Method of Moments and Circuit Network Theory in Antenna Analysis

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Abstract:-The Method of Moment (MoM) is a numerical technique for the analysis of linear antennas. Circuit network principles describe the analysis of circuit components in a given system. The two techniques find application in the analysis and synthesis of antennas. With this combination, antenna problem formulation tends to be distinct with clear method of solution. This paper therefore describes the relationship between the two solution methods with emphasis on the fundamentals they share in common. The implications of making any choice in the solution approach are also investigated.

Keyword*: Antenna, circuit components, transmission lines, Moment Method.*

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1. Introduction

Antenna principles are described by Electromagnetic field (EM) theory and circuit network theory. These theories are both linked via Method of Moments [1]. Nevertheless, network theories are applied to Electromagnetic field problems to improve problem formulation and contribute to the solution methodology [2]. Since Antennas are capable of radiating and receiving EM waves, they can be modeled using shunt admittance circuit representation. In this form the generator is modeled as a current source of which the principle of Thevein's and Norton's equivalent circuit can be applied to obtain the current necessary for EM radiation to occur. The circuit can in other words be described by basic circuit theory principles. A center fed dipole antenna behaves like a balanced transmission line which is open at the far end(s). An applied voltage at the source terminals gives rise to a standing wave which must have zero current at its open circuit ends. This is an equivalent condition that applies to numerical analysis of linear antenna. By applying a known basis function on the MoM solution, the current distribution on the antenna can be determined using Ohm's law. This current is equal to zero at the end(s) of the wire. Other guided waves structures are also modeled by segmenting the circuits into elementary block, which is represented by matrix expressions [3]. These blocks are characterized in form of transfer functions switch, so that for a given excitation, a response is obtained. Furthermore, Network oriented modeling for radiating electromagnetic structures has been investigated showing that for each radiation mode, a circuit representation is given [4].

Analysis involving antennas also describes circuit parameters which are vital information for Engineering application. For instance the value of the resistance that governs the power transferred to the antenna for outward Electromagnetic wave propagation is the radiation resistance of the antenna and is similar to the resistance of a transmission line that tends to limit the current flow through the conductor.

The remaining part of the paper describes the areas where MoM application in antennas and circuit network equation are related. The work is concluded in the section 9.

2. Formulation

The MoM formulation of antenna analysis is well known but salient remarks which are necessary are highlighted. The MoM expression for the analysis of linear antenna is represented by an inhomogeneous equation give by

C(f) = h

Where;

C represent the operator, h the excitation and f the unknown response. The unknown is described by a set of basis or expansion function which could be the pulse, preciswise linear or piecewise sinusoidal functions (fn) respectively

$$fn = \begin{cases} 1 & \dfrac{z - z_{n-1}}{z_n - z_{n-1}} \quad z_{n-1} < z < z \\ & \dfrac{z_{n+1} - z}{z_{n+1} - z_n} \quad z_n < z < z_{n+1} \\ 0 & \text{elsewhere} \end{cases}$$

$$fn = \begin{cases} 1 & \text{in subdomain} \\ 0 & \text{elsewhere} \end{cases}$$

$$fn = \begin{cases} 1 & \dfrac{Sin\beta(z - z_{n+1}}{Sin\beta\Delta z} \\ & \dfrac{Sin\beta(z_{n+1} - z)}{Sin\beta\Delta z} \\ 0 & \text{elsewhere} \end{cases}$$

$$f = \sum_{n=1}^{N} a_n f_n$$

an = unknown expansion coefficient

hence $\sum_{n=1}^{N} a_n C(f_n) = h$

The above expression is weighted by a testing function and on applying the inner product rule of two functions, a uniform equation of equal dimension is achieved.

$$\sum_{n=1}^{N} a_n \langle w_m \ C(f_n) \rangle = \langle w_m, h \rangle$$

The above expression can thus be represented by a circuit network equation (ohm law).

Thus $\sum_{n=1}^{N} z_{mn} I_n = V_m$

ZI = V

The matrix equation of MoM resembles an N port generalization of ohm's law.

In other words Pocklingtons or Hallen's equation describing the fields of an antenna with N integral equation in N unknown can be reduced numerically to linear algebraic equation whose presentation is analogous to that of Kirchoff's network equation of N mesh circuit problem. Also analytical model which are based on either field theory or network theory expresses solution for independent variables. These variables include electric and magnetic field components or voltages and current. They can collectively be expressed by analytical functions such as sine functions, cosine functions, Bessel, Fourier and Hankel series so that the solution to a problem can be well defined.
A network equation which involves resistors (R), capacitors (C) and inductors (L) can be represented by the expression

$$\sum_{n=1}^{N} \left(j\omega L_{gn} + \frac{1}{jwC_{gn}} + R_{gn} \right) I_n = V_g$$

The above expression satisfies Kirchoff's theory for voltage and current for the given set of meshes (n) with L_{gn}, C_{gn}, R_{gn} known as the mutual inductances, capacitances and resistances respectively for a linear circuit, V_g the voltage in the gth mesh and I_n the unknown circulating current in nth mesh [5].
The above equation can be expressed in a single equation given as

$$I\left[j\omega L - j(\omega c)^{-1} + R\right] = IZ = V$$

Where
Z is the complex impedance of the circuit. The equation expresses in general a complex form of ohm's law for a time dependent case. This draws some close relationship in MoM and circuit theory.

3. Source model

The magnetic frill source is a type of feed model which finds more application in loop antennas. In describing this feed model for MoM analysis, the outer and inner conductor radius feeding the loop must to be determined .This is achieved by applying a circuit network equation for determining the characteristic impedance expression for the transmission line. This expression is given by;

$$Zc = \sqrt{\frac{\mu_o}{\varepsilon_o}}\, In \frac{(b/a)}{2\pi}$$

Where b/a is the ratio of the outer conductor radius to the inner, μ_o and ε_o are the permeability and permittivity of free space respectively. This shows the interdependence between the MoM and circuit network equation.

4. Loading

Transmission lines can be loaded depending on the essence of its design, so also are antennas. Both can be loaded either in a distributive or in a lumped form. The overall effect of loading in antennas and transmission lines is seen to affects the current distribution on them [6]. Other effects of loading in antennas include; lowering the resonant frequency of the antenna, increasing peak admittance and narrowing the bandwidth of operation (especially when antennas has inductive loads) [7]. This implies that the overall effect of loading in antennas is to improve its radiation parameters. In transmission lines, by adding inductive loads, the performance of the line is improved thereby reducing the attenuation along the line.

5. Superposition Theory

The superposition theorem is a very important concept used to analyze D.C and A.C circuits. It helps to determine some circuit components which ordinarily may be difficult with other simple approach. This theorem also finds application in MoM for antenna analysis. It is used in describing radiation property of identical antenna elements oriented in the same direction with uniform characteristics. This type of antenna configuration is referred to as array antennas. The total field radiated by this array antenna is determined by applying superposition theory. It sums the contributions of the field radiated by the individual antennas independently. .

$$E\theta_{total} = E\theta_1 + E\theta_2 + E\theta_3 + --- + E\theta_N$$

6. Continuity Equation

The continuity equation; $div\, J = \frac{-d\rho}{dt}$ is a key expression used in deriving either pocklington's or Hallen's equations in MoM solution. In circuit theory, the continuity equation describes the conservation of charges which correspond to Kirchoff's current law. In the analysis of linear antennas therefore, the subdomain basis function as applied in MoM analysis satisfies this continuity equation at the sub-element interconnections, ends or edges of the antenna under consideration. This helps to obtain the current distribution at specific points along the antenna.

7. Matching

Matching is a vital process in circuit analysis to aid maximum power transfer from the source to the load. In antennas matching improves the system performance to maximize antenna efficiency. In numerical analysis of antennas, matching is highly achieved when antennas are operated at their resonant frequencies. At this instant, the reactive part of their terminal impedance/ admittance is small or better still negligible. It is important to note that transmitting and receiving device must be terminated in an impedance close to the characteristic impedance of the transmission line so that losses (reflections along the lines) are eliminated and an efficient transmission process is achieved.

8. Result

Table 1: Relationship between components of circuit network theory and Moment Method

Network Theorem	Method of Moment
Impedance may be known at the start of circuit analysis.	This parameter is determined in the process of numerical analysis
Kirchoff's law applies in circuit analysis.	The continuity equation is the corresponding application in antenna analysis.
Superposition theorem applies to DC and AC circuits	Superposition theorem applies in analysis of multiple identical antenna elements with same geometrical and electrical characteristics
Cancelled radiation effects are observed in circuit components.	Re-enforced radiation is observed in antennas, depending on design interest
Current and voltage describes and relates circuit components	Magnetic and Electric fields describes the element structure

9. Conclusion

In this paper is given the relationship between circuit network equation and the Method of Moment. Emphasis is laid on the characteristics they share in common with respect to antenna analysis. This ranges from the mode of expression of their fundamental equations to loading effects, superposition principles, matching and continuity expression. These outcomes are essential for thorough understanding and analysis of linear antennas to ease problem formulation and solution approach. It can thus be concluded that network models applied to field problems can generate compact expressions for Electromagnetic structures.

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