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A Capacitated Heterogeneous Vehicle Routing Problem for Catering Service Delivery with Committed Scheduled Time

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Abstract— The heterogeneous vehicle routing problem (HVRP) is a well-known combinatorial optimization problem which describes a heterogeneous set of vehicles with different capacity, in which each vehicle starts from a central depot and traverses along a route in order to serve a set of customers with known geographical locations. This paper develops a model for the optimal management of service deliveries of meals of a catering company located in Medan City, Indonesia. The HVRP incorporates time windows, deliveries, fleet scheduling in the committed scheduled time planning.. The objective is to minimize the sum of the costs of travelling and elapsed time over the planning horizon. We model the problem as a linear mixed integer program and we propose a feasible neighbourhood direct search approach to solve the problem.

Keywords- *Catering problem, logistic, scheduling, committed scheduled, direct search*

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I. INTRODUCTION

Companies such as catering service which serve their customers through delivery meals needs a logistic system to plan their services. The logistic system which involves delivery considering fleet of vehicle and their routing is called Vehicle Routing Problem (VRP). This well known combinatorial optimization problem aims at minimizing the total travel cost (proportional to the travel times or distances) and operational cost (proportional to the number of vehicles used) such that customers' demand is fulfilled in time. At the beginning [5] introduced VRP for solving truck dispatching problem. Afterwards there are a lot of real world logistic problems can be solved using the VRP model. The very recent application proposed by [20] for delivery perishable food products. They developed a nonlinear optimization model with the objective is maximize the total level of customer satisfaction. A comprehensive interesting survey of the Vehicle Routing Problem can be found in ([22], [1], [4], [10], [24], [14]), and [23]. [8] addressed a thorough review of past and recent developments of VRP.

The catering company uses various type of vehicle in its operation to deliver the meals for customers. Each type of vehicle has different capacities. The variant of VRP which considers mixed fleet of vehicles is called Heterogeneous VRP (HVRP), introduced firstly by [9]. This generalization is important in practical terms, for most of customers demand are served by several type of vehicles ([11], [12]).The objective of the HVRP is to find fleet composition and a corresponding routing plan that minimizes the total cost.

Due to its combinatorial nature of the problem, most approach proposed for solving the HVRP is heuristics. [16] were the first to tackle the HVRPTW and developed a number of parallel insertions heuristics based on the insertion scheme of [19],and embedding in the calculations of the relevant criteria the acquisition costs of [9]. [7] proposed a sequential construction algorithm, extending Solomon's1heuristic with vehicle insertion savings calculations based again on the criteria of [9]. [6] proposed a 3-phase algorithm for the multi-depot HVRPTW motivated by cluster-based optimization, while [17] presented a two-phase solution frame- work relying on a hybridized tabu search integrated within a new reactive variable neighborhood search meta-heuristic algorithm, with very good results. Another Tabu Search approach was used by [12]. They extended an existing tabu search specially designed for the m-VRPTW developed by [15]. [3] presented a deterministic annealing metaheuristic for the HVRPTW, outperforming the results of [16], and then [2] developed a linearly scalable hybrid threshold-accepting and guided local search meta heuristic for solving large scale HVRPTW instances. [18] presented an Adaptive Memory Programming solution approach for the HVRPTW that provides very good results in the majority of the benchmark instances examined. [21] proposed a hybrid algorithm for the problem. Their algorithm is composed by an Iterrated Local Search (ILS) based heauristic and a Set Partitioning (SP) formulation.

This paper is about to deliver daily meals from a catering company to customers spread across the city of Medan, Indonesia. The requests from customers are varied in the amount of meals and time of delivery. Therefore the company needs to schedule the time of delivery, and type of vehicle to be used.It should be noted that the scheduled time for the catering problem is so tight. Coverage area of the operation of this catering company is large, in such a way, the company divides the whole area into several sub-area, with a consequence that it is necessarily to include the scheduling of the sub-area in the HVRPTW. The catering company has a limited number of fleet of vehicles. Therefore it needs to plan a schedule that can organize these vehicles in order to satisfy their customers. Due to the fact that heterogeneous vehicle with different capacities are available, the basic framework of the vehicle routing can be viewed as a Heterogeneous Vehicle Routing Problem with Time Windows (HVRPTW). We address a mixed integer programming formulation to model the problem. A feasible neighbourhood heuristic search is proposed to get the integer feasible solution after solving the continuous model of the problem.

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II. PROBLEM FORMULATION OF THE CATERING PROBLEM

The basic frame work model of HVRP for the catering problem can be defined as follows. Let $G=(V,A)$ be a directed graph, where V = {0, 1, ... , *n*} is the vertex set and $A = \{(i, j) : i, j \in V, i \neq j\}$ is the set of route. For each route $(i, j) \in A$ a distance (or travel) cost c_{ij} is defined. The depot vertex, center of service , i.e., vertex 0 $(i=0)$, where the vehicle fleet is located. Define V_c □ is the set of customers' vertex. Each customer $i \in V_c$ has a known fixed daily meal demand $d_i \geq 0$ within the planning horizon of time T□ in a day. A fleet of K vehicles is composed by m different type of vehicles, each with capacity Q_m . The number of vehicles available for vehicle type m is n_m. Define K_m as the set of vehicle type m. At the center of catering ($i=0$), a time window for vehicles to leave and to return to depot is given by $[a_0, b_0]$. Vehicle routes are restricted to a maximum duration of H_k , $k = 1, ..., K$. Each type of vehicle is associated with a fixed cost, f_m . Another cost occurs for travelling through route $(i,j) \in A$, defined as $\alpha_{ij}^{m} = d_{ij} g_{ij}$, where d_{ij} is the distance travelled between route I to route j and g_{ij} a factor cost for travelling, for m type of vehicle.

Vehicles used to deliver are initially located at the central catering depot. Each customer $i \in V_c$ requires a service time, s_i, has a time window $[a_i,b_i]$, and is served by exactly one vehicle with the associated type. Furthermore ,a fixed acquisition cost fk is incurred for each of vehicle k in the routes. Each route originates and terminates at the central depot and must satisfy the time window constraints, i.e., a vehicle cannot start servicing customer i before a_i and after b_i; however, the vehicle can arrive before a_i and wait for service.
The catering company considered in this research is a well known one. It has a lot of customers. Therefore the time scheduled for deliveries is so tight. The model formulated is necessarily imposed a tight time window for each delivery . The problem can be called as HVRP with committed scheduled time.

The decision variables for the problem defined as

- $x_{ijk}^{t} = \begin{cases} 1 & \text{if vehicle type } k \in K \text{ to deliver f} \\ & (i,j) \in V_c, \text{on time } t \in T; \\ 0 & \text{otherise;} \end{cases}$
- $x_{ijm}^{t} = \begin{cases} 1 & \text{if vehicle type } m \in K_m \text{ to deliver fo} \\ & (i,j) \in V_c, \text{on time } t \in T; \\ 0 & \text{otherise;} \end{cases}$
- $x_{ijk}^{t} = \begin{cases} 1 & \text{if vehicle type } m \in K \text{ is availble an} \\ & \text{active at depot on time } t \in T; \\ 0 & \text{otherise;} \end{cases}$

- l_{im} Arrival time for vehicle type m at customer I (non-negative continuous variable)
- u_{im} Duration of service of vehicle type m at customer i (non-negative continuous variable)

III. THE MODEL

The basic model of HVRP with committed scheduled time for catering problem can be written mathematically as follows.

In this basic framework of HVRP the manager of the catering company wants to use the available vehicle for each type efficiently, such that the total cost is minimized. The total cost consists of traveling cost of all vehicle used and the cost for the availability of vehicle in the planning horizon time of a day.

$$minimize \sum_{t\in T} \sum_{(i,j)\in V_c} \sum_{k\in K} c_{ij} x_{ijk}^{t} + \sum_{(i,j)\in V_c} \sum_{m\in K_m} \sum_{t\in T} \alpha_{ijm}^{t} x_{ijm}^{t} + \sum_{m\in K} \sum_{t\in T} f_m z_{om}^{t} \quad (1)$$

Subject to

$$\sum_{(i,j)\in V_c, i\neq j} x_{ijk}^{t} = 1, \quad \forall m \in K, t \in T \quad (2)$$

$$\sum_{(i,j)\in V_c, i>j} \sum_{k\in K} x_{ijk}^{t} = 1, \quad \forall t \in T \quad (3)$$

These two constraints (Eq. (2) and (3)) is to ensure that exactly one vehicle regardless their type enters and departs from every customer and from the central depot.

$$x_{ijm}^{t} \leq z_{0m}^{t}, \quad (i, j) \in V_c, \forall m \in K_m, t \in T \quad (4)$$

Constraint (4) represents that each customer is served only by the available and active vehicle of type m.

$$\sum_{j\in V_c} x_{1jk}^{t} \leq 1 \quad \forall k \in K, t \in T \quad (5)$$

$$\sum_{i\in V_c, i>1} x_{i1k}^{t} \leq 1 \quad \forall k \in K, t \in T \quad (6)$$

Constraints (5) and (6) state the availability of vehicles by bounding the number of route, related to vehicle k for each type, directly leaving from and returning to the central depot, not more than one, respectively.

$$\sum_{(i,j)\in V_c, i<j} x_{ijk}^{t} - \sum_{(i,j)\in V_c, j<i} x_{ijk}^{t} \leq 1 \quad \forall k \in K, t \in T \quad (7)$$

A flow conservation equation is necessarily needed to maintain the continuity of each vehicle route on each period of time. This equation is presented in Constraint (7).

$$\sum_{i\in V_c} d_i \sum_{j\in V_c} x_{jim}^{t} \leq Q_m \quad \forall m \in K_m, t \in T \quad (8)$$

Constraint (8) is to ensure that each delivery does not exceed the capacity of each type of vehicle.

$$x_{ijk}^{t}(l_{ik} + u_{ik} + s_i + t_{ij} - l_{jk}) = 0 \quad \forall k \in K, (i, j) \in A, t \in T \quad (9)$$

Constraint (9) establishes the precedence relationship among the arrival times of vehicles at customers in the routes in the assigned period of time.

$$l_{ik} \le a_i \sum_{j \in V_c} x_{ijk}^t \quad \forall k \in K, i \in V_c, t \in T \tag{10}$$

$$a_i \sum_{j \in \Delta_i^+} x_{ijk}^t \le l_{ik} + u_{ik} \le b_i \sum_{j \in V_c} x_{ijk}^t \quad \forall k \in K, i \in V_c, t \in T \tag{11}$$

Constraints (10) and (11) present time window as the committed scheduled time for each customer.

$$\sum_{k \in S_c} \sum_{j \in N} x_{0j}^k \le n_c \quad \forall c \in C \tag{12}$$

Constraint (12) guarantees that the number availability of active vehicle does not exceed the number of vehicle available at the central depot of catering company.
The formulation of the catering problem is in the form of Mixed Integer Linear Programming Problem (MILP).

IV. The Framework of the Approach

The basic framework of a Mixed Integer Linear Programming (MILP) problem can be written as follows.

$$\text{Minimize} \quad Z = c^T x \tag{13}$$
$$\text{Subject to} \quad Ax = b \tag{14}$$
$$x \ge 0 \tag{15}$$
$$x_j \text{ integer for some } j \in J \tag{16}$$

with J as the index set.

Firstly, we solve the relaxed form of the MILP, Eq. (13)-(16) A component of the optimal basic feasible vector $(x_B)_k$, to MILP solved as continuous can be written as

$$(x_B)_k = \beta_k - \alpha_{k1}(x_N)_1 - \cdots - \alpha_{kj}(x_N)_j - \cdots - \alpha_{kn} - m(x_N)_N n - m \tag{17}$$

It should be noted that, this expression can be found in the final tableau of Simplex procedure. If $(x_B)_k$ is an integer variable and we assume that β_k is not an integer, from the idea of neighborhood solution, we do the partitioning of β_k into the integer and fractional components such that

$$\beta_k = [\beta_k] + f_k, 0 \le f_k \le 1 \tag{18}$$

suppose we wish to increase $(x_B)_k$ to its nearest integer, $([\beta] + 1)$. Based on the idea of suboptimal solutions we may elevate a particular nonbasic variable, say $(x_N)_{j^*}$, above its bound of zero, provided α_{kj^*}, as one of the element of the vector α_{j^*}, is negative. Let Δ_{j^*} be amount of movement of the non variable $(x_N)_{j^*}$, such that the numerical value of scalar $(x_B)_k$ is integer. Referring to Eqn. (18), Δ_{j^*} can then be expressed as

$$\Delta_{j^*} = \frac{1-f_k}{-\alpha_{kj^*}} \tag{19}$$

while the remaining nonbasic stay at zero. It can be seen that after substituting (18) into (19) for $(x_N)_{j^*}$ and taking into account the partitioning of β_k given in (19), we obtain

$$(x_B)_k = [\beta] + 1 \tag{20}$$

Thus, $(x_B)_k$ is now an integer.

It is now clear that a nonbasic variable plays an important role to integerize the corresponding basic variable. Therefore, the following result is necessary in order to confirm that must be a non-integer variable to work with in integerizing process.

Theorem 1. Suppose the MILP problem (13) - (16) has an optimal solution, then some of the nonbasic variables $(x)_{n_j}$,j = 1,…,n-m must be non-integer variables.

Proof:
Solving problem as a continuous of slack variables (which are non-integer, except in the case of equality constraint). If we assume that the vector of basic variables consists of all the slack variables then all integer variables would be in the nonbasic vector x_N and therefore integer valued.

V. The Algorithm

For the integrizing process it is necessarily to partition the non-feasible integer basic variable.
Let

$$x = [x] + f, \quad 0 \le f \le 1$$

be the (continuous) solution of the relaxed problem, $[x]$ is the integer component of non-integer variable x and f is the fractional component.

Stage 1.

Step 1. Get row i^* the smallest integer infeasibility, such that

$$\delta_{i^*} = \min\{f_i, 1 - f_i\}$$

(This choice is motivated by the desire for minimal deterioration in the objective function, and clearly corresponds to the integer basic with smallest integer infeasibility).

Step 2. Do a pricing operation

$$v_{i^*}^T = e_{i^*}^T B^{-1}$$

Step 3. Calculate $\sigma_{ij} = v_{i^*}^T \alpha_j$

With corresponds to

$$\min_j \left\{ \left| \frac{d_j}{\alpha_{ij}} \right| \right\}$$

Calculate the maximum movement of nonbasic j at lower bound and upper bound.
Otherwise go to next non-integer nonbasic or superbasic j (if available). Eventually the column j^* is to be increased form LB or decreased from UB. If none go to next i^*.

Step 4.

Solve $B\alpha_{j^*} = \alpha_{j^*}$ for α_{j^*}

Step 5. Do ratio test for the basic variables in order to stay feasible due to the releasing of nonbasic j^* from its bounds.

Step 6. Exchange basis

Step 7. If row $i^* = \{\varnothing\}$ go to Stage 2, otherwise Repeat from step 1.

Stage 2. Pass1 : adjust integer infeasible superbasics by fractional steps to reach complete integer feasibility.

Pass2 : adjust integer feasible superbasics. The objective of this phase is to conduct a highly localized neighbourhood search to verify local optimality.

VI. CONCLUSIONS

The catering company has a lot of customers to be served with a variety of volume of meal container. Therefore the company needs several type of vehicle to carry out the deliveries. This paper is to develop a model of Heterogeneous Vehicle Routing with Time Windows Problem This model is used for solving a catering problem of a company located in Medan city, Indonesia. The result model is in the form of mixed integer linear programming problem. We solve the model using a nearest neighbor heuristic algorithm.

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