

Dynamic Behaviour of Rotor Roller Bearing System with Bearing Defects

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Abstract— Rotating machinery is becoming faster and light weight due to the advanced technologies made in engineering and materials sciences. It is required to run them for longer periods of time. The detection, location and analysis of faults are highly recommended in highly reliable operations. The high speed rotor bearing system often shows unpredictable dynamic response due to manufacturing defects. As it is not possible to produce perfect surface or contour even with the best available machine tools so imperfection such as surface waviness in the rolling element and races developed during manufacturing process cannot be avoided. The radial and axial clearance provided in the design of bearings to compensate for thermal expansion, can also be a source of vibrations and introduce nonlinearity in the dynamic system. Using vibration analysis, the condition of a machine can be periodically monitored. In this study, dynamic behaviour of rotor roller bearing system with bearing defects, like radial clearance, declining of roller and localized inner race defect. The nonlinear bearing forces of a roller bearing under two dimensional loads and develops 2-DOF dynamics equation of a rotor-roller bearing system. The contacts between the rolling elements and the races are considered as non-linear springs, whose stiffness is obtained by using Hertzian elastic contact deformation theory. The system shows the nonlinear characteristics under dynamic condition. The equation of motion in radial and axial condition is obtained for shaft and rolling elements and they are solving by numerical integration technique Newmark- β method. Vibration of rolling element in the radial direction is analyzed time and frequency domain. Characteristics defects frequency and their components can be seen in the frequency spectrum of roller element vibration. As the clearance increases the dynamic behaviour becomes complicated with the number and the scale of instable region becoming larger.

Keywords- Nonlinear dynamic analysis, Chose, Poincare map, Newmark- β , ball passage frequency, Stiffness

1. INTRODUCTION

Rotary machines are recognized as crucial equipment in power stations, petrochemical plants, and automotive industry that require precise and efficient performance. Bearings are the most widely used mechanical parts in rotational equipment and are primary cause of breakdowns in machines. Such malfunctions can lead to costly shutdowns, lapses in production, and even human casualties. To minimize machine downtimes, a sensitive and robust monitoring system is needed to detect faults in their early stages and to provide warnings of possible malfunctions. Such a monitoring system can reduce maintenance costs, avoid catastrophic failures and increase machine availability. To develop an effective diagnostic and prognostic system, a comprehensive understanding of the bearing behavior is required. [18] Bearing vibrations play an important role in the dynamics of machines. As the demand for high speed rolling element bearings increases, it is important the bearings run smoothly, with long life. Because of the non linear behavior of the bearings, vibrations are hard to predict. A major source of non linear behavior in a bearing is attributed to the Hertzian contact. The analysis of vibrations caused by rolling element bearing gives important information to analyze the rotor dynamics of the system. The non linear behavior of the bearing makes the system extremely sensitive to initial conditions. The sources of vibrations in a bearing are due to varying compliance of the structure, external disturbances, and internal excitations. Internal excitations stem from geometric deviations of the interacting surfaces from their idle geometry. These deviations in turn are the results of either manufacturing limitations or

normal wear of the bearing surface. [18]

Typically, a rolling element bearing consists of two rings with a set of elements running in the tracks between the rings. The standard shapes of a rolling element include ball, cylindrical roller, tapered roller, needle, and barrel roller, encased in a cage that provides equal spacing and prevents internal strikes. [16] Even a normally loaded, properly lubricated, and correctly assembled bearing fails due to material fatigue after a certain running time. The typical fatigue life of a bearing can be significantly shortened due to manufacturing defects, improper handling and installation, or lack of lubrication. The result is either a localized or a distributed defect in the components of the bearings.

2. Dynamic model of roller bearing

2.1 Geometric parameter of the bearing

Bearing Considered: NJ 305

D_1	Outer diameter	62 mm
D_2	Inner diameter	25 mm
D_{r1}	Outer race diameter	54 mm
D_{r2}	Inner race diameter	34 mm
l	length of roller	11 mm
d	Diameter of roller	9.25 mm
Z	No. of rollers	11
D_m	Pitch Diameter	$\frac{(D_1+D_2)}{2} = 43.5$ mm
P_d	Internal clearance	0.25 mm
m	Mass of the bearing	0.29 kg

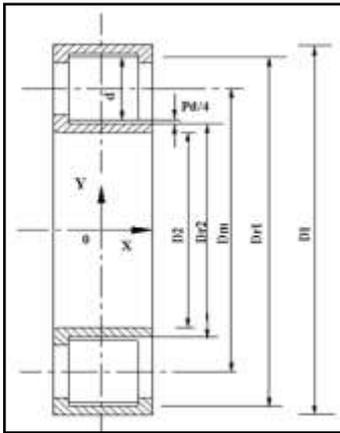


Figure no: 1 cross section of a roller bearing model

2.2 Spring force

According to the definition of the stiffness, the spring force can be defined as below Spring force or contact force = (Stiffness) \times (Deflection or Deformation) Palmgren [15] developed empirical relation from laboratory test data which define relationship between contact force and deformation for line contact for roller bearing as below.

$$\delta = 3.84 \times 10^{-5} \times \frac{Q^{0.9}}{l^{0.8}}$$

Where, Q = Contact force or spring force, l = Contact length

δ = Deformation, Take, $q = Q/l$

Where, q = Contact force per length

Above equation reduced to,

$$\delta = 3.84 \times 10^{-5} \times \frac{Q^{0.9} \times l^{0.1}}{l^{0.8} \times l^{0.1}} = 3.84 \times 10^{-5} \times q^{0.9} \times l^{0.1}$$

(Eq.1)

Contact length = $l = kW$, Equation 1.1 becomes,

$$\delta = 3.84 \times 10^{-5} \times q^{0.9} \times (kw)^{0.1}$$

Rearranging the above equation to define q yields,

$$q = \left[\frac{\delta}{3.84 \times 10^{-5} \times (kw)^{0.1}} \right]^{1.11} = \frac{\delta^{1.11}}{1.24 \times 10^{-5} \times (kw)^{0.11}} \quad (\text{Eq.2})$$

2.3 Contact Model

The behaviour of a single rolling bearing contact, i.e. between a rolling element and the race way of the inner ring or outer ring is represented spring-mass- damper. The model describes the nonlinear stiffness of the dry contact (Hertz) whereas due to the lubrication of constant damping value is introduced. As a result the contact force is given by

$$F_c = k \cdot \delta^{1.11} + c \cdot \dot{\delta}$$

Where δ is the contact indentation. The deflection co-efficient is k is determined by the material properties and geometric properties of the elastic bodies. The viscous damping c is determined by lubricant properties.

2.4 Contact Deformation (δ)

The elastic deformations (δ) in two bodies in contact, having radius of curvature R_1 and R_2 is given by, change in distance between the centres of curvature of the contacting bodies. In the case of an unloaded contact $\delta = 0$, the corresponding initial distance between the two centres of curvature is given by $R_1 + R_2$.

3.4.1 Component of Contact deformation (δ)

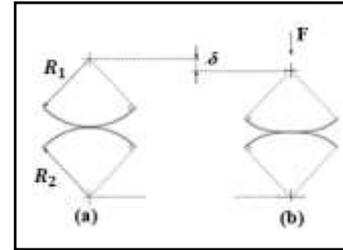


Figure No: 2 Definition of the contact deformation by means of an unloaded contact (a) and a contact loaded with an external force F (b).

When roller bearing subjected to radial and axial load, contact deformation become function of the following component to be considered in this work for localized defect.

- (1) Contact deformation due to ideal normal loading
- (2) Radial deflection due to thrust loading
- (3) Radial internal clearance
- (4) Localized defect

3.4.1.1 Contact deformation due to ideal normal loading (Δ_j)

The loads applied to roller bearings are transmitted through the rolling elements from the inner ring to the outer ring or vice versa. The magnitude of the load carried by the individual roller depends on the internal geometry of the bearing.

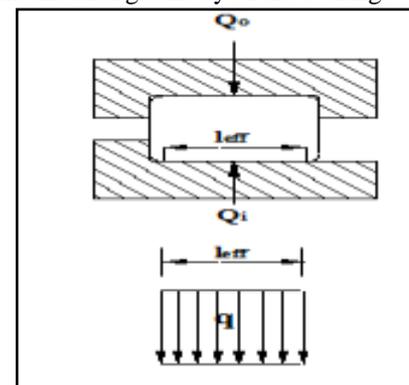


Figure N0:3 cylindrical rollers under simple applied radial load and uniformly load distribution along the effective contact length

It is apparent from this radial normal load distribution that rolling element raceway contact load is,

$$Q = q \times l_{eff}$$

3.4.2 Radial deflection due to thrust loading

When radial cylindrical roller bearings have fixed flanges on both inner and outer rings, they can carry some thrust load in addition to radial load. To accommodate the axial load, the roller will tilt due to moment couple $Q_a \cdot h$ caused by the opposing axial loads. For a straight-raceway contact, this results in the non uniform load distribution illustrated in Figure 4. It is apparent from this axial load distribution, that rolling element raceway contact load is,

$$Q = \int_0^{l_{eff}} q \, dx$$

3.4.2.1 Equation of the Radial deflection

The relative radial movement of the bearing rings caused the thrust loading as well as to radial loading. Figure 5 shows schematically a thrust loaded roller-ring assembly.

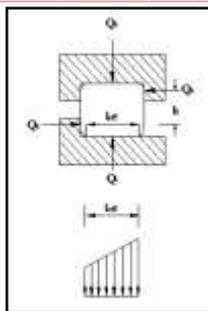


Figure No: 4 loading of cylindrical roller bearing under applied combined radial and axial loads

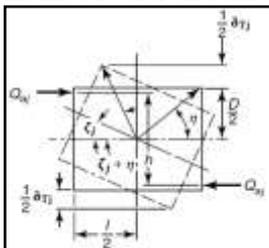


Figure No: 5 thrust loaded roller-ring assembly
Radial deflection due to roller tilting for jth roller

$$\delta_{Tj} = D \left[\frac{\sin(\zeta_j + \eta_j)}{\sin(\eta_j)} - 1 \right] \quad (\text{Eq.3.2})$$

$$\delta_{Tj} = D \times \left[\frac{\left(\frac{D}{2} + \frac{\delta_{Tj}}{2}\right) / \sqrt{\left(\frac{D}{2}\right)^2 + \left(\frac{l}{2}\right)^2}}{\left(\frac{D}{2}\right) / \sqrt{\left(\frac{D}{2}\right)^2 + \left(\frac{l}{2}\right)^2}} - 1 \right]$$

$$\delta_{Tj} = w(k - 0.5) \times \zeta_j$$

For λ^{th} laminae above equation can be written as,

$$\delta_{Tj\lambda} = w(\lambda - 0.5) \times \zeta_j$$

Same, Axial deflection due to roller tilting for jth roller is given by,

$$\delta_{aj} = D \times \zeta_j \quad (\text{Eq.3})$$

3.4.3 Radial internal clearance:

Clearance is provided in the design of bearing to compensate the thermal expansion, Clearance is considered as negative deformation, and that is, no roller raceway loading can occur at a lamina until clearance is overcome by the radial deformation.

3.4.5 Localized defect

Waves are described in terms of two parameters: the wavelength (λ), which is the distance taken by a single cycle of the wave and its amplitude (A). It is assumed that the wave geometry is to be unaffected by contact distortion. The amplitude of the sinusoidal wave at contact angle as shown in Figure 6 is given by [10]

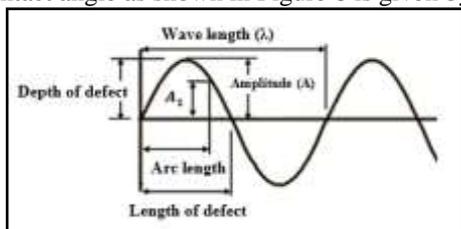


Figure No: 6 Defect model

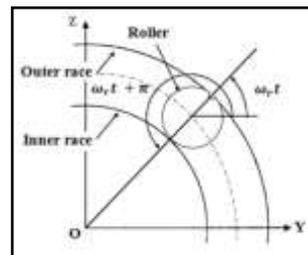


Figure No: 7 Roller defect model

For outer race the contact angle for jth roller is given by,

$$\psi_j = (\omega_r t)$$

And corresponding amplitude of roller defect at contact angle due to interaction with outer race is given by,

$$P_{ro} = A_1 + \left(D_h \times \left(\sin \left(\pi \times \frac{R_r}{D_L} \times (\omega_r t) \right) \right) \right) \quad (\text{Eq.4})$$

for inner race ψ_j is given by,

$$\psi_j = (\omega_r t + \pi)$$

And corresponding amplitude of roller defect at contact angle due to interaction with inner race is given by,

$$P_{ri} = A_1 + \left(D_h \times \left(\sin \left(\pi \times \frac{R_r}{D_L} \times (\omega_r t + \pi) \right) \right) \right) \quad (\text{Eq.5})$$

3.5 Total roller raceway deformation

The total roller raceway deformation is given by,

Total roller raceway deformation =
[Contact deformation due to ideal normal loading] +
[Radial deflection due to thrust loading]- [Radial internal Clearance]- [roller Declining] - [localised Defects]

$$\delta_{2jk} = Y2\sin\Phi_j + Z2\cos\Phi_j - \Delta_j + w \left(\lambda - \frac{1}{2} \right) \zeta_j - \frac{\delta}{4} - P2j$$

$$- w2j - ck$$

The above equation contain two unknowns [Δ_j, ζ_j], to find these unknowns we formulate the following equilibrium equations.

$$Q_j = \sum_{\lambda=1}^{\lambda=k} \frac{[Y2\sin\Phi_j + Z2\cos\Phi_j - \Delta_j + w \left(\lambda - \frac{1}{2} \right) \zeta_j - \frac{\delta}{4} - P2j - w2j - ck]^{1.11}}{1.24 \times 10^{-5} \times (k)^{0.11}} \times w^{0.89} \quad (\text{Eq.6})$$

Radial load Equilibrium

$$\frac{0.62 \times 10^{-5} F_r}{w^{0.89}} - \sum_{j=1}^{j=\frac{z}{2}+1} \frac{\tau_j \cos\psi_j}{k^{0.11}} \sum_{\lambda=1}^{\lambda=k} [Y2\sin\Phi_j + Z2\cos\Phi_j - \Delta_j + w \lambda - 12\zeta_j - \delta - P2j - w2j - ck]^{1.11} \quad (\text{Eq.7})$$

Thrust load Equilibrium

$$\frac{0.31 \times 10^{-5} F_a \times h}{w^{0.89}} - \sum_{j=1}^{j=\frac{z}{2}+1} \frac{\tau_j}{k^{0.11}} \left\{ \sum_{\lambda=1}^{\lambda=k} [\delta_{2j}]^{1.11} \left(\lambda - \frac{1}{2} \right) w - 12\lambda - 12\lambda = k\delta_{2j} \right\}^{1.11} \quad (\text{Eq.8})$$

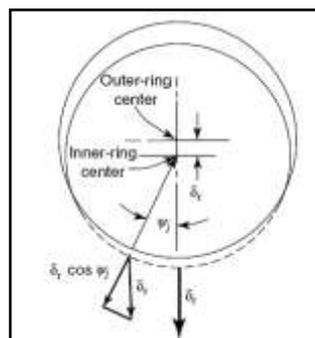


Figure No: 8 Displacement of inner ring centre relative to Outer ring centre cause by the radial loading.

Now, Sum of the radial deflection due to radial loading and radial interference cause by the axial deflection due to thrust loading minus the radial clearance is equal to the sum of the inner and outer raceway maximum contact deformations.

$$\left[\delta_a \times \frac{1}{D} \right] + [\delta_r \cos \Psi_j] - \frac{\delta}{2} - 2 \sum_{\lambda=1}^{\lambda=k} [Y2 \sin \Phi_j + Z2 \cos \Phi_j - \Delta_j + w \lambda - 12 \zeta_j - Pd2 - \delta4 - P2j - w2j - ck] = 0 \quad (\text{Eq.9})$$

Now considering the centrifugal force of the roller the equilibrium of the j^{th} roller in the z direction is represented by as under.

$$\left[\delta r \cos \Phi_j - \frac{\delta}{2} - \delta 2j \right]^{1.11} - [\delta 2j]^{1.11} - Fc/K = 0 \quad (\text{Eq.10})$$

Equation 6, 7, 8, 9 and 10 are set of nonlinear simultaneous equation which can be solved for unknown. $[r_j, \Delta_j, \zeta_j, \delta_a, \delta_r]$ to find out total roller raceway deformation. Above set of nonlinear simultaneous equations solved by MATLAB software with the help of FSOLVE function. This function is used to solve the nonlinear simultaneous equations. Nonlinear bearing force can be expressed as under

$$Q_{ry} = 1/n \sum_{j=1}^n Q2jk \sin \Phi_j \quad Q_{rz} = 1/n \sum_{j=1}^n Q2jk \cos \Phi_j$$

Where

Q_{ry} = Bearing force in y direction

Q_{rz} = Bearing force in z direction

3.5 Dynamics model of a rigid rotor system

The general form of the equations of motion for a multi-degree of freedom system is written as

$$[M]\ddot{X} + [C]\dot{X} + [K]X = f(X, t)$$

Where,

M = Mass vector of the rotor and inner ring of bearing

C = Damping vector of the system

K = stiffness vector of the system

X = Displacement vector of the rotor

f= it include the nonlinear bearing force, external load, gravity load and unbalance load

For a 2- degree of freedom (DOF) system the equation of motion can be expressed as under.

$$m\ddot{Y}2 + c\dot{Y}2 + Q_{ry} = F_y + m\omega^2 \cos \omega t$$

$$m\ddot{Z}2 + c\dot{Z}2 + Q_{rz} = F_z - mg - m\omega^2 \sin \omega t$$

Using the Newmark- β method the deferential equations of the motions can be solved and the transient response at every time obtained.

3.6 Result of mathematical modeling

Taking the rigid rotor supported by roller bearing as an example. The mass of the rotor roller bearing system is 1 kg. Damping factor of the bearing is 200 N.s/m. the rotor mass eccentricity e is $20 * 10^{-6}$ m. Shaft speed is 2000 rpm. Unbalance mass is 50g. Defect height is 0.5mm. Defect length is 0.5mm. The dynamic Behaviour is as under.

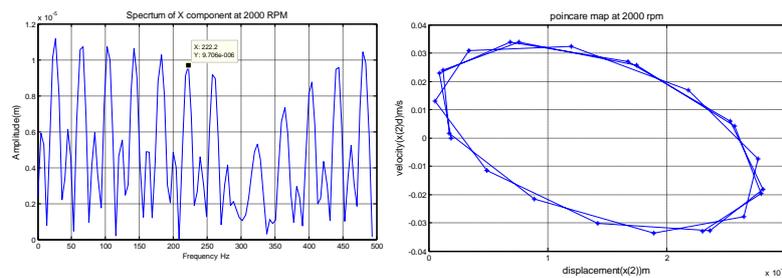
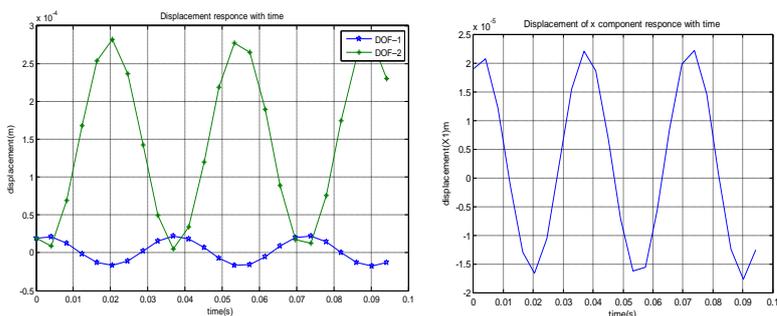


Figure No 9: Response at 2000 rpm speed

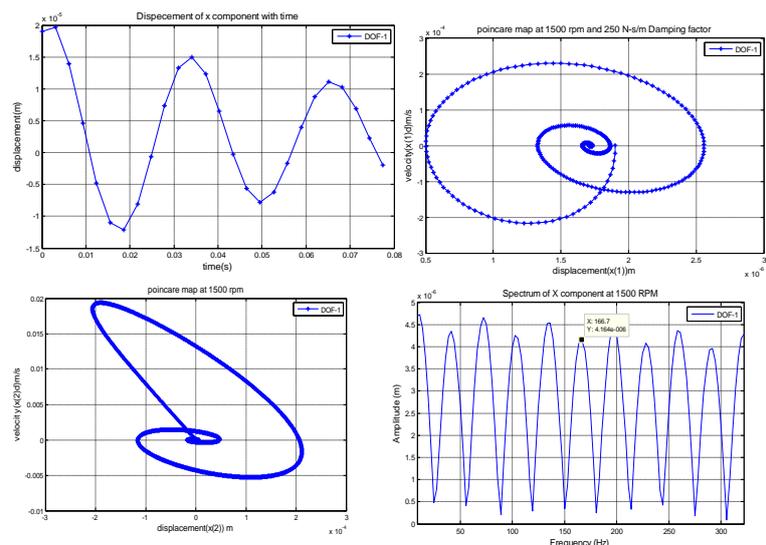


Figure No 10: Response at 1500 rpm speed

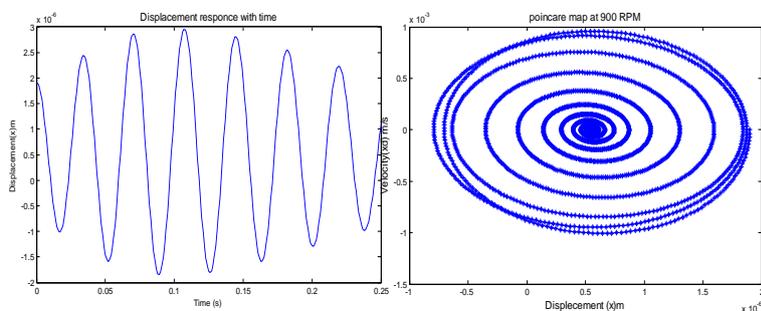


Figure No 11: Response at 1500 rpm speed

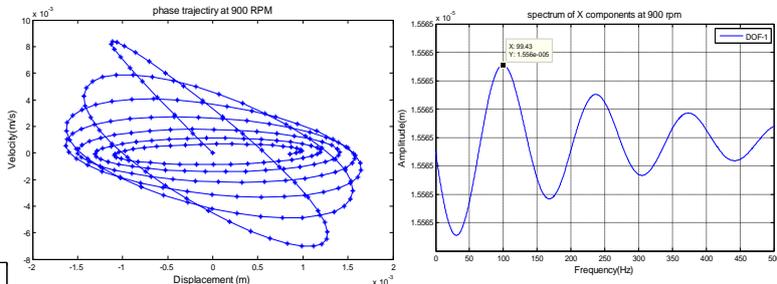


Figure No 11: Response at 900 rpm speed

4. Experimental-setup

Experimental work is carried out for the validation of the theoretical model. For validation number of simulated results

compared with the experimental work. The experimental set-up used for this study is shown in figure 12.

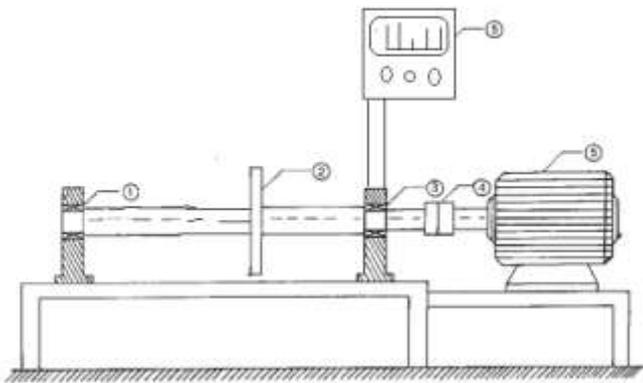


Figure no: 12 Experimental set-up of bearing rotor system
 1–roller bearing; 2–rotor; 3–Defective bearing; 4–flexible coupling; 5–motor and 6– FFT Analyzer



Figure no: 13 bearing defect at inner race

4.1 Experimental result

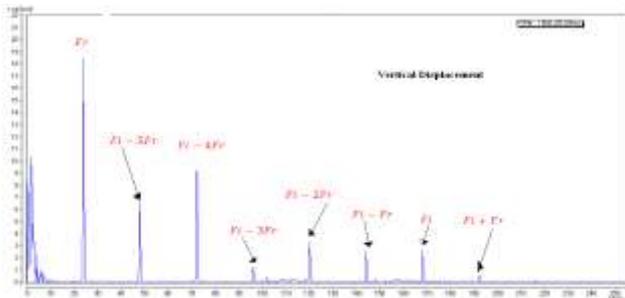


Figure No 14: Response at 1500 rpm, Frequency vs. vertical displacement

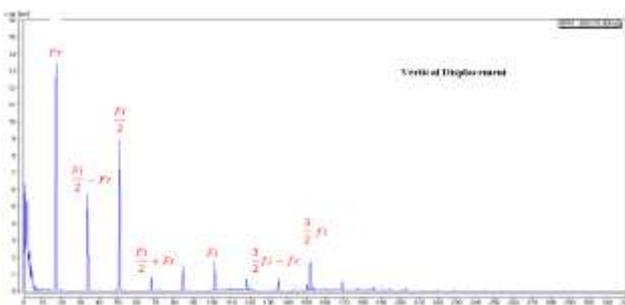


Figure No 15: Response at 900 rpm, Frequency vs. Vertical displacement

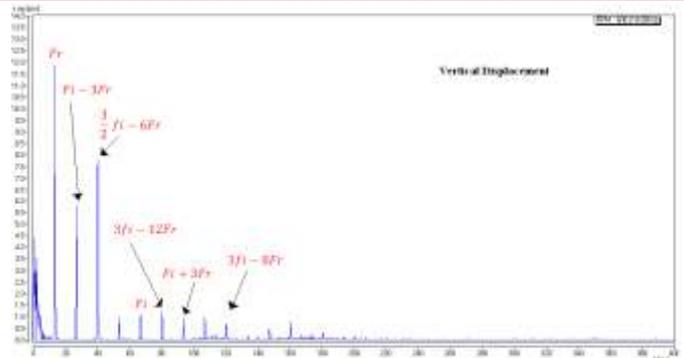


Figure No 16: Response at 600 rpm, Frequency vs. Vertical displacement

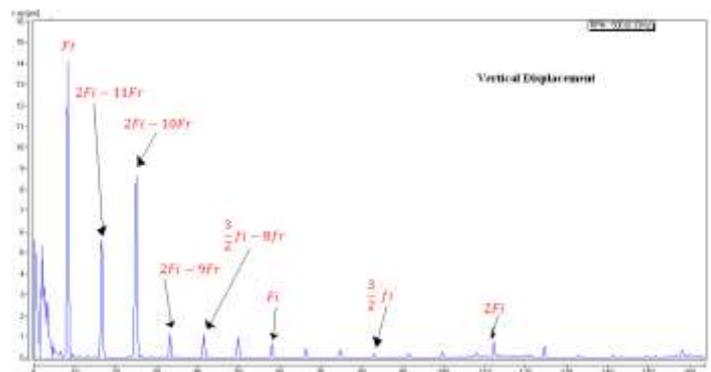


Figure No 17: Response at 500 rpm, Frequency vs. Vertical displacement

5. Results and Discussion

Vibration signature monitoring and analysis is one of the main techniques used to predict and diagnose various defects in antifriction bearings [5]. Vibration signature analysis provides early information about progressing malfunctions and forms the basic reference or base line signature for future monitoring purpose. Signature Defective rolling elements in roller bearings generate vibration frequencies at rotational speed of each bearing component and rotational frequencies are related to the motion of rolling elements, cage and races. Components flaws (inner race, outer race and rolling elements) generate a specific defect frequencies calculated from equations, mentioned by Chaudhary and Tandon [20] Inner face varying compliance frequency

$$fi = \frac{n}{2} fr [1 + \left(\frac{BD}{PD}\right) \cos\beta] \text{ Hz}$$

Inner race varying compliance frequencies are different at different speeds.

At 1500 rpm speed, $fi = 166.73 \text{ Hz}$

At 900 rpm speed $fi = 100.04 \text{ Hz}$

At 600 rpm speed $fi = 66.69 \text{ Hz}$

At 500 rpm speed $fi = 55.55 \text{ Hz}$

5.1 Result table

Shaft Speed (Rpm)	Experimental Result		Theoretical Result	
	Amplitude (µm)	Inner race varying compliance frequency, Hz	Amplitude (µm)	Inner race varying compliance frequency, Hz
1500	3.5	167.75	4	166.7
900	2.5	101.75	1.67	99.43
600	1.5	67	2	65.6
500	0.7	56.25	1.3	55

6. Conclusions

- ✓ A rotor-roller bearing system may have chaos, period doubling bifurcation and quasi periodic nature as rotational speed increase.
- ✓ Close attention should be paid to the effective chose of structural and operational parameter of the bearing in design a rotor roller bearing system.
- ✓ The model predicts the frequency spectrum having peaks at characteristics defect frequencies and the amplitudes at these frequencies emanating for the bearing. The frequency components obtained from the proposed model are similar to those appearing in the frequency spectra of the experimental data for defective bearing condition. These verify the validity of the theoretical model.
- ✓ The theoretical model can be used for design, predictive maintenance and also condition monitoring of machines.
- ✓ Trend, spectrum and waveform analyses effectively used in fault diagnosis for rotating mechanical systems, are able to guess the probable fault type.

6.1 Future scope

Roller skidding is the function of lubrication. Lubrication traction has dominant effect on the roller skidding. Slip arise when the moment due to the drag force on the roller ,which is created by viscous shearing resistance of the grease exceed the traction moment at the race way contact.

Predict the dynamic behavior of the rotor roller bearing system having 4-DOF system with consideration of skewing, declining, thickness of lubrication film, waviness, internal clearance and the localized defect of bearing.

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