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A Common Fixed Point Theorem in Intuitionistic Menger Spaces

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Abstract: The aim of this paper is to consider intuitionistic Menger Spaces and prove a common fixed point theorem for six mappings using compatibility of type (P_1) and (P_2).

Keywords: *Intuitionistic Menger Spaces; Common fixed point; Compatibility of type (P_1) and (P_2).*

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I. INTRODUCTION

There have been a number of generalizations of metric spaces. One such generalization is Menger space introduced in 1942 by Menger [5] who used distribution functions instead of nonnegative real numbers as values of the metric. This space was expanded rapidly with the pioneering works of Schweizer and Sklar[8,9]. Modifying the idea of Kramosil and Michalek [3], George and Veeramani[1] introduced fuzzy metric spaces which are very similar that of Menger space. Recently Park [7] introduced the notion of intuitionistic fuzzy metric spaces as a generalization of fuzzy metric spaces.

Kutukcu et. al [4] introduced the notion of intuitionistic Menger Spaces with the help of t-norms and t-conorms as a generalization of Menger space due to menger [5]. Further they introduced the notion of Cauchy sequences and found a necessary and sufficient condition for an intuitionistic Menger Space to be complete. Sessa [10] initiated the tradition of improving coomutativity in fixed point theorems by introducing the notion of weakly commuting maps in metric spaces. Jungck [2] soon enlarged this concept to compatible maps. The notion of compatible mapping in a Menger space has been introduced by Mishra [6].

II. PRELIMINARIES

Definition 2.1. A binary operation : [0,1] [0,1] [0,1] is a t-norm if is satisfying the following conditions:

- is commutative and associative,
- is continuous,
- a 1 = a ,for all a [0,1],
- a b c d whenever a c and b d, for all a,b,c,d [0,1].

Definition 2.2. A binary operation : [0,1] [0,1] [0,1] is a t-conorm if is satisfying the following conditions:

- is commutative and associative,
- is continuous,
- a 0 = a ,for all a [0,1],
- a b c d whenever a c and b d, for all a,b,c,d [0,1].

Remark 2.3. The concept of triangular norms (t-norms) and triangular conforms (t-conorms) are known as the axiomatic skeletons that we use for characterizing fuzzy intersectiob and union respectively. These concepts were originally introduced by Menger [1] in his study of statistical metric spaces.

Definition 2.4. A distance distribution function is a function F : R R^+ which is non-decreasing, left continous on R and inf {F(t) : t R} = 0 and sup {F(t) : t R} = 1. We will denote by D the family of all distance distribution functions while H will always denote the specific distribution function defiend by

$$H(x) = \begin{cases} 0, & x \leq 0 \\ 1, & x > 0. \end{cases}$$

$$H(x) = \begin{cases} 0, & x \leq 0 \\ 1, & x > 0. \end{cases}$$

If X is a non-empty set , F : X × X D is called a probabilistic distance on X and F(x,y) is usually denoted by $F_{x,y}$.

Definition 2.5. A non-distance distribution function is a function L : R R^+ which is non-increasing, right continous on R and inf {L(t) : t R} = 1 and sup {L(t) : t R} = 0. We will denote by E the family of all non-distance distribution functions while G will always denote the specific distribution function defiend by

$$H(x) = \begin{cases} 0, & x \leq 0 \\ 1, & x > 0. \end{cases}$$

$$G(t) = \begin{cases} 1, & t \leq 0 \\ 0, & t > 0. \end{cases} \quad G(t) = \begin{cases} 1, & t \leq 0 \\ 0, & t > 0. \end{cases}$$

If X is a non-empty set , L : X × X E is called a probabilistic non-distance on X and L(x,y) is usually denoted by $L_{x,y}$.

Definition 2.6. [4] A 5-tuple (X, F, L, ,) is sais to be an intuitionistic Menger space if X is an arbitrary set, is a continuous t-norm, is continuous t-conorm, F is a probabilistic distance and L is a probabilistic non-distance

on X satisfying the following conditions: for all x, y,z X and t, s 0
(1) $F_{x,y}(t) + L_{x,y}(t)$ 1 ,
(2) $F_{x,y}(0) = 0$,
(3) $F_{x,y}(t) = H(t)$ if and only if x = y,
(4) $F_{x,y}(t) = F_{y,x}(t)$,
(5) if $F_{x,y}(t) = 1$ and $F_{y,z}(s) = 1$, then $F_{x,z}(t+s) = 1$,
(6) $F_{x,z}(t+s)$ $F_{x,y}(t)$ $F_{y,z}(s)$,
(7) $L_{x,y}(0) = 1$,
(8) $L_{x,y}(t) = G(t)$ if and only if x = y,
(9) $L_{x,y}(t) = L_{y,x}(t)$,
(10) if $L_{x,y}(t) = 0$ and $L_{y,z}(s) = 0$, then $L_{x,z}(t+s) = 0$,
(11) $L_{x,z}(t+s)$ $L_{x,y}(t)$ $L_{y,z}(s)$.

The function $F_{x,y}(t)$ and $L_{x,y}(t)$ denote the degree of nearness and degree of non-nearness between x and y with respect to t, respectively.

Remark 2.7. Every Menger space (X, *F*,) is intuitionistic Menger space of the form
(X, *F*,1 – *F*, ,) such that t-norm and t-conorm are associated , that is x y = 1- (1-x) (1-y) for any x , y X.
Example 2.8. Let (X,d) be a matric space. Then the metric d induces a distance distribution function *F* defined by $F_{x,y}$ (t) = *H* (t - d (x,y)) and a non-distace function *L* defined by $L_{x,y}$ (t) = *G* (t – d (x,y)) for all x,yX and t ≥ 0. Then (X,*F*,*L*) is an intuitionistic probabilistic metric space. We call this instutionistic probabilistic metric space induced by a metric d the induced intuitionistic probabilistic metric space. If t-norm is a b = min{a,b} and t-conorm is a b = min {1,a + b} for all a, b [0,1] then (X,*F*,*L*, ,) is an intuitionistic Menger space.

Remark 2.9. Note that the above example holds even with the t-norm ab = min{a,b}and t-conorm a b = max {a,b}and hence (X,*F*,*L*, ,) is an intuitionistic Menger space with respect to any t-norm and t-conorm. Also note t-norm and t-conorm are not associated.

Definition 2.10. [4] Let (X,*F*,*L*, ,) be an intuitionistic Menger space with t t ≥ t and (1 – t) (1 - t) ≤ (1 – t). Then:

- A sequence $\{x_n\}$ in X is said to be convergent to x in X if, for every > 0 and (0,1), there exists positive integer N such that $F_{xn,x}$ () > 1 - and $L_{xn,x}$ () < whenever n ≥ N.
- A sequence $\{x_n\}$in X is called Cauchy sequence if, for every > 0 and (0,1), there exists positive interger N such that $F_{xn,xm}$ () > 1- and $L_{xn,xm}$ () < whenever n,m ≥ N.
- An intuitionistic Menger space (X,*F*,*L*, ,) is said to be complete if and only if every Cauchy sequence in X is convergent to a point in X.

The proof of the following lemmas is on the lines of Mishra [6].

Lemma 2.11. Let (X,*F*,*L*, ,) be an intuitionistic Menger space with tt ≥ t and (1 - t) (1 - t) ≤ (1 - t) and $\{y_n\}$ be a sequence in X. If there exists a number k (0, 1) such that:

- $F_{yn+2,yn+1}$ (kt) ≥ $F_{yn+1,yn}$ (t),
- $L_{yn+2,yn+1}$ (kt) ≤ $L_{yn+1,yn}$ (t) for all t > 0 and n =1,2,3,4,… Then$\{y_n\}$ is a Cauchy sequence inX.

Proof. By simple induction with the condition (1), we have for all t > 0 and n = 1,2,3,…,

$F_{yn+1,yn+2}(t) \geq F_{y1,y2}(t/k^n)$, $L_{yn+1,yn+2}(t) \leq L_{y1,y2}(t/k^n)$.

Thus by Definition 2.6 (6) and (11), for any positive integer m ≥ n and number t > 0, we have

$$F_{yn,ym}(t) \geq F_{yn,yn+1}\left(\frac{t}{m-n}\right)\left(\frac{t}{m-n}\right) F_{yn+1,yn+2}\left(\frac{t}{m-n}\right)\left(\frac{t}{m-n}\right) \ldots F_{ym-1,ym}\left(\frac{t}{m-n}\right)\left(\frac{t}{m-n}\right)$$

$$\geq \overbrace{(1-\lambda)*(1-\lambda)*\cdots\cdots*(1-\lambda)}^{m-n} > (1-\lambda),$$

and

$$L_{yn,ym}(t) \leq L_{yn,yn+1}\left(\frac{t}{m-n}\right)\left(\frac{t}{m-n}\right) L_{yn+1,yn+2}\left(\frac{t}{m-n}\right)\left(\frac{t}{m-n}\right) \ldots L_{ym-1,ym}\left(\frac{t}{m-n}\right)\left(\frac{t}{m-n}\right)$$

$$\leq \overbrace{\lambda \diamond \lambda \diamond \cdots\cdots \diamond \lambda}^{m-n} < \lambda,$$

which implies that $\{y_n\}$ is a Cauchy sequence in X. This completes the proof.

Lemma 2.12. Let (X,*F*,*L*, ,) be an intuitionistic Menger space with t t ≥ t abd (1 –t) (1 –t) ≤ (1 – t) and for all x,y X , t > 0 and if for a number k (0,1)

$F_{x,y}$ (kt) ≥ $F_{x,y}(t)$ and $L_{x,y}$ (kt) ≤ $L_{x,y}$ (t) (I)

then x = y.

Proof. Since t > 0 abd k (0 , 1) we get t > kt. In intuitionistic Menger space (X,*F*,*L*, ,) , $F_{x,y}$ is non decreasing and $L_{x,y}$ is non-increasing for all x,y X, then we have

$F_{x,y}$ (t) ≥ $F_{x,y}$(kt) and $L_{x,y}(t)$ ≥ $L_{x,y}$(kt).

Using (I) and the definition of intuitionistic Menger space, we have x = y.

Definition 2.13. The self-maps A and B of an intuitionistic Menger space (X,*F*,*L*, ,) are said to be compatible if for all t > 0,

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lim┬n(($\lim_{n\to\infty} \lim_{n\to\infty} F_{ABxn,BAxn}(t) = 1$

and $\lim_{n\to\infty} \lim_{n\to\infty} L_{ABxn,BAxn}(t) = 0$,

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} \lim_{n\to\infty} Ax_n$ $= \lim_{n\to\infty} \lim_{n\to\infty} Bx_n = z$ for some z X.

Definition 2.14. Two self-maps A and B of an intuitionistic Menger space (X,F,L, ,) are said to be weakly compatible if they commute at their coincidence points ,that is if Ax = Bx for some x X then ABx = BAx.

Remark 2.15. If self-maps A and B of an intuitionistic Menger space (X,F,L, ,) are compatible then they are weakly compatible.

Definition 2.16. [4] Two self mappings A and B of an intuitionistic Menger space (X,F,L, ,) are said to be

(i) Compatible of type (P) if

$F_{ABxn,BBxn}(t) \to 1$ and $F_{BAxn,AAxn}(t) \to 1$ for all t > 0

where $\{x_n\}$is a sequence in X such that $Ax_n, Bx_n \to z$ for some z in X as $n \to\infty$.

(ii) Compatible of type (P_1) if

$F_{ABxn,BBxn}(t) \to 1$ for all t > 0.

where $\{x_n\}$ is a sequence in X such that $Ax_n, Bx_n \to z$ for some z in X as $n \to\infty$.

(iii) Compatible for type (P_2) if

$F_{BAxn,AAxn}(t) \to 1$ for all t > 0

where $\{x_n\}$ is a sequence in X such that $Ax_n, Bx_n \to z$ for some z in X as $n \to\infty$.

III. MAIN RESULTS

Theorem 3.1. Let (X, F, L, ,) be a complete intuitionistic Menger space with tt t and (1-t) (1-t) (1-t) and let A, B, S, T, P and Q be selfmappings of X such that the following conditions are satisfied :

- A(X) ST(X) , B(X) PQ(X) ,
- There exists k (0,1) such that for every x,y X and t > 0,

  $F_{Ax,By}(kt)$ { $F_{PQx,STy}(t)$ $F_{Ax,PQx}(t)$ $F_{By,STy}(t)$ $F_{Ax,STy}(t)$}

and $L_{Ax,By}(kt)$ { $L_{PQx,STy}(t)$ $L_{Ax,PQx}(t)$ $L_{By,STy}(t)$ $L_{Ax,STy}(t)$} ,

- Either A or PQ is continuous,
- The pair {A,PQ} and {B,ST} are both compatible of type (P_1) or type (P_2),
- PQ = QP, ST = TS, AQ = QA, BT = TB.

Then A, B, S,T, P and Q have a unique common fixed point in X.

Proof. By (1) since A(X) ST(X) for any point x_0X, there exists a point x_1 in X such that $Ax_0 = STx_1$. Since B(X) PQ(X) , for this point x_1 we can choose a point x_2 in X such that $Bx_1 = PQx_2$ and so on. Inductively, we can define a sequence $\{y_n\}$ in X such that for n = 0,1,2,3,…

$y_{2n} = Ax_{2n} = STx_{2n+1}$ and $y_{2n+1} = Bx_{2n+1} = PQx_{2n+2}$.

By (2), for all t > 0, we have

$F_{y2n,\, y2n+1}(kt) = F_{Ax2n,\, Bx2n+1}(kt)$

{ $F_{PQx2n,ST\,x2n+1}(t)$ $F_{Ax2n,\, PQx2n}(t)$ $F_{B\,x2n+1,ST\,x2n+1}(t)$ $F_{Ax2n,ST\,x2n+1}(t)$}

= { $F_{y2n-1,y2n}(t)$ $F_{y2n,\, y2n-1}(t)$ $F_{y2n+1,y2n}(t)$ $F_{y2n,y2n}(t)$}

$F_{y2n-1,y2n}(t)$ $F_{y2n+1,y2n}(t)$,

and

$L_{y2n,\, y2n+1}(kt) = L_{Ax2n,\, Bx2n+1}(kt)$

{ $L_{PQx2n,ST\,x2n+1}(t)$ $L_{Ax2n,\, PQx2n}(t)$ $L_{B\,x2n+1,ST\,x2n+1}(t)$ $L_{Ax2n,ST\,x2n+1}(t)$}

= { $L_{y2n-1,y2n}(t)$ $L_{y2n,\, y2n-1}(t)$ $L_{y2n+1,y2n}(t)$ $L_{y2n,y2n}(t)$}

$L_{y2n-1,y2n}(t)$ $L_{y2n+1,y2n}(t)$.

Similarly, we also have

$F_{y2n+1,\, y2n+2}(kt)$ $F_{y2n,y2n+1}(t)$ $F_{y2n+2,y2n+1}(t)$,

and

$L_{y2n+1,\, y2n+2}(kt)$ $L_{y2n,y2n+1}(t)$ $L_{y2n+2,y2n+1}(t)$.

Thus it follows that for m = 1,2,3,…

$F_{ym+1,\, ym+2}(kt)$ $F_{ym,ym+1}(t)$ $F_{ym+1,ym+2}(t)$,

and

$L_{ym+1,\, ym+2}(kt)$ $L_{ym,ym+1}(t)$ $L_{ym+1,ym+2}(t)$.

Consequently, it follows that for m = 1,2,3,… , p = 1,2,3,…

$F_{ym+1,\, ym+2}(kt)$ $F_{ym,ym+1}(t)$ $F_{ym+1,ym+2}(t / k^p)$,

and

$L_{ym+1,\, ym+2}(kt)$ $L_{ym,ym+1}(t)$ $L_{ym+1,ym+2}(t / k^p)$.

By noting that $F_{y_{m+1},y_{m+2}}$ (t / k^p) 1 and $L_{y_{m+1},y_{m+2}}$ (t / k^p) 0, as p, we have for m = 1,2,3,…

$F_{y_{m+1},y_{m+2}}$ (kt) $F_{y_m,y_{m+1}}$ (t)

and

$L_{y_{m+1},y_{m+2}}$ (kt) $L_{y_m,y_{m+1}}$ (t).

Hence by Lemma 2.11, {y_n} is a Cauchy sequence in X. Since X is complete, the sequence {y_n} converges to a point z in X. Also its subsequences

{ Ax_{2n}}z, { PQx_{2n}}z, { Bx_{2n+1}}z and { STx_{2n+1}}z

Case (i): PQ is continuous, the pair {A,PQ} and {B,ST} are both compatible of type (P_2),

$PQPQx_{2n}PQz$, $PQAx_{2n}PQz$ (since PQ is continuous)

$AAx_{2n}PQz$ (since {A,PQ} is compatible of type (P_2))

By taking x = Ax_{2n} , y = x_{2n+1} in (2), we get

$F_{AAx_{2n},B x_{2n+1}}$ (kt) {$F_{PQAx_{2n},ST x_{2n+1}}$ (t) $F_{AAx_{2n},PQ\ Ax_{2n}}$ (t) $F_{B x_{2n+1},ST x_{2n+1}}$ (t) $F_{AAx_{2n},ST x_{2n+1}}$ (t)}

$F_{PQz,z}$ (kt) {$F_{PQz,z}$ (t) $F_{PQz,PQz}$ (t) $F_{z,z}$ (t) $F_{PQz,z}$ (t)}

$F_{PQz,z}$ (kt) $F_{PQz,z}$ (t)

and

$L_{AAx_{2n},B x_{2n+1}}$ (kt) {$L_{PQAx_{2n},ST x_{2n+1}}$ (t) $L_{AAx_{2n},PQ\ Ax_{2n}}$ (t) $L_{B x_{2n+1},ST x_{2n+1}}$ (t) $L_{AAx_{2n},ST x_{2n+1}}$ (t)}

$L_{PQz,z}$ (kt) {$L_{PQz,z}$ (t) $L_{PQz,PQz}$ (t) $L_{z,z}$ (t) $L_{PQz,z}$ (t)}

$L_{PQz,z}$ (kt) $L_{PQz,z}$ (t)

Therefore by lemma 2.12, we have PQz = z. Similarly by taking x = z , y = x_{2n+1} in (2), we get

$F_{Az,B x_{2n+1}}$ (kt) {$F_{PQz,ST x_{2n+1}}$ (t) $F_{Az,PQz}$(t) $F_{B x_{2n+1},ST x_{2n+1}}$ (t) $F_{Az,ST x_{2n+1}}$ (t)}

$F_{Az,z}$ (kt) {$F_{z,z}$ (t) $F_{Az,z}$ (t) $F_{z,z}$ (t) $F_{Az,z}$ (t)}

$F_{Az,z}$ (kt) $F_{Az,z}$ (t)

and

$L_{Az,B x_{2n+1}}$ (kt) {$L_{PQz,ST x_{2n+1}}$ (t) $L_{Az,PQz}$(t) $L_{B x_{2n+1},ST x_{2n+1}}$ (t) $L_{Az,ST x_{2n+1}}$ (t)}

$L_{Az,z}$ (kt) {$L_{z,z}$ (t) $L_{Az,z}$ (t) $L_{z,z}$ (t) $L_{Az,z}$ (t)}

$L_{Az,z}$ (kt) $L_{Az,z}$ (t)

Therefore by lemma 2.12, we have Az = z.

Since A(X) ST(X), there exists wX such that z = Az = STw

By taking x = x_{2n} , y = w in (2) , we get

$F_{A x_{2n},Bw}$(kt) { $F_{PQ\ x_{2n},STw}$(t) $F_{A x_{2n},PQ\ x_{2n}}$ (t) $F_{Bw,STw}$(t) $F_{A x_{2n},STw}$(t)}

$F_{z,Bw}$(kt) { $F_{z,z}$(t) $F_{z,z}$ (t) $F_{Bw,z}$(t) $F_{z,z}$(t)}

$F_{z,Bw}$(kt) $F_{Bw,z}$(t)

and

$L_{A x_{2n},Bw}$(kt) {$L_{PQ\ x_{2n},STw}$(t) $L_{A x_{2n},PQ\ x_{2n}}$ (t) $L_{Bw,STw}$(t) $L_{A x_{2n},STw}$(t)}

$L_{z,Bw}$(kt) { $L_{z,z}$(t) $L_{z,z}$ (t) $L_{Bw,z}$(t) $L_{z,z}$(t)}

$L_{z,Bw}$(kt) $L_{Bw,z}$(t)

Therefore by lemma 2.12, we have Bw = z. Hence STw = Bw = z.

Since (B,ST) is compatible of type (P_2), we have STBw = BBw , Therefore STz = Bz.

Now by taking x = x_{2n} , y = z in (2) , we get

$F_{A x_{2n},Bz}$(kt) { $F_{PQ\ x_{2n},STz}$(t) $F_{A x_{2n},PQ\ x_{2n}}$ (t) $F_{Bz,STz}$(t) $F_{A x_{2n},STz}$(t)}

$F_{z,Bz}$(kt) { $F_{z,z}$(t) $F_{z,z}$ (t) $F_{Bz,z}$(t) $F_{z,z}$(t)}

$F_{z,Bz}$(kt) $F_{Bz,z}$(t)

and

$L_{A x_{2n},Bz}$(kt) {$L_{PQx_{2n},STz}$(t) $L_{A x_{2n},PQ\ x_{2n}}$ (t) $L_{Bz,STz}$(t) $L_{A x_{2n},STz}$(t)}

$L_{z,Bz}$(kt) { $L_{z,z}$(t) $L_{z,z}$ (t) $L_{Bz,z}$(t) $L_{z,z}$(t)}

$L_{z,Bz}$(kt) $L_{Bz,z}$(t) .

Therefore by lemma 2.12, we have Bz = z.

Az = Bz = PQz = STz = z.

i.e. z is a common fixed point for A, B, PQ and ST.

Case (ii): A is continuous, the pair {A,PQ} and {B,ST} are both compatible of type (P_2),

$AAx_{2n}Az$, $APQx_{2n}Az$ (since A is continuous)

$PQAx_{2n}Az$ (since {A,PQ} is compatible of type (P_2))

By taking x = Ax_{2n} , y = x_{2n+1} in (2) and letting n, we get $F_{Az,z}$ (kt) $F_{Az,z}$ (t) and $L_{Az,z}$ (kt) $L_{Az,z}$ (t) Therefore by lemma 2.12, we have Az = z. Since A(X) ST(X), there exists wX such that z = Az = STw . By taking x = x_{2n} , y = w in (2) , we get STw = Bw = z. Since (B,ST) is compatible of type (P_2), we have STBw = BBw , therefore STz = Bz. Now by taking x = x_{2n} , y = z in (2) , we get z =

Bz = STz. Since B(X) PQ(X), there exists uX such that z = Bz = PQu. By taking x = u, y = x_{2n+1} in (2) and letting n, we get $F_{Au,z}$ (kt) $F_{Au,z}$ (t) and $L_{Au,z}$ (kt) $L_{Au,z}$ (t) Therefore by lemma 2.12, we have Au = z. Since z = Bz = PQu, hence Au = PQu. Since (A,PQ) is compatible of type (P_2),we have PQAu = AAu PQz = Az.

Az = Bz = PQz = STz = z.

i.e. z is a common fixed point for A, B, PQ and ST.

Now PQz = z

Q(PQz) = Qz QPQz = Qz PQQz = Qz i.e. Qz is a fixed point for PQ.

Since STz = z TSTz = Tz STTz = Tz i.e. Tz is a fixed point for ST.

Similarly, STz = z SSTz = Sz STSz = Sz

Sz is a fixed point for ST. Hence Sz and Tz are fixed point for ST.

Now Az = z QAz = Qz AQz = Qz i.e. Qz is a fixed point for A.

Since Bz = z TBz = Tz BTz = Tz. i.e. Tz is a fixed point for B.

Now we prove that Tz = Qz. By taking x = Qz , y = Tz in (2) , we get

$F_{AQz,BTz}$(kt) { $F_{PQQz,STTz}$(t) $F_{AQz,PQQz}$(t) $F_{BTz,STTz}$(t) $F_{AQz,STTz}$(t)}

$F_{Qz,Tz}$(kt) { $F_{Qz,Tz}$(t) $F_{Qz,Qz}$(t) $F_{Tz,Tz}$(t) $F_{Qz,Tz}$(t)}

$F_{Qz,Tz}$(kt) $F_{Qz,Tz}$(t)

and

$L_{AQz,BTz}$(kt) { $L_{PQQz,STTz}$(t) $L_{AQz,PQQz}$(t) $L_{BTz,STTz}$(t) $L_{AQz,STTz}$(t)}

$L_{Qz,Tz}$(kt) { $L_{Qz,Tz}$(t) $L_{Qz,Qz}$(t) $L_{Tz,Tz}$(t) $L_{Qz,Tz}$(t)}

$L_{Qz,Tz}$(kt) $L_{Qz,Tz}$(t)

Therefore by lemma 2.12, we have Qz = Tz. Qz is a common fixed point for A, B, PQ and ST.

By taking x = Qz and y = z in (2) , we get

$F_{AQz,Bz}$(kt) { $F_{PQQz,STz}$(t) $F_{AQz,PQQz}$(t) $F_{Bz,STz}$(t) $F_{AQz,STz}$(t)}

$F_{Qz,z}$(kt) { $F_{Qz,z}$(t) $F_{Qz,Qz}$(t) $F_{z,z}$(t) $F_{Qz,z}$(t)} (since z = Bz = STz)

$F_{Qz,z}$(kt) $F_{Qz,z}$(t)

and

$L_{AQz,Bz}$(kt) { $L_{PQQz,STz}$(t) $L_{AQz,PQQz}$(t) $L_{Bz,STz}$(t) $L_{AQz,STz}$(t)}

$L_{Qz,z}$(kt) { $L_{Qz,z}$(t) $L_{Qz,Qz}$(t) $L_{z,z}$(t) $L_{Qz,z}$(t)}

$L_{Qz,z}$(kt) $L_{Qz,z}$(t)

Therefore by lemma 2.12, we have Qz = z. Therefore z = Qz = Tz is a common fixed point for A, B, PQ and ST.Since STz = z Sz = z and PQz = z Pz = z

z is a common fixed point for A, B, S, T, P and Q.

For uniqueness, let v be a common fixed point for A,B,S,T,P andQ.By taking x=z, y=v in (2),we get

$F_{Az,Bv}$(kt) { $F_{PQz,STv}$(t) $F_{Az,PQz}$(t) $F_{Bv,STv}$(t) $F_{Az,STv}$(t)}

$F_{z,v}$(kt) { $F_{z,v}$(t) $F_{z,z}$(t) $F_{v,v}$(t) $F_{z,v}$(t)}

$F_{z,v}$(kt) $F_{z,v}$(t)

and

$L_{Az,Bv}$(kt) { $L_{PQz,STv}$(t) $L_{Az,PQz}$(t) $L_{Bv,STv}$(t) $L_{Az,STv}$(t)}

$L_{z,v}$(kt) { $L_{z,v}$(t) $L_{z,z}$(t) $L_{v,v}$(t) $L_{z,v}$(t)}

$L_{z,v}$(kt) $L_{z,v}$(t)

Therefore by lemma 2.12, we have z = v.

z is a unique common fixed point for A, B, S, T, P and Q.

If we put A = B in theorem 3.1, we have the following result:

Corollary 3.2. Let (X, F, L, ,) be a complete intuitionistic Menger space with tt t and (1-t) (1-t) (1-t) and let A, S, T, P and Q be selfmappings of X such that the following conditions are satisfied :

- A(X) ST(X) , A(X) PQ(X) ,
- There exists k (0,1) such that for every x,y X and t > 0,

  $F_{Ax,Ay}$(kt) { $F_{PQx,STy}$(t) $F_{Ax,PQx}$(t) $F_{Ay,STy}$(t) $F_{Ax,STy}$(t)}

and $L_{Ax,Ay}$(kt) { $L_{PQx,STy}$(t) $L_{Ax,PQx}$(t) $L_{Ay,STy}$(t) $L_{Ax,STy}$(t)} ,

- Either A or PQ is continuous,
- The pair {A,PQ} and {A,ST} are both compatible of type (P_1) or type (P_2),
- PQ = QP, ST = TS, AQ = QA, AT = TA.

Then A, S,T, P and Q have a unique common fixed point in X.

If we put T = Q = Ix (The identity map on X) in theorem 3.1, we have the following:

Corollary 3.3. Let (X, F, L, ,) be a complete intuitionistic Menger space with tt t and (1-t) (1-t) (1-t) and let A, B, S and P be selfmappings of X such that the following conditions are satisfied :

- A(X) S(X) , B(X) P(X) ,
- There exists k (0,1) such that for every x,y X and t > 0,

$F_{Ax,By}$(kt) { $F_{Px,Sy}$(t) $F_{Ax,Px}$(t) $F_{By,Sy}$(t) $F_{Ax,Sy}$(t)}

and $L_{Ax,By}$(kt) { $L_{Px,Sy}$(t) $L_{Ax,Px}$(t) $L_{By,Sy}$(t) $L_{Ax,Sy}$(t)} ,

- Either A or P is continuous,
- The pair {A,P} and {B,S} are both compatible of type (P_1) or type (P_2),

Then A, B, S and P have a unique common fixed point in X.

If we put S = T = P = Q = Ix (the identity map on X) in corollary 3.2, we have the following:

Corollary 3.4. Let (X, F, L, ,) be a complete intuitionistic Menger space with tt t and (1-t) (1-t) (1-t) and let A be a continuous mapping from X into itself .There exists k (0,1) such that for every x,y X and t > 0,

$F_{Ax,Ay}$(kt) { $F_{x,y}$(t) $F_{Ax,x}$(t) $F_{Ay,y}$(t) $F_{Ax,y}$(t)}

and $L_{Ax,Ay}$(kt) { $L_{x,y}$(t) $L_{Ax,x}$(t) $L_{Ay,y}$(t) $L_{Ax,y}$(t)} , then A has a unique fixed point in X.

Now , we give an example to illustrate Corollary 3.3

Example 3.5. Let X = [0,1] with the metric *d* defined by *d*(x,y) = x - y and for each t [0, 1] define

$$F_{x,y}(t) = \begin{cases} \frac{t}{t+|x-y|} & , \text{ if } t > 0 \\ 0 & , \text{ if } t = 0 \end{cases}$$

$$\begin{cases} \frac{t}{t+|x-y|} & , \text{ if } t > 0 \\ 0 & , \text{ if } t = 0 \end{cases}$$

$$\begin{cases} \frac{t}{t+|x-y|} & , \text{ if } t > 0 \\ 0 & , \text{ if } t = 0 \end{cases} \quad \text{and} \quad L_{x,y}(t)$$

$$\begin{cases} \frac{t}{t+|x-y|} & , \text{ if } t > 0 \\ 0 & , \text{ if } t = 0 \end{cases} =$$

$$\begin{cases} \frac{|x-y|}{t+|x-y|} & , \text{ if } t > 0 \\ 1 & , \text{ if } t = 0 \end{cases}$$

$$\begin{cases} \frac{|x-y|}{t+|x-y|} & , \text{ if } t > 0 \\ 1 & , \text{ if } t = 0 \end{cases}$$

for all x,y X. Clearly (X, *F*, *L*, ,) is a complete intuitionistic Menger space where is defined by tt t and is defined by (1-t) (1-t) (1-t) . Define A, P, B and S : X X by

Ax = $\frac{xx}{44}$, Sx = $\frac{xx}{22}$, Bx = $\frac{xx}{88}$, Px = x respectively. Then A, P, B and S satisfy all the conditions of Corollary 3.3 with k[$\frac{11}{22}$,1) and have a unique common fixed point 0 X.

REFERENCES

[1] A. George and P. Veeramani, On some results in Fuzzy metric spaces, Fuzzy sets and systems, 64 (1994) , 395-399.

[2] G. Jungck, Compatible mappings and common fixed points, Internat. J. Math. Sci. (1986) 771-779.

[3] O. Kramosil and J. Michalek, Fuzzy metric and statistical spaces,Kybernetica,11(1975),326-334.

[4] S. Kutukcu, A. Tuna,and A. T.Yakut, Generalized contraction mapping principal in Intuitionistic menger spaces and application to diiferential equations, Appl. Math. And Mech., 28 (2007)799-809.

[5] K. Menger, Statistical metric, Proc. Nat, Acad. Sci. U. S. A, 28 (1942) , 535-537.

[6] S. N. Mishra, Common fixed points of compatible mappings in PM-spaces, Math. Japon.36 (1991) 283-289.

[7] J. H. Park, Intuitionistic fuzzy metric spaces, Chaos, Solitions and Fractals,22 (2004)1039-1046

[8] B. Schweizer and A. Sklar, Statistical metric spaces, Pacific J. Math. , 10 (1960), 313-334.

[9] B. Schweizer and A. Sklar, Probabilistic metric spaces, Elsevier,North-Holland, NewYork,1983.

[10] S. Sessa, On a weak commutative condition in fixed point consideration , Publ. Inst. Math. (Beograd) 32 (1982) 146-153.