Performance Analysis of the Detection Methods for SFBC-OFDM Communication Systems in a Fast Fading Channel

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Abstract: MIMO-OFDM systems have increased the diversity gain and provide higher data rates. For high performance 4G wireless communication, we use MIMO-OFDM employing Space Frequency Block Codes (SFBC) which provides spatial diversity in fast fading environments. By converting the codes from time domain to frequency domain, the MIMO-OFDM schemes apply Alamouti coding directly to OFDM technique. SFBC technique encodes a pair of input bits wherein each symbol is transmitted from two antennas over two sub-carriers. On the receiver side we analyze four detection methods including Simple maximum-likelihood (SML), Joint maximum-likelihood (JML), Zero-forcing (ZF) and Decision-feedback (DF). The evaluation of the detection methods is done on the basis of the SNR (Signal to Noise Ratio), BER (Bit Error Rate) and complexity of implementation in fast fading channels under three vehicular speeds i.e. 30km/hr, 60km/hr and 120km/hr. Using the results obtained from both mathematical expressions and numerical simulations, we compare the presented schemes and show their significant advantages.

Index terms- *Multiple-input–multiple-output Orthogonal Frequency Division Multiplexing (MIMO-OFDM), Alamouti code, Space Frequency Block Code, Simple Maximum Likelihood, Joint Maximum Likelihood, Zero Forcing, Decision Feedback, Signal to Noise Ratio, Bit error rate(BER).*

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I. INTRODUCTION

In OFDM, a high data rate signal is broken down into multiple narrowband carriers each behaving as a flat fading sub-channel, where each carrier is less sensitive to frequency selective fading. The subcarriers have the minimum frequency separation required to maintain orthogonality of their corresponding time domain waveforms, yet the signal spectra corresponding to the different subcarriers overlap in frequency[1]. The available bandwidth is used very efficiently as the sidebands from each carrier overlap. A guard interval having the cyclic prefix is added to each symbol to minimize the channel delay spread and intersymbol interference. An inverse FFT (Fast Fourier Transform) is performed on the frequency-domain subcarriers to produce the OFDM symbol in the time-domain and FFT acts as demodulator [2].

To fulfill the increasing demand for high-performance 4G wireless communication, OFDM may be combined with antenna arrays at the transmitter and receiver to increase the system capacity as well as antenna and frequency diversity gain resulting in a multiple-input multiple-output (MIMO) configuration [3].

In MIMO systems, a transmitter sends multiple signals by multiple transmit antennas. These streams go through a channel matrix having rows and columns equal to the number of transmit and receive antennas respectively. Then, the receiver gets the received symbols by the multiple receive antennas and decodes the received signal vectors into the original information.

An MIMO system is modelled as

$$y = Hx + n \qquad \text{……..(1)}$$

where y and x are the receive and transmit vectors, respectively, and H and n are the channel matrix and the noise vector, respectively.

II. SFBC BLOCK DIAGRAM AND WORKING

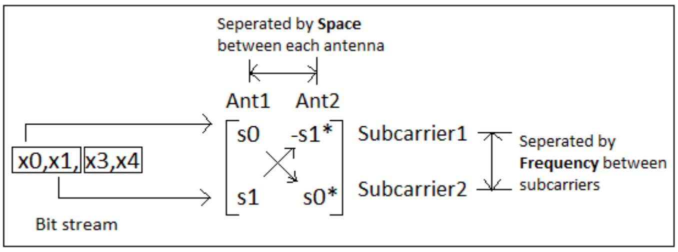

Fig 1. Space frequency block code

SFBC is represented as a matrix with each row representing a subcarrier and each column representing a transmitting antenna. To avoid fast fading in the time domain, two orthogonal symbols are mapped over two adjacent subcarriers of the same OFDM symbol and over two antennas. This provides transmission diversity and reduces the transmission delay. SFBC encodes a pair of bits, x0 and x1 into four symbols s0, s1, -s1* and s0* and transmits s0 and –s1* over a one sub-carrier from the two antennas Ant1 and Ant2. The other two vectors, s1 and s0*, are transmitted over the subsequent sub-carrier [4][5].

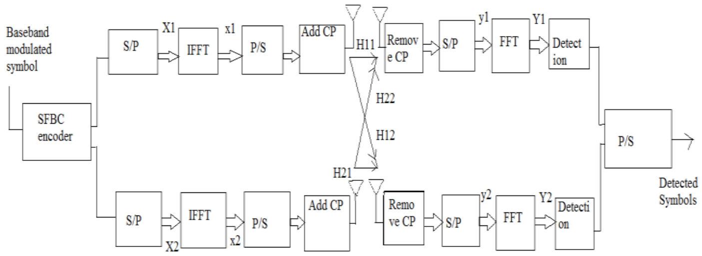

Fig 2. SFBC-OFDM Transmitter-receiver system

The block diagram consists of an SFBC encoder, S/P (Serial to parallel converter), IFFT (Inverse Fast Fourier Transform) block which converts the symbols to the time domain, P/S (Parallel to Serial converter) and a block to add cyclic prefix having a length greater than the expected delay spread to maintain orthogonality among subcarriers[2]. The fast fading channel matrix and parameters are given further down in this paper. At the receiver end, the CP is removed and FFT is performed to get the symbols in the frequency domain. The different detection methods mentioned in the abstract are then used to get the original data bits which were transmitted.

The procedure with mathematical equations is given below:

- At the transmitter, a block of N modulation symbols $x_1, x_2 \dots x_N$ arrives at the SFBC encoder.

- Space-frequency encoder codes each data symbol **X**(*n*),

  X(n)=[X(n,0) X(n,1)….X(n,K-2) X(n,K-1)]

  Into two vectors X1(*n*) and X2(*n*) as

  X1(n)=[X(n,0) –X*(n,1)….X(n,K-2) -X*(n,K-1)] ....……(2)
  X2(n)=[X(n,1) X*(n,0)….X(n,K-1) X*(n,K-2)] ……………….....................................(3)

  where *K* is the number of subcarriers, and * denotes complex conjugate.

- The output of the SFBC encoder is the two K× 1 vectors ***X***1 and ***X***2 that represent the frequency-domain OFDM symbols transmitted from each antenna which are fed to IFFT block.

- The IFFT is assumed to have a block size of N, so it takes N/2 Alamouti block codes to perform one IFFT. IFFT produces the time-domain signal vectors **x**[n] which is written as

$$x[n]=\left(\sum_{n=0}^{N-1} X[k].\exp(j2\pi nk/N)\right)/N \quad ....(4)$$

- Because of the frequency domain interpretation of the Alamouti symbols, when Alamouti coding is used with OFDM, code blocks are referred as space frequency block codes. Hence, the samples at the output of the IFFT stage are "time" domain samples of the transmitted waveform.
- The cyclic prefix is then added with length equal to or larger than the length of the channel impulse response to prevent inter-symbol interference [5].
- The channel impulse response between transmit antenna *m* and the receive antenna at the *nth* OFDM block instance can be expressed as

$$h_m(n) = \sum_{l=0}^{L-1} hm, l(n)\, z^{-1} \quad ……..(5)$$

  where z^{-1} and *hm*,*l*(*n*) denote a unit sample delay and the complex channel gain of the *lth* resolvable multipath component, respectively.
- The received signals are combined at the receiver using ZF detection.

  The odd and even components of the signal at the receiver are
  R1= h11y1 + h21y2 ..…..(6)
  R2= -h12y1* + h22y2* ..…...(7)

  Channel matrix, h = h11h12* - h21h22*
  Where, h11, h12, h21 and h22 are the channel coefficients and y1 and y2 are the received symbols.

- The received signals after IFFT are then estimated as shown below:

$\tilde{x}1$ = h12*R1 + h21R2* ......(8)
= h12*(h11Y1+h21Y2)+ (h21(-h12*Y1+h22*Y2))
= h12*h11Y1+<u>h12*h21Y2</u>- <u>h12*h21Y2</u>+h21h22*Y1
= Y1 (h12*h11+h21h22*) / h
= Y1
= X1

Similarly,

$\tilde{x}2$ = h22*R1 – h11R2* ……(9)
= h22*(h11Y1+h21Y2) - (h11(-h12*Y1+h22*Y2))
= <u>h22*h11Y1</u>+h22*h21Y2+h11h12*Y2 - <u>h11h22*Y1</u>
= Y2 (h22*h21+h11h12*) / h
= Y2
= X2

The degradation caused by the channel variation in high-mobility systems is proportional to the Doppler frequency, which may exceed 20% of the subcarrier spacing. Consequently, space-frequency block coded (SFBC) systems are proposed to combat the channel variations in time-domain because the SFBC blocks are transmitted simultaneously over multiple adjacent subcarriers.

III. SFBC DETECTION METHODS

For any communication system to perform satisfactorily it requires a proper transmitter, a suitable channel and a detector that is sensitive to the received signals. The detector should be capable to accept only the required signals and remove the errors to get the correct data that had been transferred.

For SFBC detection, the four novel detectors to combat the fast fading channel and hence obtain better performance are:

- Simple maximum-likelihood (SML) detector
- Joint Maximum Likelihood (JML) detector
- The Zero-forcing (ZF) detector
- The Decision-feedback (DF) detector

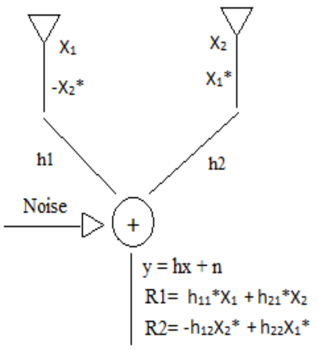

Fig.3 MISO technique used for SFBC detection

We are using 2×1 MISO (Multiple input Single Output) where we have 2 transmitters and one receiving antenna. The symbol X1 from first TX is transmitted along with symbol X2 from second TX on the first subcarrier and symbol –X2* from first TX and X2* from second TX on second subcarrier and so on for the remaining 128 subcarriers.
In $h_{i,j}$ 'i' is the transmitter number and 'j' the subcarrier number.
So h1 = [$h_{1,1}$ $h_{1,2}$ $h_{1,3}$ ....$h_{1,128}$]
and h2 = [$h_{2,1}$ $h_{2,2}$ $h_{2,3}$ ....$h_{2,128}$]

Additive white Guassian noise (AWGN) is added to the message symbol on the transmission fast fading Rayleigh channel. The received symbol is 'y' contains message symbol and noise. The detector filters the noise out and divides 'y' in the even and odd parts R1 and R2 respectively.

The various detection methods stated above can be used to extract the transmitted symbols X1 and X2 from R1 and R2.

The received symbol y in matrix form is y = Hs + v
where, H is the channel matrix and v is the additive white Gaussian noise. Thus, the objective of the receiver will be to estimate 's' from the pair (y,H).
The objective of the receiver is to obtain an estimate of the symbol x, from the given data in y and H.

Thus, the likelihood of $\hat{x}$ is obtained by the Maximum Likelihood estimate of x, i.e. $\hat{x}_{ML}$, is given by

$$\hat{x}_{ML} = \arg_{min} \| y - H\hat{x} \| \qquad ....(10)$$

In this expression $\arg_{min}$ is argument of the minimum.
$\| y - H\hat{x}\|$ attains minimum value among all values of $H\hat{x}$ and given y. Thus, the ML detector chooses the message $\hat{x}$ which yields the smallest distance between the received vector, y, and hypothesized message, $H\hat{x}$.

i) Simple maximum-likelihood (SML) detector:

SML(Simple Maximum Likelihood) detector does detection of X_{2i} and X_{2i+1} seperately. It doesn't consider noise and cross talk (off diagonal elements of matrix H).

Algorithm:

1. Received symbol matrix, R = h1x1+h2x2 + n
2. Channel matrix, h1 = $|h11|^2 + |h22|^2$
   h2 = $|h12|^2 + |h21|^2$
3. Two received signals: R1=h11x1 + h21x2
   R2= -h12x1*+h22x2*
4. Take estimate of received symbols:
   $\widetilde{x1}$= h11*R1 + h22R2*
   $\widetilde{x2}$ = h21*R1 – h12R2*
5. Equalize the symbols in the frequency domain.

ii) Joint Maximum Likelihood (JML) detector:

It detects two symbols jointly. Decoding complexity increases exponentially as number of transmitting antennas and receiving antennas increases.

Algorithm:

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1.Calculate the A posteriori probability (APP) of the received symbol pairs

P(1,1,:) = $|R1 + h11 + h21|^2$ + $|R2 + h22 – h12|^2$ ……………….(4.13)

P(1,2,:) = $|R1 + h11 - h21|^2$ + $|R2 + h22 + h12|^2$ ……………….(4.14)

P(2,1,:) = $|R1 - h11 + h21|^2$ + $|R2 - h22 – h12|^2$ ……………….(4.15)

P(2,2,:) = $|R1 - h11 - h21|^2$ + $|R2 - h22 + h12|^2$

2. Make 64 matrices of the order 2*2 .

3. First find the index of the row and then of the column of the minimum element in the APP matrices.

4. To recover the sent bits use the following equation:
X_receive=[X_ receive min_row(min_col) – 1 min_col - 1]
which gives the element required from the matrix.

iii) The Zero-forcing (ZF) detector:

Linear ZF detection is usually employed in MIMO detection, due to the simplicity of implementation without any a priori knowledge of the noise in the channel. The Zero Forcing (ZF) detector applies the inverse of the frequency response of the channel to the received vector which sets the linear and successive interference amplitude to zero and then rounds the result to the to the closest symbol in the alphabet considered. It thus removes the effect of the channel like inter-symbol interference (ISI) from the received signal. When the MIMO channel matrix is square ($n_R = n_T$) and non-singular (invertible) we simply have to invert the cannel matrix (H^{-1}) in the inversion step:

$\hat{s}$= Q{$H^{-1}x$}.

However, if the channel matrix is tall ($n_R > n_T$), we need to take the pseudo-inverse of H, which leads to the following inversion step:

$\hat{s}$ = Q{$(H^H H)^{-1} H^H x$}

The received symbols are pre-multiplied with a matrix often called as nulling matrix. In ZF detection, the nulling matrix is either the inverse or the pseudo-inverse of the channel matrix (H), depending on the matrix dimensions. The ZF detector has a couple of disadvantages like, in some cases, finding singular channel matrices which are not invertible and the fact that it completely cancels the interferences, at the cost of noise enhancement.

Algorithm:
1. Received symbol matrix, R = h1x1+h2x2 + n
2. Channel matrix, h = h11h12* - h21h22*
3. Two received signals: R1=h11x1 + h21x2
R2=-h12x1*+h22x2*
4. Take estimate of received symbols:
$\widetilde{x1}$ = h12*R1 + h21R2*
$\widetilde{x2}$ = h22*R1 – h11R2*
5. $\widetilde{x1}$ = h12*R1 + h21R2*

= h12*(h11X1+h21X2) + (h21(-h12*X1+h22*X2))
= h12*h11X1+<u>h12*h21X2 - h12*h21X2</u>+h21h22*X1
= X1 (h12*h11+h21h22*) / h ……………(4.16)

$\widetilde{x1}$ = X1

$\widetilde{x2}$ = h22*R1 – h11R2*
=<u>h22*h11X1</u>+h22*h21X2+h11h12*X2 - <u>h11h22*X1</u>

= X2 (h22*h21+h11h12*) / h

$\widetilde{x2}$ = X2

7. Take inverse of channel matrix:
h^{-1} = $(h11h12^* + h21h22^*)^{-1}$
8. Final estimated symbols are: $\widetilde{x1}$ = x1 h^{-1}
$\widetilde{x2}$ = x2 h^{-1}
9. Equalize the symbols in the frequency domain.
X_receive(1:2:Ncarr) = X_est1;
X_receive(2:2:Ncarr) = X_est2;

iv) The Decision-feedback (DF) detector:

The DF detector uses feedback of the received symbols $X_{(2i+0)}$ and $X_{(2i+1)}$ to help make an estimate of the channel. It first estimates a signal and then uses it to help make a decision about the next estimated signal. The decision fed back is then subtracted from the detected output signal received. The algorithm of DF detection method is given below. ………………(4.18)

1.Estimate the first variant $\hat{X}$ 1 using ZF detection method.

2.Nullify the contribution of $\hat{X}$ 1 by subtracting it from the linear equalizer output signal. Then take the hard decision to obtain the estimated signal $\hat{X}$ 2 . ……….(4.19)

Algorithm:
1. Received symbol matrix, R = h1x1+h2x2 + n
2. Channel matrix,
h1 = h11h12* + h21h22* ...…… (h_ZF)
h2 = $|h12|^2$ + $|h21|^2$ ……… (h_SML)
h = h11h21* - h12h22*
3. Two received signals: R1 = h11x1 + h21x2
R2= -h12x1*+h22x2*
$\widetilde{x1}$=h11*R1 + h22R2
=h11*h11X1+h11*h21X2-h22h12*X2+h22h22*X1
= $(h11^2 + h22^2)$ X1 + (h11*h21 - h22h12*) X2

Assuming h12 ≅ h11 and h22 ≅ h21,

$\widetilde{x1}$= (h11^2 + h22^2) X1 / h

= X1

$\widetilde{x2}$= h21*R1 + h12R2

=h21*h11X1+h21*h21X2-h12h12*X2*+h12h22*X1*

= X2

4. Take estimate of received symbols:

$\widetilde{x1}$= (h11*R1 + h22R2*)/ h1

5. Calculate X_sink1 by taking signum of the real part of $\widetilde{x1}$, adding 1 to it and dividing by 2.

6. $\widetilde{x2}$ = {h21*R1 – h12R2 – h(2X_sink1 - 1)}/ h2.

7. Calculate X_sink1 by taking signum of the real part of $\widetilde{x1}$, adding 1 to it and dividing by 2.

8. Equalize the symbols in the frequency domain.

X_receive(1:2:Ncarr) = X_sink1;

X_receive(2:2:Ncarr) = X_sink2;

IV. SIMULATION RESULTS:

After performing analysis of SFBC –OFDM system for 3 different channels such as 30kmph, 60kmph and 120kmph channel using 4 detection methods, we obtained the following results using BPSK modulation, no. of subcarriers =128, cyclic prefix =32, 64 point FFT and IFFT.

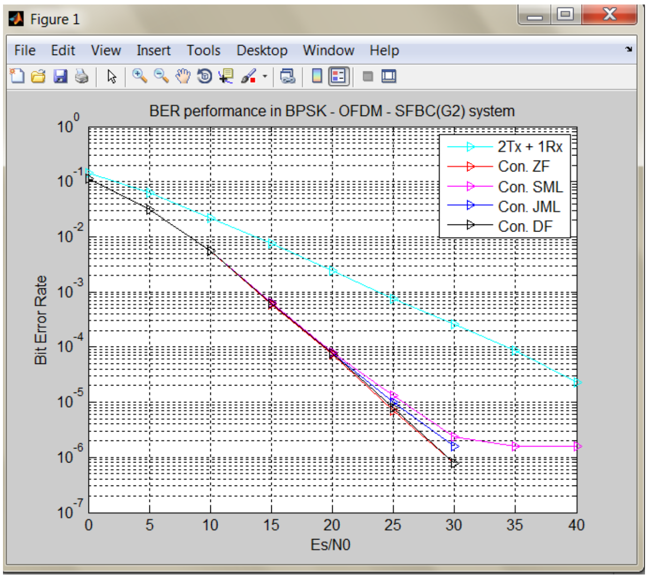

Fig 4. BER performance at 25Hz(30kmph)

Above figure shows the BER performance of SFBC OFDM system for slow fading channel. For slow fading channel Doppler shift=25Hz and vehicle speed=30kmph. From figure we have analysed that DF detection method gives better BER performance for Es/N$_0$=30 which is BER=10^{-6} to 10^{-7}.

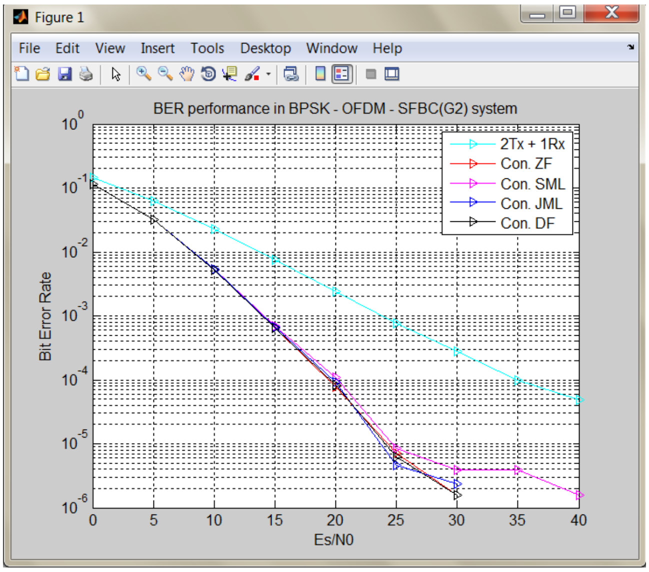

Fig 5. BER performance at 50 Hz(60kmph)

Above figure shows the BER performance of SFBC OFDM system for slow fading channel. For slow fading channel Doppler shift=50Hz and vehicle speed=60kmph. From figure we have analysed that conventional DF detection method gives better BER performance for Es/N$_0$=30 which is BER=10^{-6}

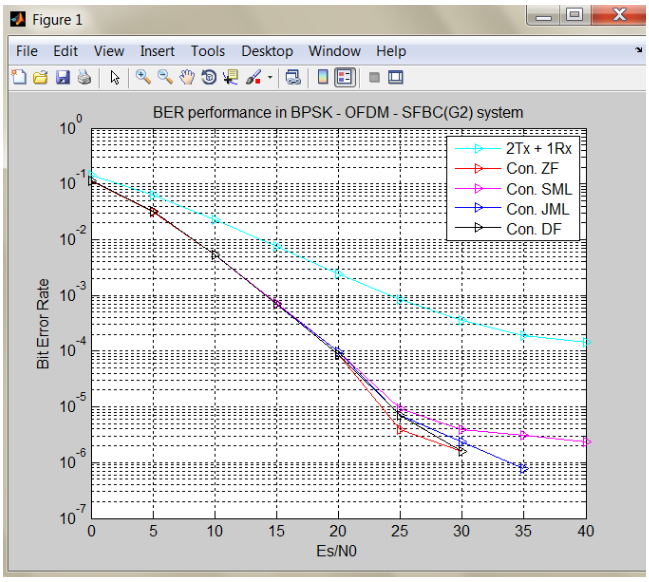

Fig 6. BER performance at 100Hz(120kmph)

Above figure shows the BER performance of SFBC OFDM system for slow fading channel. For slow fading channel Doppler shift=100Hz and vehicle speed=120kmph. From figure we have analysed that ZF detection method gives better BER performance for E$_s$/N$_0$=30 which is BER=10^{-6} to 10^{-7}

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V. COMPARISON BETWEEN DETECTION TECHNIQUES:

Based on SNR v/s BER:
Taking the reference Es/No (SNR) to be 30 we compare the BER of the different detection methods.

SR. NO.	DETECTION METHODS	SNR (Eb/No)	BER
1.	ZF	30	10^{-6}
2	JML	30	10^{-5}-10^{-6}
3.	DF	30	10^{-5}-10^{-6}
4.	SML	30	10^{-5}-10^{-4}

Up till SNR= 20, all detections methods show similar performances (BER values overlap). Then, complexity of the system plays an important role in choosing the detection technique.

V1. CONCLUSION

From the numerical expressions and graphical representations, we can conclude that JML detector reproduces the input symbols with least distortion, thus reducing crosstalk and noise but it has the highest complexity. The ZF detector, has the lowest complexity, drives the crosstalk to zero and has best performance with lowest BER. The DF detector has good performance and it attenuates the interference between adjacent channels by whitened-matched filtering, thus having lesser bits in error. The SML detector has the highest number of bits in error and hence least performance.

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