Optimum Design of Linear Phase Paraunitary Filter Bank & its Applications in Signal Processing

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Abstract-Filter Banks plays crucial role in signal processing and image processing as subband processing gives dominant results in time critical applications. In formal years, various Para unitary Linear Phase Filter Banks are proposed by following conventional and computational complex factorization and lattice approaches consisting of complex nonlinear optimization problems. One of the recent methods to design Filter Bank having properties of Linear Phase and Paraunitary is via Singular value decomposition technique which leads to optimum results compared to existing methods as most of the time it deals with matrix operations. In this paper, design benchmark is evaluated as two dominant optimization queries and reasonable key of each optimization query is solved by performing Singular Value Decomposition. Proposed Paper discusses linear phase condition of filter banks satisfying mirror image symmetry at analysis side and perfect reconstruction property at synthesis side. Singular Value Decomposition approach leads to fast and efficient simulation results compared to existing filter banks designs. Proposed method of filter bank design deals with any arbitrary channels and every length of the filters.

Keywords—*Filter Bank, Linear Phase, Paraunitary, SVD, Least Square Filters, FIR, DC Leakages, PR Errors, Tree Structure Filter Bank, Compression*

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I. INTRODUCTION

Filter Bank is the systematic arrangement of filters consisting of low pass filters, band pass filters and high pass filters connected one after another for the spectral decomposition of the signal. These FBs most of the time deals with concept of sampling rates hence they generally expressed as multirate systems. These filter banks are widely used in speech coding, image processing, communication, RF antennas, and analog voice privacy system and digital audio industries.

Due to the property of Paraunitary it is easy to obtain synthesis filters once we got the analysis filters.FB exhibits Linear Phase condition that is there is no any phase distortions in sub band components hence reliable to extract Perfect Reconstruction property at receiver side. Due to the maximally decimated property there is reduction in total number of samples in sub band components hence used in image processing applications like compression.Maximally decimated condition also avoid the aliasing occurred in Filter Banks.FIR filter bank, due to the finite number of filter coefficients in FB, by default stability is guaranteed in our proposed FB design method.

FBs decompose the input signal and allot different frequency bands to each sub band components satisfying appropriate frequency characteristics. Processing on different subband components of narrow bandwidths is easy. Different combinations of Decimators and Interpolators are used followed by digital filters like decimation filters and interpolation filters. Decimation filters ensures that signal being decimated by decimator is bandlimited satisfying proper aliasing conditions whereas interpolation filters takes care of only highlighted portion of the signal and rejects the mirror images created by interpolators.

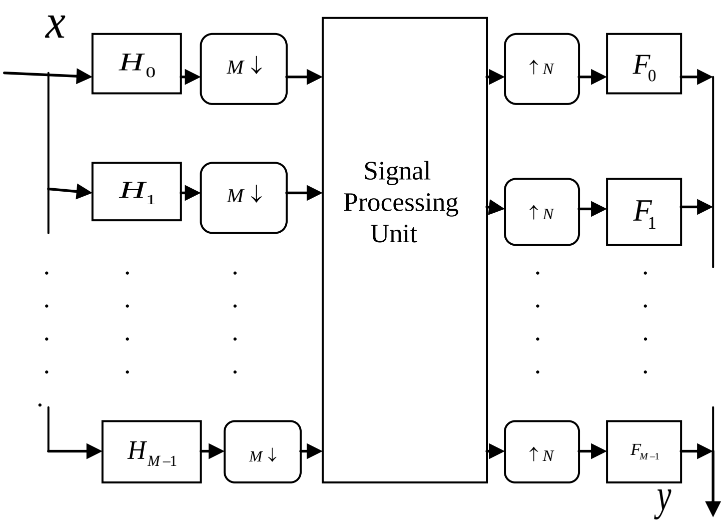

Fig.1.General Design of Filter Bank

II. LINEAR PHASE PARAUNITARY FILTER BANK

The author [1] proposes design of 2-channel QMF banks exhibiting sufficient PR condition via lattice approach. These lattice structures fulfil PR property along with good stop band attenuations. Proposed methodology is said to be robust because there is adaptive selection of filter coefficients. Initially lower order prototype filter is derived and using lattice structures all other higher order filters are obtained simply by adding lattice sections one after another. But for generalize M-channel cases there is still lots of difficulties while adding lattice sections as there is no any specific spectral factorization approach to design 2-channel FB design.

To address this difficulty author [2] proposes method of factorization and parameterization of lattice

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structures to design filter bank.FBs are represented as cascade of the lattices. Every lattice consists of the cascade combination of unitary matrix and delay element. Each unitary matrix is formulated by set of the arbitrary gyratory angles. Lengths of the filters are decided by number of lattices. As author supposed to take the product of all these unitary matrices one after another, leads to highly non-linear trigonometric functions of rotational angles. Hence optimization is not traceable as taking gradients of such non-linear functions is complex. Only line search algorithm is employed to obtain minimal optimization results which results in increased computational power and hence computational complexity.

The author [3] starts with given filter length and investigate factorization algorithm to obtain causal FIR PU filter bank. For general PU system fundamental First order factorization form is obtained by performing SVD on polyphase matrix coefficients of filter banks. Then author [3] develops new structure of PU FB by the decomposition of hermitian unitary matrices. Number of filter coefficients to be derived is half compared with existing method of FB design via factorization approach hence it reduces computational complexity and yields more efficient results.

.In paper [5] author proposes design of M-channel PULPFB with narrow transition width and maximum stopband attenuation such that symmetric polarities of linear phase filters should satisfy particular pattern. Author [5] defines the limiting boundary of filters in FB as design method exhibits linear phase property. To achieve linear phase property for M-channel PUFB, author [5] deals with optimization which minimizes a quadratic point function subject to these quadratic matrix equality constraints. However, as each quadratic matrix equality constraint corresponds to a condition defined in a high dimensional ball and there is more than one high dimensional ball for every filter in FB, the optimization problem is still not achievable, to find the solution for such a non-traceable optimization problem highly complex computer aided design tool is required which further increases computational complexity at great extent.

Author [6] proposes improved method of design of maximally decimated FIR PR QMF FB with polyphase decomposition matrices of FBs .In previous work parameters characterizing polyphase component matrix E (z) are arbitrary rotational angles which results in delayed convergence of the above mentioned optimization problem, hence to counter attack that problem, author [6] initialize these parameters based on spectral factorization approach. This factorization is done by eigenfilters approach without root-finding techniques in an efficient manner starting with initialization of one of the M analysis filters, as a spectral factor of an Mth band filter. These modified characterization of lossless FIR E (z) is free from the rotational angles yield more efficient results.

To counter attack on this difficulty, author [7] proposes new factorization method to design PUFBs via matrix decomposition approach in terms of projection matrices.PUFBs decomposes into a product of elementary building blocks with each building block represents arbitrary set of angles that can be adaptively varied to build particular FB satisfying specific criteria and particular application. Less number of free parameters are required to design PUFB via this matrix decomposition approach compared to the conventional lattice factorization which is used in existing FB design techniques. Design of longer channel filters with less numerical computation and more economical PUFB representation is possible using this method of PUFB matrix decomposition without addition computational complexity.

To remedy on all these problems discussed above, proposed method generates optimization solution and analytical solution is obtained by using SVD approach performing matrix operations. Algorithm of proposed method is iterative as number of random iterations are performed until the best results are obtained in terms of desired filters in FB.The proposed design method is remedy of FB design in time critical applications. Proposed method starts with design of Linear Phase optimum least square filter and ends with actual desired filter which is close enough to the least square filter. As most of the time it has to perform matrix operations by eliminating the use of computer aided design tool, proposed method is free from computational complexity and best suits in time critical applications of FBs.

III. PROPOSED METHODOLOGY

STEP I.

Design of optimum linear phase least square filter

a) To extract filter coefficients by minimizing square of the error between the responses of desired filter and filter to be designed by solving linear equations as follows

$$\overline{h}_m^* = -\overline{Q_m}^{-1}\left(\overline{q}_m - \frac{v_m + i^T - \overline{Q_m}^{-1}\overline{q}_m}{i^T Q_m^{-1} i}\, i\right) \qquad (1).$$

b) Obtain least square objective functions that is intermediate matrices as follows

$$\overline{Q_m} = 2\int_{\omega\in\beta_{p,m}\cup\beta_{s,m}} \psi_m(\omega)\psi_m{}^T(\omega)\,d\omega \qquad (2)$$

$$\overline{q_m} = -2\int_{\omega\in\beta_{p,m}\cup\beta_{s,m}} D_m(\omega)\psi_m(\omega)\,d\omega \qquad (3)$$

Where

$$\psi_m(\omega) = 2\left[\cos\left(\frac{N-1}{2}\right)\omega\ldots\ldots\cos\left(\frac{\omega}{2}\right)\right]^T \qquad (4)$$

$$\text{For } \omega\in\beta_{p,m} \quad D_m(\omega) = \pm\sqrt{M} \qquad (5)$$

$$\text{For } \omega\in\beta_{s,m} \quad D_m(\omega) = 0 \qquad (6)$$

c) Design of cutoff bands in passbands and stopbands.

For m=0

$$\beta_{p,0} = \left[\Delta - \frac{\pi}{M}, \frac{\pi}{M} - \Delta\right] \qquad (7)$$

$$\beta_{s,0} = \left[\frac{\pi}{M} + \Delta, \pi\right] \cup \left[-\pi, -\frac{\pi}{M} - \Delta\right] \qquad (8)$$

For m=1,2,…M-2

$$\beta_{p,m} = \left[\frac{k\pi}{M} + \Delta, \frac{(k+1)\pi}{M} - \Delta\right] \cup \left[-\frac{(k+1)\pi}{M} + \Delta, -\frac{k\pi}{M} - \Delta\right] \quad (9)$$

$$\beta_{s,m}=\left[\frac{(k+1)\pi}{M}+\Delta,\pi\right]\cup\left[-\pi,-\frac{(k+1)\pi}{M}-\Delta\right]\cup\left[\frac{-k\pi}{M}+\Delta,\frac{k\pi}{M}-\Delta\right] \quad (10)$$

For m=M-1

$$\beta_{p,M-1}=\left[\frac{(M-1)\pi}{M}+\Delta,\pi\right]\cup\left[-\pi,-\frac{(M-1)\pi}{M}-\Delta\right] \quad (11)$$

$$\beta_{s,M-1}=\left[-\frac{(M-1)\pi}{M}+\Delta,\frac{(M-1)\pi}{M}-\Delta\right] \quad (12)$$

d) Design of DC leakage parameter

For m=0 $$v_m=\pm\sqrt{\frac{M}{2}} \quad (13)$$

STEP II.

Obtain H by assuming arbitrary rotational angles following GIVENs rotational matrix and perform SVD on H..Extract sub matrix of U_H for further computations that is $U_{H,1}$ of order $N\times M$

$$H=U_H D_H V_H \quad (14)$$

STEP III.

Initialize arbitrary unitary matrix U_ϕ

Without any prior knowledge of U_Φ initially start computation by considering it as an identity matrix of order $M\times M$

Obtain diagonal matrix of U_ϕ

$$D_\phi=diag\left(U_\phi,........,U_\phi\right) \quad (15)$$

STEP IV.

Introduce a new parameter B and apply SVD on obtained B

$$B=U_{H,1}^T D_\phi^T \overline{H} \quad (16)$$

$$B=U_B D_B V_B \quad (17)$$

STEP V.

Compute adaptive $\hat{V}_H$ and $\overline{\lambda}$ as follows.

$$\hat{V}_H=U_B V_B^T \quad (18)$$

$$\overline{\lambda}=U_B D_B V_B^T \quad (19)$$

STEP VI.

Evaluate Ω and A then compute $\hat{B}$

$$\Omega=U_{H,1}\hat{V}_H \quad (20)$$

$$A=2\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\Omega_{m,n}\Omega_{m,n}^T \quad (21)$$

Let $h_{m,n,i}$ be the singular elements LSF designed in step I

$$\hat{b}_i=2\sum_{m=0}^{M-1}\sum_{n=0}^{\frac{N}{M}-1}h_{m,n,i}\Omega_{m,n}^T \quad (22)$$

$$\hat{B}=\left[\hat{b}_0,\hat{b}_1,.......\hat{b}_{M-1}\right] \quad (23)$$

$$\hat{B}=U_{\hat{B}}D_{\hat{B}}V_{\hat{B}}^T \quad (24)$$

STEP VII.

Compute modified U_ϕ and $\overline{\lambda}^*$

$$U_\phi=V_{\hat{B}}U_{\hat{B}}^T \quad (25)$$

$$\overline{\lambda}^*=\frac{1}{2}V_{\hat{B}}\left(D_{\hat{B}}-U_{\hat{B}}^T*A*U_{\hat{B}}\right)V_{\hat{B}}^T \quad (26)$$

STEP VIII.

Obtain new analysis filter bank matrix H using previously designed U_ϕ and $\hat{V}_H$ such that

$$H=D_\phi U_{H,1}\hat{V}_H \quad (27)$$

Iterate repeatedly until the energy differences between two successive iterations is bounded by some acceptable range.

$$tr((H_{n+1}-H_n)^T(H_{n+1}-H_n))\le cutoff \quad (28)$$

IV. DESIGN EXAMPLE OF PARAUNITARY LINEAR PHASE FILTER BANK

In this section design example of filter bank by selecting N=8 and M=4 is presented. As designed filter bank supports intermediate channels, transition width Δ should be large so in this example we choose Δ=0.11pi.

Table I

H_0	H_1	H_2	H_3
-0.0945	0.1561	-0.1610	0.1023
0.1610	-0.1023	-0.0945	0.1561
0.3453	-0.5705	0.5882	-0.3737
0.5882	-0.3737	-0.3453	0.5705
0.5882	0.3737	-0.3453	-0.5705
0.3453	0.5705	0.5882	0.3737
0.1610	0.1023	-0.0945	-0.1561
-0.0945	-0.1561	-0.1610	-0.1023

According to the Table I, filter bank of size 8×4 satisfies paraunitary condition as it yields to the validation of shift orthogonality property

$$H^T H=\delta(\beta)I_{M\times M} \quad (29)$$

Hence we can conclude that analysis filter bank designed by our proposed methodology is paraunitary and we can easily designed synthesis filter bank by flipping the coefficients of designed analysis filter bank.

Designed Filter bank also satisfies the property of linear phase as each and every filter out of four filters satisfies the condition of linear phase as for every filter in the designed filter bank there is equivalence in first and last coefficients. According to the table, filters H_0 and H_2 satisfies the symmetric linear phase property whereas filters H_1 and H_3 satisfies linear phase condition with asymmetric responses.

V. SIMULATION RESULTS

DC leakages in provided by analysis filters in designed in above example tells the information about the average intensities of an image. The part of the image which occurs less

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frequently is determined by the DC leakages provided by the filters of analysis filter bank. To have proper reconstruction of an image at the synthesis side, DC leakages should be as low as possible. DC leakages can be carried out by the following equation

$$DCL_m = 20\log_{10}\left(\left|i^T \overline{h_m} - v_m\right|\right) \quad (30)$$

Where v_m computed by the equation (13) and $\overline{h_m}$ be the filter coefficients of designed filter bank and filter banks designed in ref [2].Table II concludes that the DC leakages introduced by the filters designed in our filter bank is low compared to the filters in Filter bank designed by author of ref [2].

Table II

	m=0	m=1	m=2	m=3
FB Designed in ref [2]	-53.22dB	-296.3dB	-20.59dB	-299.6dB
FB Designed by proposed methodology	-76.08dB	-325.1dB	-32.03dB	-337.2dB

Filter bank basically consists of the bank of analysis filters and synthesis filters. By default filter bank may causes some errors because of imperfect design. These errors basically occur due to the truncations in filter coefficients, errors due to the excessive imperfect quantization. Most of the errors in filter bank occur due to the use of sampling rate alteration blocks in between as they violate the nyquist criteria. These type of errors termed as aliasing errors and they can be computed by the following equations

$$ATI(\omega) = \frac{1}{M}\sum_{l=0}^{M-1} H_l\left(\omega - \frac{2\pi l}{M}\right) * F_l(\omega) \quad (31)$$

Where $ATI(\omega)$ determines the aliasing term and $H_l(\omega)$ And $G_l(\omega)$ represents the responses of analysis and synthesis filter banks in Fourier domain.Finally aliasing error can be computed by the following equation

$$AEI = 10\log_{10}\left(\max_{\omega\in[-\pi,\pi]}\left|\sum_{l=1}^{M-1} ATI(\omega)\right|\right) \quad (32)$$

Table III

	Aliasing Error
FB Designed in ref [2]	-144.1956dB
FB Designed by proposed methodology	-146.7204dB

According to the Table III aliasing errors introduced by the filter bank designed by proposed methodology is significantly less compared to the ref [2].

VI. SIGNAL DECOMPOSITION USING TREE STRUCTURE FILTER BANK

Multilevel Filter Bank that is tree structured Filter bank designed by iterating mother Filter Bank of such as two-channel QMF or four-channel QMF banks. However as corresponding mother Filter bank satisfies perfect reconstruction condition hence the every filter banks in the branch of its tree structure definitely satisfies perfect reconstruction property. Any test signal such as triangular signal when passed through such tree structure filter bank leads to the signal decomposition at various levels. Tree structure filter bank is designed by using the filter bank already designed in previous section of length N=8 and M=4.After passing the test signal of triangular wave through this tree structure filter bank we get signal decomposed into different frequency bands as shown in the following figures. As designed filter bank is paraunitary at synthesis side we get back reconstructed test signal consisting of some delay shown in following figure.

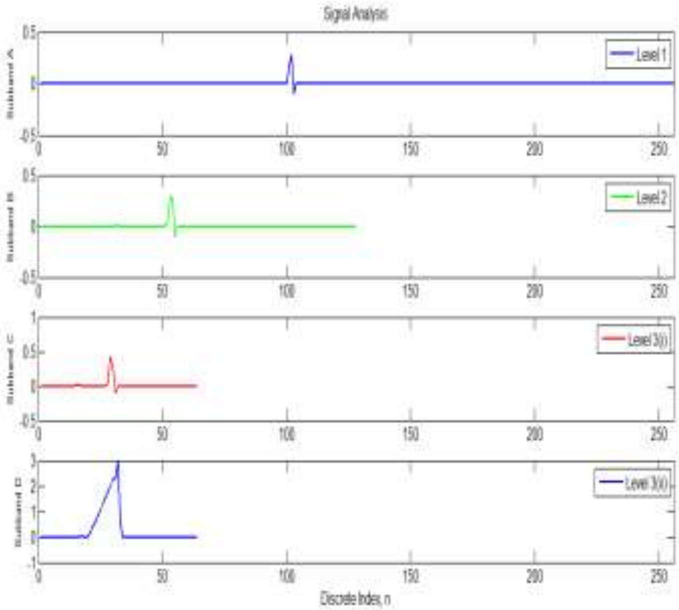

Fig.2.Signal Analysis through Tree Structure analysis Filter Bank

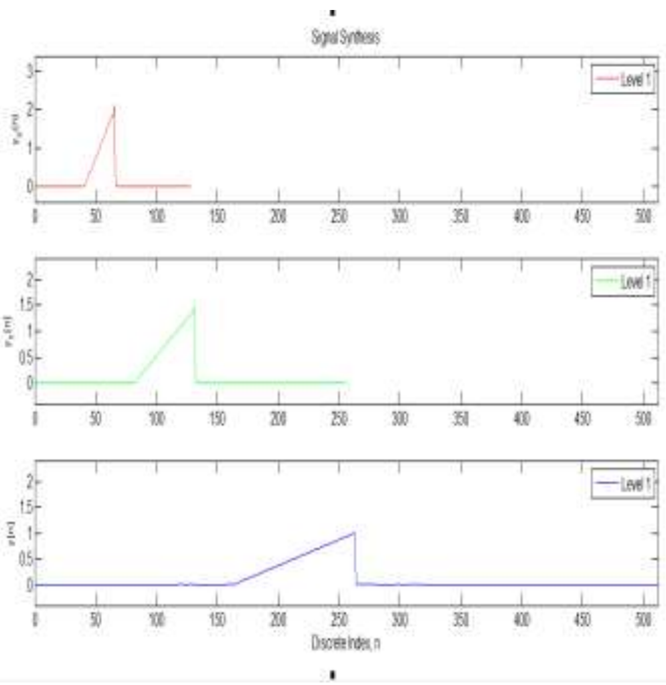

Fig.3.Signal Synthesis through Tree Structure synthesis Filter Bank

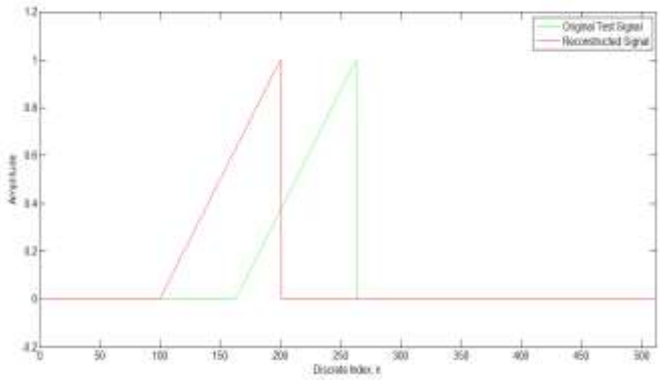

Fig.4.Delayed Perfect Reconstruction of the signal at the synthesis Side

VII. CONCLUSION

Proposed method introduces FB design by using iterative SVD approach instead of going by conventional factorization and lattice structures. Maximally Decimated Paraunitary Liner Phase Filter Bank is designed by performing simple matrix operations and hence reduces computational complexity at great extent.Proposed methodology of FB design leads to better performance in terms of decision parameters like ripple magnitudes in pass bands and stopbands,total aliasing distortion,transtion widths,DC leakages and computational time. Various FB design methods have different advantages and disadvantages based on applications in multiple domains, this proposed FB design method is to counterattack on the existing computational complex lattice realization design approach for time critical applications.FB design via SVD approach deals with any number of channels and obtains any lengths of filters in FB, satisfying Paraunitary and Linear Phase Properties.

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