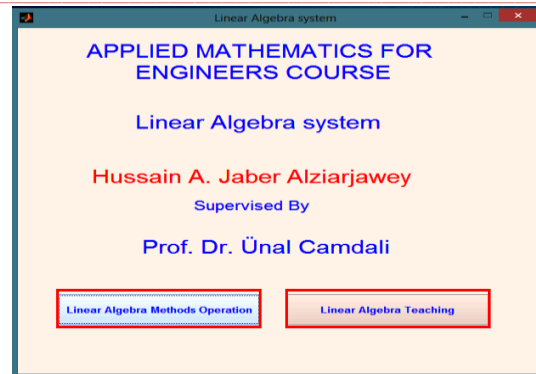


Figure 2. Main window of linear package system.

- Display input matrix \mathbf{A} (multi dimensions) and input vector that used in this work. The “Fig.3” shows how enter these input parameters and display it.

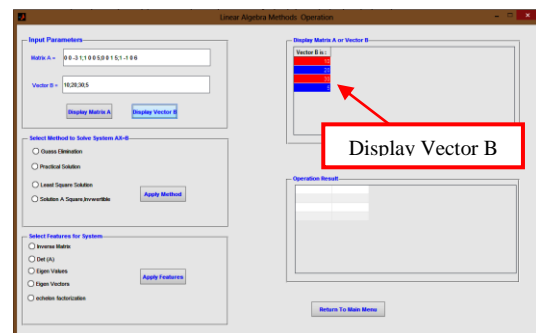


Figure 3. Example of entering input parameters (matrix \mathbf{A} & vector \mathbf{B}).

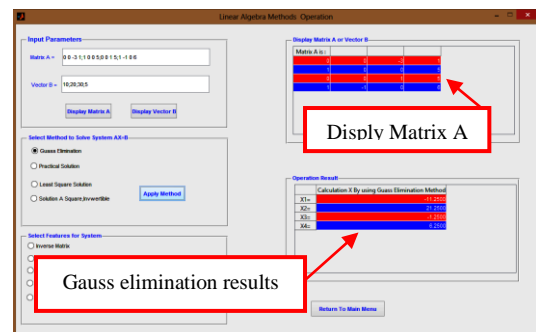


Figure 4. Example of obtaining results of Gauss elimination.

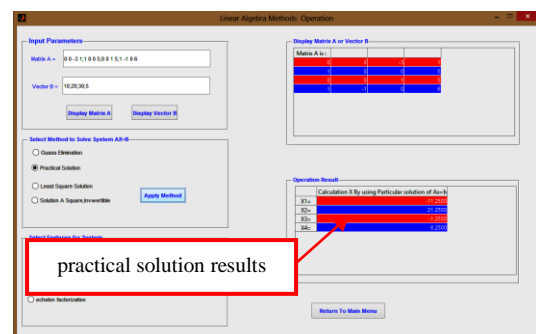


Figure 5. Example of obtaining results of practical solution.

- Presented the most important methods and that mostly used in linear algebra concepts such that (Gauss elimination, practical solution, least square solution and solution a square, Invertible). The “Fig.4” and “Fig.5” shows the main important methods of linear package system.
- Showing some features such that (Inverse Matrix, Determinant, Eigen Values, Eigen Vector and echelon factorization). The “Fig.6” and “Fig.7” demonstrates all these features.

“Fig.9-11” shows some example window of explanation and testing of Linear Algebra Teaching concepts.

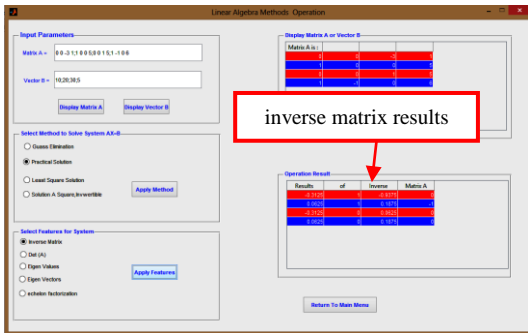


Figure 6. Example of obtaining results of inverse matrix results.

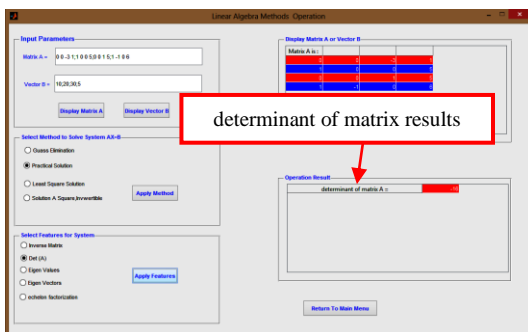


Figure 7. Example of obtaining results of determinant of matrix results.

In the second module of this system, that called linear algebra teaching module that give short review of explanation of most subjects of linear algebra concepts with a good examples as well as useful tests of linear algebra concepts for deep clearing when anyone who use this system. The “Fig.8” shows the main window of second module (Linear Algebra Teaching).

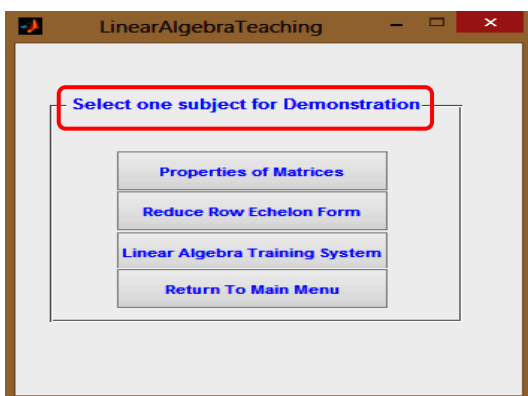


Figure 8. Main window of Linear Algebra Teaching.

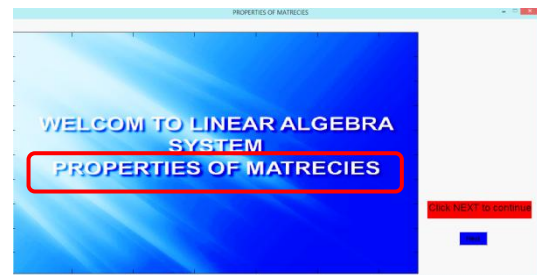


Figure 9. Properties of matrices.



Figure 10. Reduce Row Echelon Form.

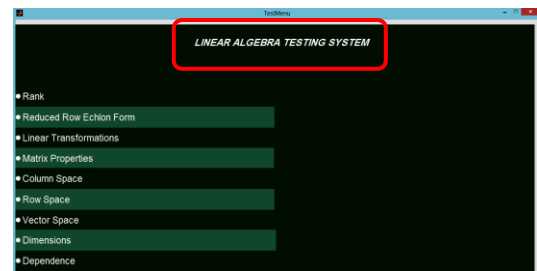


Figure 11. Linear Algebra Teaching Testing.

IV. CONCLUSION AND FUTURE WORK

This paper has presented a thorough Matlab program which it is easy and very useful for any person who interested in mathematical research studies and most of engineering field in addition to students who works in mathematical area. The graphical user interface is very easy to use and enables to calculate unknown values or other calculation that related to all subjects related to linear algebra. This package consists of two main modules using the MATLAB Graphical User Interface (GUI); the first one deals with the most of important linear algebra methods (Gauss elimination, practical solution, least square solution) to calculate unknown values as well as some features such that (inverse matrix calculation, determinant, Eigen value and Eigen vector and echelon form). In the second module, describes and illustrates most of important linear algebra methods in short review that presents important points for each methods, furthermore, this module creates some examination tests for students to increase the ability of students to understanding these method as well as make them familiar with mathematics.

For future work, some topics are suggested as:

- Expanding of this system package by adding some topics as ordinary differential equation, second order

differential equation, nth order differential equation, partial differential equation, Laplace equation and Fourier series.

- Also the system package results could be shared to website using ASP .NET, which can give facility for users or students by sharing applications which requires no installation of MATLAB or any special program just a Web browser.

- [29] H. Kassim AlJobouri, H. A. Alziarjawey, I. Cankatya, "Biosignal Processing, Medical Imaging and fMRI (BSPMI) Software Package Based on MATLAB GUI for Education and Research, Vol. 1, No. 2, pp.2380-8128, 2015.
- [30] MATLAB, "Creating Graphical User Interfaces", The Mathworks, Inc, R2014a.

REFERENCES

- [1] Meyer, Carl D. , "Matrix Analysis and Applied Linear Algebra," Society for Industrial and Applied Mathematics (SIAM), 2001.
- [2] E. Kreyszing, H. Kreyszing, E. Norminton, "Advance Engineering Mathematics," John Wiley & Sons, Inc, USA, 2011.
- [3] A.Jeffrey, "Advance Engineering Mathematics," Academic Press, 2002.
- [4] Axler, Sheldon Jay, "Linear Algebra Done Right," Springer-Verlag, 1997.
- [5] Poole, David, "Linear Algebra: A Modern Introduction," Brooks/Cole, 2006.
- [6] Lay, David C., "Linear Algebra and Its Applications," Addison Wesley, 2005.
- [7] Axler, Sheldon, "Linear Algebra Done Right," Springer, 2004.
- [8] Anton, Howard, "Elementary Linear Algebra (Applications Version) ," Wiley International, 2005.
- [9] Demmel, James W., "Applied Numerical Linear Algebra," SIAM, 1997.
- [10] Leon, Steven J., "Linear Algebra With Applications," Pearson Prentice Hall, 2006.
- [11] P. o'neil, "Advance Engineering Mathematics," Cengage Learning, USA.
- [12] Bhatia, Rajendra, "Matrix Analysis, Graduate Texts in Mathematics," Springer, 1996.
- [13] Gelfand, I. M., "Lectures on Linear Algebra," Dover Publications, 1989.
- [14] Gantmacher, F.R., "Applications of the Theory of Matrices," 2005.
- [15] Gantmacher, Felix R., "Matrix Theory Vol. 2 (2nd ed.)," American Mathematical Society, 2000.
- [16] Dym, Harry, "Linear Algebra in Action," AMS, 2007.
- [17] Strang, Gilbert, "Linear Algebra and Its Applications," 2005.
- [18] Gantmacher, Felix R., "Matrix Theory Vol. 1," American Mathematical Society, 1990.
- [19] Golan, Johnathan S., "The Linear Algebra a Beginning Graduate Student Ought to Know," Springer, 2007.
- [20] Golub, Gene H.; Van Loan, Charles F., "Matrix Computations, Johns Hopkins Studies in Mathematical Sciences," The Johns Hopkins University Press, 1996.
- [21] Glazman, I. M.; Ljubic, Ju. I., "Finite-Dimensional Linear Analysis, Dover Publications," 2006.
- [22] Golan, Johnathan S., "Foundations of Linear Algebra," Kluwer, 1995.
- [23] Friedberg, Stephen H.; Insel, Arnold J.; Spence, Lawrence E., "Linear Algebra," Prentice Hall, 2002.
- [24] Horn, Roger A.; Johnson, Charles R., "Topics in Matrix Analysis," Cambridge University Press, 1994.
- [25] Greub, Werner H., "Linear Algebra, Graduate Texts in Mathematics," Springer, 1981.
- [26] Horn, Roger A.; Johnson, Charles R., "Matrix Analysis," Cambridge University Press, 1990.
- [27] Halmos, Paul R., "Finite-Dimensional Vector Spaces," Undergraduate Texts in Mathematics, Springer, 1993.
- [28] H. A. Alziarjawey, I. Cankatya, "Heart Rate Monitoring and PQRST Detection Based on Graphical User Interface with Matlab," International Journal of Information and Electronics Engineering, Vol. 5, No. 4, pp. 311- 316, 2015.