

A fuzzy approach for unequal workers-task assignment with heptagonal fuzzy numbers

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Abstract—The problem of assigning n tasks to n workers with minimum assignment cost often arises in many practical applications. In real life, the cost of assignment may not be certain and also the number of tasks may not be always equal to the number of workers and an unbalanced situation may arise often. In this paper, the representation of the Heptagonal fuzzy number (HFN) and its arithmetic operations are reviewed and a fuzzy approach called as fuzzy row penalty method is extended within the context of Heptagonal fuzzy numbers to deal with the unbalanced Fuzzy Assignment Problems. The cost of assignment is represented by Heptagonal fuzzy numbers which are capable of incorporating the degree of satisfaction about the assignment cost. So the use of HFN is more suitable. The proposed method overcomes the limitations of existing methods and it is illustrated through numerical example.

Keywords-Fuzzy Assignment Problem; Heptagonal fuzzy numbers; Unbalanced Fuzzy Assignment Problem; Value and Ambiguity

I. INTRODUCTION

In most of the real life problems, information available are sometimes insufficient or sometimes available as vague linguistic descriptions such as “about 10 hours”, “high profit” etc. To deal with this, the problem has to be modeled with approximately available data. This can be done using Zadeh’s fuzzy set theory [17]. Lei [8] represented the processing time of operation in the fuzzy job shop scheduling problem with availability constraints using Triangular fuzzy numbers. A.Kumar, A.Gupta and A.Kaur [7] used Triangular fuzzy numbers whereas Sagaya & Henry [14] used Trapezoidal fuzzy numbers to represent the fuzzy cost or fuzzy times in the Fuzzy Assignment Problem. Mahapatra [10] considered the cost of components as triangular fuzzy numbers in series system models with system reliability and cost. A.Kaur and A.Kumar [6] presented a model to solve the transportation problems in fuzzy environment using non-normal Generalized Trapezoidal fuzzy numbers. There are several papers in the literature in which the authors used triangular or trapezoidal fuzzy numbers in order to characterize the vague parameters that arise in real life problems. But in few cases, it is not possible to restrict the membership functions to take either triangular or trapezoidal form. In such cases, Heptagonal fuzzy number (HFN) can be used to solve the problems. K.Rathi&S.Balamohan [11] introduced the concept of HFN.

Although a significant amount of research work has already been carried out by the former researchers on Fuzzy Assignment Problems (FAPs), there is still a need to employ a simple and systematic direct approach to handle the ambiguity and fuzziness in assignment. M.Chen [2]

proposed a fuzzy assignment model that considers all persons to have same skills. X.Wang [16] solved a similar model by graph theory. Dubois & Fortemps [3] surveyed refinements of the ordering of solutions supplied by the max-min formulation and they have given a general algorithm which computes all maximal solutions in the sense of these relations. M.Sakawa, I.Nishizaki and Y.Uemura [15] presented interactive fuzzy programming approach to solve two levels Assignment Problem. C.J.Lin & U.P.Wen [9] solved the assignment problem with fuzzy interval number costs by a labeling algorithm.

In this paper, the uncertainty in assignment cost is represented by Heptagonal fuzzy numbers [11] and a new direct approach is proposed for solving unbalanced FAP without reformulation. Numerical illustrations are given to show the advantages of the proposed method.

Rest of the paper is organized in the following way: In Section II, the basic concept of fuzzy set theory and fuzzy numbers are briefly introduced. In Section III, IV Heptagonal fuzzy number and its arithmetic operations are reviewed from [11]. In Section V, ranking approach for comparing HFNs is presented. In Section VI, fuzzy row penalty method [12] is extended within the context of HFN and its ranking approach to solve the FAP. In Section VII, numerical examples are given and it is shown that the proposed method offers an effective way for handling balanced FAP as well as unbalanced FAP. Finally, Section VIII presents the concluding remarks.

II. PRELIMINARY

In this section, some basic definitions of fuzzy set theory and fuzzy numbers are reviewed [4].

Definition 2.1 A fuzzy set is characterized by a membership function mapping the elements of a domain space or universe of discourse X to the unit interval [0, 1]. (i.e.) $\mu_{\tilde{A}} : X \rightarrow [0,1]$.

Definition 2.2 The support of a fuzzy set \tilde{A} in the universal set X is the set $Supp(\tilde{A}) = \{x \in X \mid \mu_{\tilde{A}}(x) > 0\}$ that contains all the elements of X that have non-zero membership grade in \tilde{A} .

Definition 2.3 The core of a fuzzy set \tilde{A} in the universal set X is the set $Core(\tilde{A}) = \{x \in X \mid \mu_{\tilde{A}}(x) = 1\}$ that contains all the elements of X that exhibit a unit level of membership in \tilde{A} .

Definition 2.4 An α -cut of a fuzzy set \tilde{A} is a crisp set A_{α} defined as $A_{\alpha} = \{x \in X \mid \mu_{\tilde{A}}(x) \geq \alpha\}$

Definition 2.5 A fuzzy set \tilde{A} is a convex fuzzy set if and only if each of its α -cut A_{α} is a convex set.

Definition 2.6 A fuzzy set \tilde{A} is a fuzzy number iff (i) For all $\alpha \in (0,1]$ the α -cut sets A_{α} is a convex set (ii) $\mu_{\tilde{A}}$ is an upper semi continuous function (iii) $Supp(\tilde{A})$ is bounded set in R (iv) The height of $\tilde{A} = \max_{x \in X} \mu_{\tilde{A}}(x) = \omega > 0$.

III. HEPTAGONAL FUZZY NUMBERS

In this section, the concept of Heptagonal fuzzy number (HFN) introduced in [11] is reviewed.

Definition 3.1 The Heptagonal fuzzy number is defined as $\tilde{H} = (f_1(r), g_1(t), g_2(t), f_2(r))$ for $r \in [0, k]$ and $t \in [k, 1]$ where $f_1(r)$ and $g_1(t)$ are bounded left continuous non decreasing functions over $[0, \omega_1]$ and $[k, \omega_2]$ respectively, $f_2(r)$ and $g_2(t)$ are bounded left continuous non increasing functions over $[0, \omega_1]$ and $[k, \omega_2]$ respectively and $0 \leq \omega_1 \leq k, k \leq \omega_2 \leq 1$.

Definition 3.2 The membership function for the heptagonal fuzzy number $\tilde{H} = (h_1, h_2, h_3, h_4, h_5, h_6, h_7; k, \omega)$ is defined as follows

$$\mu_{\tilde{H}}(x) = \begin{cases} 0 & \text{for } x < h_1 \\ k \left(\frac{x-h_1}{h_2-h_1} \right) & \text{for } h_1 \leq x \leq h_2 \\ k & \text{for } h_2 \leq x \leq h_3 \\ k + (\omega - k) \left(\frac{x-h_3}{h_4-h_3} \right) & \text{for } h_3 \leq x \leq h_4 \\ k + (\omega - k) \left(\frac{h_5-x}{h_5-h_4} \right) & \text{for } h_4 \leq x \leq h_5 \\ k & \text{for } h_5 \leq x \leq h_6 \\ k \left(\frac{h_7-x}{h_7-h_6} \right) & \text{for } h_6 \leq x \leq h_7 \\ 0 & \text{for } x \geq h_7 \end{cases}$$

where $0 < k < 1, k \leq \omega \leq 1$. (See Fig. 1)

Remarks 3.1

- (i) The above defined heptagonal fuzzy number becomes normal heptagonal fuzzy number if $\omega = 1$.
- (ii) If $k = 0$, then the heptagonal fuzzy number reduces to triangular fuzzy number (h_3, h_4, h_5) and if $k = 1$, it reduces to the trapezoidal fuzzy numbers (h_1, h_2, h_6, h_7) .

Definition 3.3 For $\alpha \in (0,1]$, the α -cut of HFN $\tilde{H} = (h_1, h_2, h_3, h_4, h_5, h_6, h_7; k, \omega)$ is defined as

$$[\tilde{H}]_{\alpha} = \begin{cases} [h_1 + \frac{\alpha}{k}(h_2-h_1), h_7 - \frac{\alpha}{k}(h_7-h_6)] & \text{for } \alpha \in [0, k] \\ [h_3 + \left(\frac{\alpha-k}{\omega-k}\right)(h_4-h_3), h_5 - \left(\frac{\alpha-k}{\omega-k}\right)(h_5-h_4)] & \text{for } \alpha \in (k, \omega) \end{cases}$$

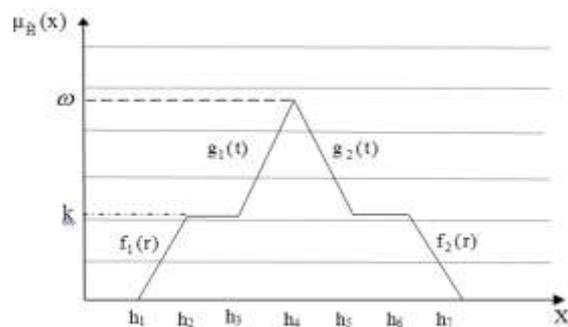


Figure.1 Graphical representation of HFN

IV. ARITHMETIC OPERATIONS ON HEPTAGONAL FUZZY NUMBERS

In this section, Chen's method based on the function principle [1] is used to develop arithmetic operations between heptagonal fuzzy numbers. The function principle is basically a point wise operation and is more useful than the extension principle for the fuzzy numbers with heptagonal membership function because the function principle preserves the membership form whereas it is not the case in extension principle.

Let $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7; k_A, \omega_A)$ and $\tilde{B} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7; k_B, \omega_B)$ be two heptagonal fuzzy numbers. Then

(i) Addition of two heptagonal fuzzy numbers

$$\begin{aligned} \tilde{A} \oplus \tilde{B} &= (c_1, c_2, c_3, c_4, c_5, c_6, c_7; k, \omega) \\ &= (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6, \\ &\quad a_7 + b_7; \min\{k_A, k_B\}, \min\{\omega_A, \omega_B\}) \end{aligned}$$

(ii) Scalar Multiplication of heptagonal fuzzy numbers

$$\lambda \tilde{A} = \begin{cases} (\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4, \lambda a_5, \lambda a_6, \lambda a_7; k_A, \omega_A) & \text{if } \lambda > 0 \\ (\lambda a_7, \lambda a_6, \lambda a_5, \lambda a_4, \lambda a_3, \lambda a_2, \lambda a_1; k_A, \omega_A) & \text{if } \lambda < 0 \end{cases}$$

(iii) Subtraction of two heptagonal fuzzy numbers

$$\begin{aligned} \tilde{A} \ominus \tilde{B} &= \tilde{A} \oplus (-\tilde{B}) \\ &= (a_1, a_2, a_3, a_4, a_5, a_6, a_7; k_A, \omega_A) \oplus \\ &\quad (-b_7, -b_6, -b_5, -b_4, -b_3, -b_2, -b_1; k_B, \omega_B) \\ &= (a_1 - b_7, a_2 - b_6, a_3 - b_5, a_4 - b_4, a_5 - b_3, a_6 - b_2, \\ &\quad a_7 - b_1; \min\{k_A, k_B\}, \min\{\omega_A, \omega_B\}) \end{aligned}$$

V. RANKING OF HEPTAGONAL FUZZY NUMBERS

The ranking method proposed in [11] is used to rank the heptagonal fuzzy numbers.

Definition 5.1 The value of the heptagonal fuzzy number $\tilde{H} = (h_1, h_2, h_3, h_4, h_5, h_6, h_7; k, \omega)$ is given by $val(\tilde{H}; r) = (1 - \lambda)V_*(\tilde{H}; r) + \lambda V^*(\tilde{H}; r)$ where the lower value and upper value of \tilde{H} are given by

$$\begin{aligned} V_*(\tilde{H}; r) &= \gamma \left(\frac{h_1 + r h_2}{r + 1} \right) + h_3(1 - \gamma) + \eta(h_4 - h_3) \text{ and} \\ V^*(\tilde{H}; r) &= \gamma \left(\frac{h_7 + r h_6}{r + 1} \right) + h_5(1 - \gamma) - \eta(h_5 - h_4) \text{ where} \\ \gamma &= \left(\frac{k}{\omega} \right)^r \text{ and } \eta = \frac{r}{r + 1} \left(1 - \frac{k(\omega^r - k^r)}{r \omega^r (\omega - k)} \right). \end{aligned}$$

Definition 5.2 The ambiguity of the heptagonal fuzzy number \tilde{H} is given by $Amb(\tilde{H}) = \frac{1}{2} (V^*(\tilde{H}; r) - V_*(\tilde{H}; r))$

Definition 5.3 For the two HFN \tilde{A} and \tilde{B} , if (i) $val(\tilde{A}) > val(\tilde{B})$ then $\tilde{A} \succ \tilde{B}$ (ii) $val(\tilde{A}) < val(\tilde{B})$ then $\tilde{A} \prec \tilde{B}$ (iii) $val(\tilde{A}) = val(\tilde{B})$ then find the ambiguity $Amb(\tilde{A})$ and $Amb(\tilde{B})$. If $Amb(\tilde{A}) < Amb(\tilde{B})$ then $\tilde{A} \prec \tilde{B}$, if $Amb(\tilde{A}) > Amb(\tilde{B})$ then $\tilde{A} \succ \tilde{B}$ or if $Amb(\tilde{A}) = Amb(\tilde{B})$ then $\tilde{A} = \tilde{B}$.

VI. FUZZY ASSIGNMENT PROBLEM

In this section, mathematical formulation of fuzzy assignment problem is given and fuzzy row penalty method [12] is extended within the context of HFN to solve FAP is proposed. The method is applicable for both balanced and unbalanced FAP.

A. Mathematical formulation of Fuzzy Assignment Problem

Let there be m Tasks and n Workers. If m=n then FAP is balanced otherwise FAP is unbalanced. Let \tilde{C}_{ij} be the cost of assigning i^{th} Worker to the j^{th} Task and the uncertainty in cost is here represented as Heptagonal fuzzy numbers. Let x_{ij} be the decision variable. Then the fuzzy assignment problem can be mathematically stated as follows

$$\begin{aligned} \text{Minimize } \tilde{Z} &= \sum_{i=1}^m \sum_{j=1}^n \tilde{C}_{ij} x_{ij} \\ \text{Subject to } \sum_{j=1}^n x_{ij} &= 1, i = 1, 2, \dots, m; \sum_{i=1}^m x_{ij} = 1, j = 1, 2, \dots, n \end{aligned}$$

$$x_{ij} = \begin{cases} 1 & \text{if the } i^{th} \text{ person is assigned to the } j^{th} \text{ job} \\ 0 & \text{otherwise} \end{cases}$$

B. Algorithm for the proposed method

Step 1: Form the fuzzy cost table for the given problem. The number of rows need not be equal to the number of columns as the method is applicable for unbalanced FAP without reformulation.

Step 2: Calculate the respective ranking value of each heptagonal fuzzy cost \tilde{C}_{ij} and form it as a separate table.

Step 3: In each row of the fuzzy cost table, encircle the fuzzy cost with minimum ranking value.

Step 4: Search for the fuzzy optimal assignment as follows
 (a) Examine all the encircled fuzzy costs and identify the fuzzy costs which are uniquely encircled both row wise and column wise. Assign all such fuzzy costs in the table and delete the corresponding row(s) and column(s).

(b) In the resulting table, identify any one row i such that the row i has single encircled fuzzy cost and the corresponding column j has more than one encircled fuzzy cost. Consider the rows of the encircled fuzzy costs in the column j and calculate the row penalty, that is, the difference between the ranking value of minimum and next to minimum fuzzy cost in the respective rows. Choose the row $*$ with maximum row penalty and assign the encircled fuzzy cost \tilde{C}_{*j} in that row. Delete the row $*$ and column j . Go to the next step.

Note: If more than one row has same maximum row penalty then calculate the difference between minimum and next to

next minimum ranking value of fuzzy costs. Assignment can be made in the row with maximum difference.

Step 5: For the resulting table, repeat the Step 3 and Step 4 until all rows and columns have been crossed out.

Step 6: Based on the optimal fuzzy assignment obtained,

$$\text{calculate the total optimum fuzzy cost } \tilde{Z} = \sum_{i=1}^n \sum_{j=1}^n \tilde{C}_{ij} x_{ij} .$$

VII. NUMERICAL EXAMPLE

In this section, numerical example is given to illustrate the proposed method and it is shown that the proposed method offers an effective way for handling balanced FAP as well as unbalanced FAP.

Example 7.1

Let us consider a fuzzy unbalanced assignment problem with rows representing the 4 areas A, B, C, D and columns representing the salesman's S1,S2,S3,S4,S5 in which the cost of assignment is represented by Heptagonal fuzzy numbers and are given in Table 1. The problem is to find the optimal assignment so that the total cost of area assignment becomes minimum.

Solution:

Step 1: The unbalanced fuzzy cost table is given in Table 1.

Step2: Calculate the ranking value of each fuzzy cost \tilde{C}_{ij} by taking parameters as $\lambda = 0.5$, $r = 1$ and is given in Table 2.

Step 3: In each row encircle the fuzzy cost with minimum ranking value as given in Table 3.

Step 4: Examine all the encircled fuzzy costs in Table 3. The fuzzy costs \tilde{C}_{12} and \tilde{C}_{31} are uniquely encircled in both row wise and column wise. Assign the fuzzy costs \tilde{C}_{12} and \tilde{C}_{31} as in Table 3. Delete the corresponding row and column and the resulting table is given in Table 4.

Step 5: In the resulting Table 4, calculate the row penalty for row 2 and 4 which are same. So calculate Penalty 2. Row 4 has highest penalty. Assign \tilde{C}_{43} and delete the corresponding row and column. The resulting Table 5 has two cells, assign the cost \tilde{C}_{24} which has least rank.

Step 6: The optimal assignment is $x_{12} = 1$, $x_{24} = 1$, $x_{31} = 1$, $x_{43} = 1$, $x_{ij} = 0$ for remaining i,j and the total optimal fuzzy cost is (6,14,21,33,42,49,59;0.2,0.4) with crisp value $V=32.125$, ambiguity=13.625.

The optimal assignment obtained for Example 7.1 by the proposed method is same as that of the optimal solution obtained by the existing method [5]. However, by the proposed method the unbalanced FAP is solved directly without introducing dummy row with fuzzy zero cost.

Table 1: Unbalanced Fuzzy Cost Table of Example 7.1

| Area | Salesman | | | | |
|------|----------------------------|----------------------------|-----------------------------|-----------------------------|-----------------------------|
| | S1 | S2 | S3 | S4 | S5 |
| A | (0,2,3,6,9,10,11;0.2,0.4) | (2,4,6,9,11,13,16;0.6,0.8) | (1,3,6,12,14,16,18;0.2,0.9) | (1,3,9,12,15,17,19; 0.6,1) | (0,2,3,6,9,10,11;0.2,0.4) |
| B | (1,3,9,12,15,17,19; 0.6,1) | (0,2,3,6,9,10,11;0.2,0.4) | (2,4,6,9,11,13,16;0.6,0.8) | (0,2,3,6,9,10,11;0.2,0.4) | (1,3,9,12,15,17,19; 0.6,1) |
| C | (2,4,6,9,11,13,16;0.6,0.8) | (4,7,9,12,15,18,20;0.5,1) | (0,2,3,6,9,10,11;0.2,0.4) | (1,3,6,12,14,16,18;0.2,0.9) | (0,2,3,6,9,10,11;0.2,0.4) |
| D | (1,3,9,12,15,17,19; 0.6,1) | (2,4,6,9,12,15,18;0.4,0.7) | (2,4,6,9,11,13,16;0.6,0.8) | (0,2,3,6,9,10,11;0.2,0.4) | (3,6,9,12,15,18,20;0.4,0.8) |

Table 2: Ranking value of fuzzy cost in Table 1

| Area | Salesman | | | | |
|------|--|---|--|--|---|
| | S1 | S2 | S3 | S4 | S5 |
| A | $V(\tilde{c}_{11}) = 5.875$, $Amb(\tilde{c}_{11}) = 3.125$ | $V(\tilde{c}_{12}) = 12.857$, $Amb(\tilde{c}_{12}) = 4.625$ | $V(\tilde{c}_{13}) = 10.67$, $Amb(\tilde{c}_{13}) = 3.22$ | $V(\tilde{c}_{14}) = 10.8$, $Amb(\tilde{c}_{14}) = 5.4$ | $V(\tilde{c}_{15}) = 5.875$, $Amb(\tilde{c}_{15}) = 3.125$ |
| B | $V(\tilde{c}_{21}) = 10.8$, $Amb(\tilde{c}_{21}) = 3.22$ | $V(\tilde{c}_{22}) = 5.875$, $Amb(\tilde{c}_{22}) = 3.125$ | $V(\tilde{c}_{23}) = 5.75$, $Amb(\tilde{c}_{23}) = 4.625$ | $V(\tilde{c}_{24}) = 5.875$, $Amb(\tilde{c}_{24}) = 3.125$ | $V(\tilde{c}_{25}) = 10.8$, $Amb(\tilde{c}_{25}) = 3.22$ |
| C | $V(\tilde{c}_{31}) = 5.75$, $Amb(\tilde{c}_{31}) = 4.625$ | $V(\tilde{c}_{32}) = 12.125$, $Amb(\tilde{c}_{32}) = 4.125$ | $V(\tilde{c}_{33}) = 5.875$, $Amb(\tilde{c}_{33}) = 3.125$ | $V(\tilde{c}_{34}) = 10.6$, $Amb(\tilde{c}_{34}) = 3.22$ | $V(\tilde{c}_{35}) = 5.875$, $Amb(\tilde{c}_{35}) = 3.125$ |
| D | $V(\tilde{c}_{41}) = 10.8$, $Amb(\tilde{c}_{41}) = 5.4$ | $V(\tilde{c}_{42}) = 9.428$, $Amb(\tilde{c}_{42}) = 4.625$ | $V(\tilde{c}_{43}) = 5.75$, $Amb(\tilde{c}_{43}) = 4.5$ | $V(\tilde{c}_{44}) = 5.875$, $Amb(\tilde{c}_{44}) = 3.125$ | $V(\tilde{c}_{45}) = 11.875$, $Amb(\tilde{c}_{45}) = 4.375$ |

Table 3: Encircled fuzzy cost

| Area | Salesman | | | | |
|------|----------------------------|----------------------------|-----------------------------|-----------------------------|-----------------------------|
| | S1 | S2 | S3 | S4 | S5 |
| A | (0,2,3,6,9,10,11;0.2,0.4) | (2,4,6,9,11,13,16;0.6,0.8) | (1,3,6,12,14,16,18;0.2,0.9) | (1,3,9,12,15,17,19; 0.6,1) | (0,2,3,6,9,10,11;0.2,0.4) |
| B | (1,3,9,12,15,17,19; 0.6,1) | (0,2,3,6,9,10,11;0.2,0.4) | (2,4,6,9,11,13,16;0.6,0.8) | (0,2,3,6,9,10,11;0.2,0.4) | (1,3,9,12,15,17,19; 0.6,1) |
| C | (2,4,6,9,11,13,16;0.6,0.8) | (4,7,9,12,15,18,20;0.5,1) | (0,2,3,6,9,10,11;0.2,0.4) | (1,3,6,12,14,16,18;0.2,0.9) | (0,2,3,6,9,10,11;0.2,0.4) |
| D | (1,3,9,12,15,17,19; 0.6,1) | (2,4,6,9,12,15,18;0.4,0.7) | (2,4,6,9,11,13,16;0.6,0.8) | (0,2,3,6,9,10,11;0.2,0.4) | (3,6,9,12,15,18,20;0.4,0.8) |

Table 4: Fuzzy Cost Assignment

| Area | Salesman | | | | | Row Penalty |
|------|----------|----|-----------------------------------|----------------------------------|------------------------------------|------------------------------------|
| | S1 | S2 | S3 | S4 | S5 | |
| A | | | | | | |
| B | | | (2, 4, 6, 9, 11, 13, 16;0.6, 0.8) | (0, 2, 3, 6, 9, 10, 11;0.2, 0.4) | (1, 3, 9, 12, 15, 17, 19; 0.6, 1) | Penalty1 = 0.125, Penalty2=5.05 |
| C | | | | | | |
| D | | | (2, 4, 6, 9, 11, 13, 16;0.6, 0.8) | (0, 2, 3, 6, 9, 10, 11;0.2, 0.4) | (3, 6, 9, 12, 15, 18, 20;0.4, 0.8) | Penalty1=0.125, Penalty2=6.125 |

Table 5: Fuzzy cost assignment

| Area | Salesman | | | | |
|------|----------|----|----|----------------------------------|-----------------------------------|
| | S1 | S2 | S3 | S4 | S5 |
| A | | | | | |
| B | | | | (0, 2, 3, 6, 9, 10, 11;0.2, 0.4) | (1, 3, 9, 12, 15, 17, 19; 0.6, 1) |
| C | | | | | |
| D | | | | | |

VIII. CONCLUSION AND FUTURE ENHANCEMENT

In this paper, fuzzy row penalty method [12] is extended to solve unbalanced fuzzy assignment problem with Heptagonal fuzzy cost. The method is applicable for both balanced and unbalanced FAP. It provides a direct approach to solve unbalanced FAP without reformulation. The optimal solution to FAP obtained by the proposed method is same as that of the optimal solution obtained by the existing methods. However the proposed method is simpler, easy to understand and it takes few steps for obtaining the fuzzy optimal solution. Numerical example shows that the proposed method offers an effective tool for handling the fuzzy assignment problem. In future, the proposed method may be modified to find the fuzzy optimal solution to travelling salesman problem [13] and maximization problem in FAP.

REFERENCES

[1] S.H.Chen, "Operations on fuzzy numbers with function principal", Tamkang Journal of Management Sciences, vol.6, pp.13-25,1985.
 [2] M. Chen, "On a fuzzy assignment problem", Tamkang Journal , vol.22, pp.407-411,1985.

[3] D.Dubois and P.Fortemps, "Computing improved optimal solutions to max-min flexible constraint satisfaction problems", European Journal of Operational Research , vol.95, pp.118-126, 1999.
 [4] George J.Klir and Tina.A.Folger, Fuzzy sets, Uncertainty and Information, Prentice Hall of India Pvt. Ltd., New Delhi , 2005.
 [5] K.KadhirvelandK.Balamurugan, "Method for Solving Unbalanced Assignment Problems Using Triangular Fuzzy Numbers", International Journal of Engineering Research and Applications, vol.3, no.5, pp.359-363, 2013.
 [6] A.Kaur and A.Kumar, "A new approach for solving fuzzy transportation problems using generalized fuzzy numbers", Applied Soft Computing, vol.12, pp.1201-1213, 2012.
 [7] A.Kumar, A.Gupta and A.Kaur, "Method for solving fully Fuzzy Assignment Problems using Triangular fuzzy numbers", International Journal of Computer and Information Engineering , pp.231-234, 2009.
 [8] D.Lei, "Fuzzy job shop scheduling problem with availability constraints", Computers and Industrial Engineering , vol.58, pp.610-617, 2010.

- [9] C.J.Lin and U.P.Wen, "A labeling algorithm for the fuzzy assignment problem", *Fuzzy Sets and Systems*, vol.142, pp.373-391, 2004.
- [10] B.M.Mahapatra and G.S.Mahapatra, "Reliability and Cost Analysis of Series System Models using Fuzzy Parametric Geometric Programming", *Fuzzy Information and Engineering*, vol.4, pp.399-411, 2010.
- [11] K. Rathi and S. Balamohan, "Representation and ranking of fuzzy numbers with heptagonal membership function using value and ambiguity index", *Applied Mathematical Sciences*, vol.8, no.87, pp.4309-4321, 2014.
- [12] K.Rathi, S.Balamohan, P.Shanmugasundaram, M.Revathi, "Fuzzy row penalty method to solve assignment problems with uncertain parameters", *Global Journal of Pure and Applied Mathematics*, vol.11, no.1, pp.39-44, 2015.
- [13] M.Revathi, R.Saravanan and K.Rathi, "A new approach to solve travelling salesman problem under fuzzy environment", *International Journal of Current Research*, vol.7, no.12, pp.24128-24131, 2015.
- [14] Sagaya Roseline and A.Henry, "New Methods to solve Fuzzy Assignment Problems using Ordering based on the magnitude of a fuzzy number", *Advances in Fuzzy Sets and Systems*, vol.13, no.1, pp.47-60, 2012.
- [15] M.Sakawa, I.Nishizaki and Y.Uemura, "Interactive fuzzy programming for two level linear and linear fractional production and assignment problems: a case study", *European Journal of Operations Research*, vol.135, pp.142-157, 2001.
- [16] X.Wang, "Fuzzy optimal assignment problem", *Fuzzy Mathematics*, vol.3, pp.101-108, 1987.
- [17] L.A.Zadeh, *Fuzzy sets, Information and Control*, vol.8, pp.338-353, 1965.