Analysis Of Shaped Dielectric Lens Array Antenna By Modal expansion

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***Abstract*—**Theradiation pattern of Spheroidal dielectric lens excited by array antennais expressed in terms of spheroidal (SDL) modal expansion. Since the SDL Vector wave functions are not orthogonal, each the SDL modes are expressed in terms of aninfinite sum of Spherical (SPL) vector modes, that exhibit orthogonality over a measurement surface surrounding the lens and array. The spheroidal modal coefficients can be determined by applying boundary condition on the lens surface. Using the determined coefficients near fields on the measurement surface is computed. This field can be easily transformed to the far field using standard modal expansion. This technique has been applied to a 1x2 array at 10 GHz radiating into a SDL lens and the radiated fields computed with 20 SDL modes, with each SDL mode expressed as 20 SPL modes. The results are compared with a well-known EM flow solver.

Keywords-*Lenses; Dielectric; Spherical Modes*

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I. INTRODUCTION

Analysis of the radiation pattern using SDL conducting scatters [1] – [2], and SDL dielectric lenses [3]-[5] are well reported in literature. However these efforts consider a single plane wave incidence on the SDL scatters. An accurate Moments based analysis employing HFSS is discussed in [6] but is computationally intensive and strongly dependent on the number of cells over the dielectric lens geometry. Ray tracing techniques are discussed in [7] for shaped lenses but are limited by far field approximation of the Primary source.

In this effort we consider a 1x2 array radiating into an SDL lens in its near field. The far field of the array is used to obtain the SDL modal coefficients through an orthogonal SPL modal expansion. The SDL-SPL modal coefficients are valid anywhere in space and are used to obtain the scattered modal coefficients inside and outside the SDL lens. The unknown modal coefficients are obtained by application of the boundary condition on the lens surface. The total field on a near field surface surrounding the lens and array is obtained as the sum of the incident and scattered field, which is easily transformed by a SPL modal expansion to the far field. This analysis is presented in Section II. In Section III the analysis is employed to compute the radiated fields of a 1x2 array radiating into a SDL lens at 10 GHz. The results have been compared with those obtained by a well-known EM flow solver HFSS

II. ANALYSIS

Consider a spheroidal dielectric lens excited by array antenna with their centers at O' and O respectively as shown in Fig.1.The field of any primary radiator is represented by an SDL modal expansion as [8]-[9]

$$\mathbf{E_i} = -\sum_m \sum_n a_{mn} \mathbf{M_{mn}}(\mathbf{c}, \boldsymbol{\eta}, \boldsymbol{\xi}, \boldsymbol{\phi}) + b_{mn} \mathbf{N_{mn}}(\mathbf{c}, \boldsymbol{\eta}, \boldsymbol{\xi}, \boldsymbol{\phi})$$
$$\mathbf{H_i} = -\frac{k}{j\varpi\mu} \sum_m \sum_n b_{mn} \mathbf{M_{mn}}(\mathbf{c}, \boldsymbol{\eta}, \boldsymbol{\xi}, \boldsymbol{\phi}) + a_{mn} \mathbf{N_{mn}}(\mathbf{c}, \boldsymbol{\eta}, \boldsymbol{\xi}, \boldsymbol{\phi}) \qquad (1)$$

Here the $\mathbf{M}_{mn}$, $\mathbf{N}_{mn}$ are the spheroidal vector wave functions (SpVWF).Since SpVWF's are not orthogonal Spheroidal modal complex coefficients (SpMCC) A_{mn} , B_{mn} cannot be computed directly.

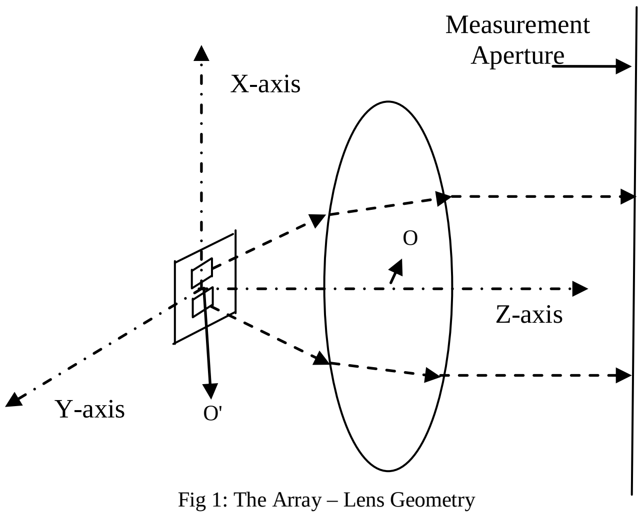

Fig 1: The Array – Lens Geometry

The SpVWF's are now expressed in terms of spherical vector wave functions[SVWF] as described below.

The spheroidal scalar wave function[8] $\psi_{mn}(\eta, \xi, \varphi)$ is

$$\psi_{mn}(\eta, \xi, \varphi) = S_{mn}\left(c, \eta\right) R_{mn}(c, \xi)_{\sin}^{\cos}(m\varphi) \; (2)$$

Here $S_{mn}\left(c, \eta\right)$ is the spheroidal angular function, and $R_{mn}(c, \xi)$ the spheroidal radial function. Spheroidal scalar wave function [8] can also be expressed as a spherical wave function series as in (3).

$$\psi_{mn}(c, \eta, \xi, \varphi; r, \theta, \varphi) = \sum_{k=1,2}^{\infty} d_k^{mn}(c) P_{m+k}^{m}(\cos\theta) Z_{m+k}^{m}(\beta r)_{\sin}^{\cos}(m\varphi) \; (3)$$

Here $d_k^{mn}(c)$ is spheroidal intermediate parameter where $c = \beta v/2$ where 'v' is the major axis of Spheroid, $P_n^m(\cos\theta)$ is Spherical Legendre function and $Z_n^m(\beta r)$ is the Radial Hankel function. The indices have their usual meanings [8]. The SpVWF are now defined in [9] as

$$\mathbf{M}_{mn}(c,\eta,\xi,\varphi;r,\theta,\varphi) = \nabla\times\mathbf{a}\psi_{mn}(c,\eta,\xi,\varphi;r,\theta,\varphi) \quad (4)$$

$$\mathbf{N}_{mn}(c,\eta,\xi,\varphi;r,\theta,\varphi) = \frac{1}{\beta}\nabla\times\mathbf{M}_{mn}(c,\eta,\xi,\varphi;r,\theta,\varphi) \quad (5)$$

where $\mathbf{a} = r\hat{\mathbf{r}}$ the SpVWF's are expanded in terms of spherical vector wave functions [SVWF] by substituting (3) in (4) and (5) to finally obtain (6) and (7).The orthogonality conditions for SVWF are now applied to the expanded SpVWFs in (6) and (7) as per the procedure defined in [9]. These are expressed as in (8) and (9) with the cross product between $\mathbf{M}_{mn}$ and $\mathbf{N}_{mn}$ being zero.

The known incident field and the unknown scattered E-fields both within the dielectric and outside the dielectric lens can now be expressed in terms of their SDL-SPL modal coefficients using (10) and (11).Consequently the Spheroidal modal complex coefficients (SpMCC) of the primary source are determined by (10) and (11) where $\mathbf{E}(\theta,\varphi)$ is field expression of primary source as described in [10].The incident field SDL-SPL modal coefficients (A_{mn}, B_{mn}) of the array is translated from the array origin at O to O' the lens origin using the procedure as in [11]-[12] .The unknown SDL-SPL scattered coefficients (A_d, B_d) and (A_s, B_s) for the fields inside and outside the dielectric lens respectively, are obtained by applying the boundary conditions on the surface of the dielectric lens viz; continuity of E and H fields.

This procedure is the same as that employed to find the scattered coefficients in [11]-[12] viz; sum of incident field and scattered field outside lens equals the scattered field inside the dielectric for each and every spherical mode (indexed by mode number). Application of this procedure yields the SDL-SPL scattered modal coefficients (A_s, B_s) as in (12) and (13) where $[A_t],[B_t]$ are row matrices with order $k\times 1$. Each of these terms expressed in (14) are themselves square matrices i.e $[DH^{(2)}(*)]$, $[DJ(*)]$ $[J(*)]$, $[H^{(2)}(*)]$ having order $n\times k$ .Here $J_n(*)$, $DJ_n(*)$ $H^{(2)}(*)$ and $DH^{(2)}(*)$ are spherical Bessel function, differential spherical Bessel function, spherical Hankel function and differential spherical Hankel function of order two respectively. Also β_d and β_r are phase constants inside and outside the lens respectively. Further 'n' and 'k' are no. of spheroidal modal coefficients and no. of spherical modes respectively. The total field may now be easily computed by summing up Incident and Scattered fields at any point on the measurement surface

$$\mathbf{M}_{mn}(c,\eta,\xi,\varphi;r,\theta,\varphi) = \sum_{k=0,1}^{\infty} d_k^{mn}(c)\frac{m}{\sin\theta}Z_{m+k}(\beta r)P_{m+1}^m(\cos\theta)\,{}^{\mathrm{Sin}}_{\mathrm{Cos}}\, m\varphi\bar{\boldsymbol{\theta}} - \sum_{k=0,1}^{\infty} d_k^{mn}(c)Z_{m+k}(\beta r)\frac{\delta\left(P_{m+1}^m(\cos\theta)\right)}{\delta\theta}\,{}^{\mathrm{Sin}}_{\mathrm{Cos}}\, m\varphi\bar{\boldsymbol{\varphi}}$$

(6)

$$\mathbf{N}_{mn}(c,\eta,\xi,\varphi;r,\theta,\varphi) = +\sum_{k=0,1}^{\infty} d_k^{mn}\frac{(m+k)(m+k+1)}{\beta r}Z_{m+k}(\beta r)P_{m+k}^m(\cos\theta)\,{}^{\mathrm{Cos}}_{\mathrm{Sin}}\, m\varphi\bar{\mathbf{r}}$$
$$+\sum_{k=0,1}^{\infty} d_k^{mn}\frac{1}{\beta r}\frac{\delta\left(rZ_{m+k}(\beta r)\right)}{\delta r}\frac{\delta\left(P_{m+k}^m(\cos\theta)\right)}{\delta\theta}\,{}^{\mathrm{Sin}}_{\mathrm{Cos}}\, m\varphi\bar{\boldsymbol{\theta}} - \sum_{k=0,1}^{\infty} d_k^{mn}\frac{m}{\beta kr\sin\theta}\frac{\delta\left(rZ_{m+k}(\beta r)\right)}{\delta r}P_{m+k}^m(\cos\theta)\,{}^{\mathrm{Sin}}_{\mathrm{Cos}}\, m\varphi\bar{\boldsymbol{\varphi}}$$

(7)

$$\int_0^{2\pi}\int_0^{\pi}\mathbf{M}_{mn}.\mathbf{M}_{mn'}\mathrm{Sin}(\theta)d\theta d\varphi = \sum_{k=0,1}^{\infty} d_k^{mn}d_k^{mn'}\frac{4\pi(2m+k)!}{(2m+2k+3)k!}\left(Z_{m+k}(\beta r)\right)^2 \quad (8)$$

$$\int_0^{2\pi}\int_0^{\pi} N_{mn}N_{mn'}\mathrm{Sin}(\theta)d\theta d\varphi =$$
$$\sum_{k=0,1}^{\infty} d_k^{mn}d_k^{mn'}\frac{2\pi(m+1)(m+k+1)(2m+k)!}{(2m+2k+3)k!}\left[(m+k+2)\left(\frac{Z_{m+k}}{kr}\right)^2\right] + (m+k+1)\frac{1}{kr}\left(\frac{\partial(rZ_{m+k})}{\partial r}\right)^2 \quad (9)$$

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$$A_{mn} = \frac{\int_{\varphi=0}^{2\pi}\int_{\theta=0}^{\pi} \mathbf{E}(\theta,\varphi).\mathbf{M}_{mn}(c,\eta,\xi,\varphi;r,\theta,\varphi)\mathrm{Sin}(\theta)d\theta d\varphi}{\int_{\varphi=0}^{2\pi}\int_{\theta=0}^{\pi} \mathbf{M}_{mn}.\mathbf{M}_{mn'}\mathrm{Sin}(\theta)d\theta d\varphi} \tag{10}$$

$$B_{mn} = \frac{\int_{\varphi=0}^{2\pi}\int_{\theta=0}^{\pi} \mathbf{E}(\theta,\varphi).\mathbf{N}_{mn}(c,\eta,\xi,\varphi;r,\theta,\varphi)\mathrm{Sin}(\theta)d\theta d\varphi}{\int_{\varphi=0}^{2\pi}\int_{\theta=0}^{\pi} \mathbf{N}_{mn}.\mathbf{N}_{mn'}\mathrm{Sin}(\theta)d\theta d\varphi} \tag{11}$$

$$[A_S] = \left[\left[DH^{(2)}(\beta_d r)\right] - \sqrt{\varepsilon_r}\,[DJ(\beta_r r)][J(\beta_d r)]^{-1}\left[H^{(2)}(\beta_r r)\right]\right]^{-1} \times \left[\sqrt{\varepsilon_r}\,[DJ(\beta_d r)][J(\beta_d r)]^{-1}[J(\beta_r r)] - [DJ(\beta_r r)]\right] \times [A_t] \tag{12}$$

$$[B_s] = \left[\left[H^{(2)}(\beta_r r)\right] - \sqrt{\varepsilon_r}\,[J(\beta_d r)][DJ(\beta_d r)]^{-1}\left[DH^{(2)}(\beta_d r)\right]\right]^{-1} \times \left[\sqrt{\varepsilon_r}\,[J(\beta_d r)][DJ(\beta_d r)]^{-1}[DJ(\beta_r r)] - [J(\beta_r r)]\right] \times [B_t] \tag{13}$$

$$[J(\beta_d r)] = \begin{bmatrix} d_0^{11}(c)J_1(\beta_d r) & 0 & d_0^{13}(c)J_1(\beta_d r) & 0 & \cdots & \cdots & 0 \\ 0 & d_1^{12}(c)J_2(\beta_d r) & 0 & d_1^{14}(c)J_2(\beta_d r) & \cdots & \cdots & d_1^{1n}(c)J_2(\beta_d r) \\ d_2^{11}(c)J_3(\beta_d r) & 0 & d_2^{13}(c)J_3(\beta_d r) & 0 & \cdots & \cdots & 0 \\ 0 & d_3^{12}(c)J_4(\beta_d r) & 0 & d_3^{14}(c)J_4(\beta_d r) & \cdots & \cdots & d_3^{1n}(c)J_4(\beta_d r) \\ d_4^{11}(c)J_5(\beta_d r) & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & d_k^{12}(c)J_{k+1}(\beta_d r) & 0 & d_k^{14}(c)J_{k+1}(\beta_d r) & \cdots & 0 & d_k^{1n}(c)J_{k+1}(\beta_d r) \end{bmatrix}_{k\times n} \tag{14}$$

Numerical computation and validation

The simulations are carried out for spheroidal dielectric lens antenna excited by 1x2 array antenna. The antenna specifications and dimensions are tabulated in Table 1.

TABLE 1. LIST OF PARAMETERS IN COMPUTATION

Operating frequency	10GHz
Array dimension	1x2
Interspacing distance between array elements	λ/2
Array Antenna dimension	34.2mm X 38.3mm
Array substrate parameters	
Substrate used	FR4
Its relative Permeability	ε_r=4.4
Height of substrate	h=1.6mm
Loss tangent	0.02
Separation distance between array and centre of spheroidal lens (00')	1cm
Spheroidal dielectric lens parameters	
Material used	Teflon
Relative permittivity	ε_r=2.08
Dimension of Spheroidal dielectric lens	105x75mm

The simulation structure of spheroidal dielectric lens excited by lens array antenna is presented in Fig. 2. The far field pattern of the 1x 2 primary array radiator was obtained using [10]. This was expanded employing a SpVWF as detailed in (10) and (11). The translation of these SpMCC was performed through a distance of 1cm as explained in [11]. The scattered fields were computed using (12) and (13) where order of matrix n x k is 20 x 20 and the total field obtained by summing this with the incident field. The E and H plane far fields are plotted in Figures 3 and 4 respectively. Also shown in the figures are a comparison with the fields computed using an EM Flow Solver HFSS 17 unit cells

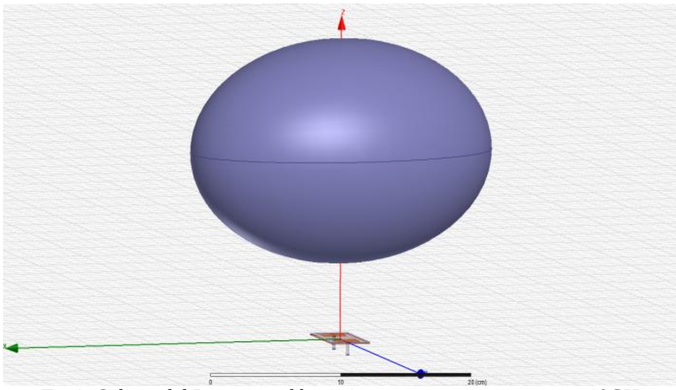

Fig 2: Spheroidal Lensexcited by 1x2 array antenna operating at 10GHz

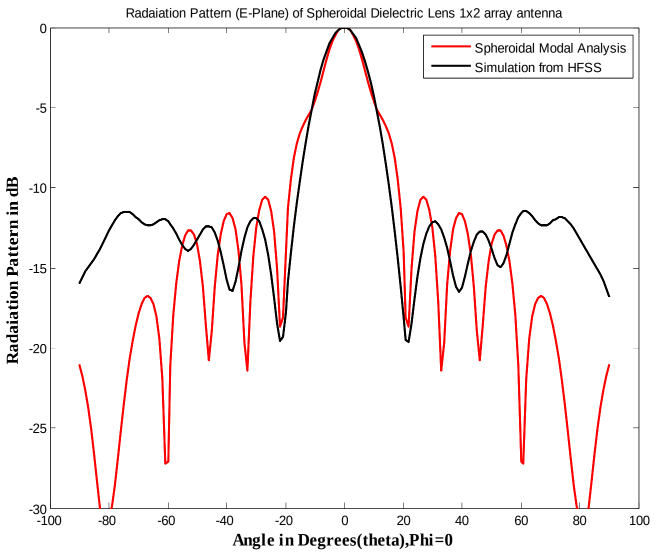

Fig 3: Radiation pattern of a Spheroidal lens array antenna in E plane

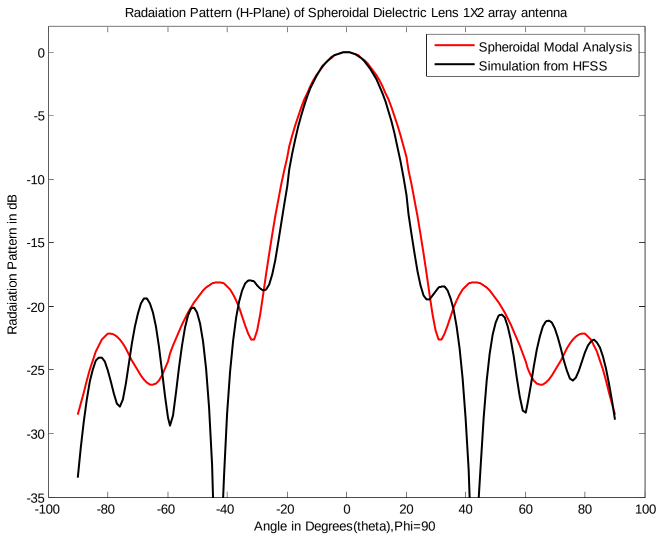

Fig. 4: Radiation pattern of a Spheroidal lens array antenna in H plane

III. CONCLUSION

Spheroidal modal analysis is an accurate method to determine fields for dielectric lens array antenna. In this paper expressions of SDL-SPLVWF'S and its orthogonality conditions are presented which can be used to solve multiple scattering problems. A novel method to derive the scattered coefficients both inside and outside the lens is derived by application of boundary condition. Using these coefficients the total field pattern of spheroidal lens array antenna is determined. To validate this results Spheroidal Lens excited by 1x2 array antenna operating at 10GHz were considered it is found that due to the collimation of lens there is an increase in gain of 7dB compared to primary radiator. There is a close agreement with the field's determined using spheroidal modal analysis and simulated using EM Flow Solver HFSS.

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