Approximate Controllability of Semi-linear Fuzzy Dynamical System

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***Abstract*—**In this paper we consider a semi-linear dynamical system with fuzzy initial condition. We discuss the results regarding the approximate controllability of the system and existence of the controller which steers the system to the desired state. The theory is substantiated with an illustrative example.

Keywords- *fuzzy systems, approximate controllability, fuzzy initial condition, fuzzy number*

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I. Introduction

In this paper we propose to establish the controllability results for the semi-linear dynamical system. A system with the coefficient of the highest order derivative is free from dependent variable given as

$$\frac{dX}{dt} = AX + BU + F(t, X) \qquad (1)$$
$$X(0) = \tilde{X}_0$$

where, $X_{n\times 1}$ is the state vector, $U_{m\times 1}$ is the control vector, $A_{n\times n}$ is the time invariant evolution matrix, $B_{n\times m}$ is the control matrix and $F(t, X)$ is E^n – valued mapping defined on $[0,\infty)\times E^n$, $F = (f_1, f_2, \ldots f_n)$.

For the dynamical system represented by form (1) when there are possibilistic uncertainties or vagueness for the entries in the evolution matrix *A*, control matrix *B* and or in the initial condition $X_{0,}$ a suitable model will require setup involving fuzzy numbers. In the most general form of the fuzzy dynamical system equivalent to (1) the entries of the matrix *A*, *B* and the initial condition X_0 are represented by fuzzy numbers. In this article, we will consider the case where the initial condition is represented by fuzzy number. Such a system occurs as intermediate one while studying controlled diffusion systems, controlled prey-predator systems, more general controlled Lotka-Volterra models.

Study of first order linear or nonlinear fuzzy dynamical equations (FDEs) is inevitable as they appear extensively in many applications. The concept of a fuzzy derivative was first introduced by S. L. Chang, L. A. Zadeh in [1]. It was followed up by D. Dubois, H. Prade in [2]. Dubey in [3] have established controllability for linear time invariant fuzzy systems. Lin [4] adopted max-product automata to model the system whereas, Qiu [5], Cao and Ying [6] modeled the system using max-min automata and for such systems they developed the supervisory control under full crisp observations. The controllability and observability criteria for fuzzy dynamical control systems were discussed by Ding and Kandel [7, 8]. We propose results for semilinear time invariant systems with fuzzy initial condition.

The paper is organized in following manner; in the initial three sections we present the concepts and introductory tools to deal with fuzzy initial value problems. In the following section we propose the results for the controllability of the system. The illustrative real life example involving fuzzy initial condition is solved using proposed scheme. The last section concludes the article with remarks and observations.

II. Preliminaries

Let $P_K(\Re^n)$ denote the family of all nonempty compact, convex subsets of $\Re^n$. If $\alpha,\beta\in\Re$ and $A,B\in P_K(\Re^n)$, then $\alpha(A+B)=\alpha(A)+\alpha(B)$, $\alpha(\beta A)=(\alpha\beta)A$, $1\cdot A=A$ and if $\alpha,\beta\ge 0$, then $(\alpha+\beta)A=\alpha(A)+\beta(A)$.

The distance between *A* and *B* is defined as Hausdroff metric

$$d_H(A,B)=\max\left[\sup_{a\in A}\inf_{b\in B}\| a-b\|, \sup_{b\in B}\inf_{a\in A}\| a-b\|\right]$$

where $\|.\|$ denotes a norm in R^n. Then it is clear that ($P_K(\Re^n), d_H$) becomes a metric space refer [9].

Definition 2.1: A fuzzy number *u* is completely determined by any pair $u=(u_l,u_u)$ of functions $u_l(r), u_u(r):[0,1]\to\Re$, satisfying the three conditions:

(i) $u_l(r)$ is a bounded, monotonic, non-decreasing left-continuous function for all $r\in[0,1]$.

(ii) $u_u(r)$ is a bounded, monotonic, non-increasing right-continuous function for all $r\in[0,1]$.

(iii) For all $r\in(0,1]$ we have: $u_l(r)\le u_u(r)$.

For every $u=(u_l,u_u)$, $v=(v_l,v_u)$ and $k > 0$ we define addition and scalar multiplication as follows:

$$(u+v)_l(r)=u_l(r)+v_l(r),\ \ (u+v)_u(r)=u_u(r)+v_u(r), \qquad (2)$$

$$(ku)_l(r)=ku_l(r),\ \ (ku)_u(r)=ku_u(r), \qquad (3)$$

The collection of all fuzzy numbers with addition and multiplication as defined by equations (2), (3) is denoted by E^1.

Let $I=[t_0,t_0+a]$, $t_0\ge 0$ and a >0 and denote by $E^n=\{u:\Re^n\to[0,1]\}$ where, *u* satisfies the conditions given below:

(i) u is normal i.e. there exist an $x_0 \in \Re^n \ni u(x_0) = 1$.

(ii) u is fuzzy convex, that is for $0 \le \lambda \le 1$, $u(\lambda x_1 + (1-\lambda)x_2) \ge \min\{u(x_1), u(x_2)\}$;

(iii) u is upper semi continuous

(iv) $[u]^0 = cl\{x \in \Re^n \mid u(x) > 0\}$ is compact.

For $0 < \alpha \le 1$ denote $[u]^\alpha = \{x \in \Re^n \mid u(x) \ge \alpha\}$. Then from (i)-(iv), it follows that the α - level set $[u]^\alpha \in P(\Re^n)$ for all $0 \le \alpha \le 1$. Especially for addition and scalar multiplication, we have $[u+v]^\alpha = [u]^\alpha + [v]^\alpha$, $[ku]^\alpha = k[u]^\alpha$. For the results wherever fuzzy number is considered we will assume the triangular fuzzy number $A = (a,b,c)$ given as

$$A(x) = \begin{cases} (x-a)/(b-a) & a < x \le b \\ (c-x)/(c-b) & b < x \le c \\ 0 & otherwise \end{cases} \quad (4)$$

For A such as given by (4), the α - cut for $0 < \alpha \le 1$ is denoted by $[A_l^\alpha, A_u^\alpha]$. For $\alpha = 0^+$, We denote the level cut by $A^0 = [A_l, A_u]$ = [a, c] and for $\alpha = 1$, $A^1 = b$.

If $g : \Re^n \times \Re^n \to \Re^n$ is a function, then according to Zadeh's extension principle we can extend g to $E^n \times E^n \to E^n$ defined as $g(u,v)(z) = \sup_{z=g(x,y)} \min\{u(x), v(x)\}$.

It is well known that $[g(u,v)]^\alpha = g([u]^\alpha, [v]^\alpha)$ for all $u, v \in E^n$, $0 \le \alpha \le 1$ and continuous function g, refer [9].

Definition 2.2: Let $d_H[[u_1]^\alpha, [u_2]^\alpha]$ be the Hausdroff distance between the set $[u_1]^\alpha$, $[u_2]^\alpha$ of $P_K(\Re^n)$. Then we define $d[u_1, u_2] = \sup_{0 \le \alpha \le 1} d_H\{[u_1]^\alpha, [u_2]^\alpha\}$ the distance between u_1 and u_2 in E^n. Then (E^n,d) is a complete space, refer [9,10]. Some properties of the metric d are as follows refer [10-15]:

$$d[u_1 + u_3, u_2 + u_3] = d[u_1, u_2] \text{ and } d[u_1, u_2] = d[u_2, u_1]$$

$$d[\lambda u_1, \lambda u_2] = |\lambda| d[u_1, u_2],$$

$$d[u_1, u_2] \le d[u_1 + u_3] + d[u_3 + u_2]$$

for all $u_1, u_2, u_3 \in E^n$ and $\lambda \in \Re$.

Definition 2.3: A fuzzy function $f : I \to E^1$ is called continuous if for $t \in \Re$ and $\varepsilon > 0$, $\exists \delta > 0$, such that $|t - t_0| < \delta \Rightarrow d|f(t) - f(t_0)| < \varepsilon$.

Definition 2.4: Consider $x, y \in E^n$, if there exists $z \in E^n$ such that $x = y + z$, then z is called the H-difference of x and y and it is denoted by $x - y$. A mapping $F : I \to E^n$ is differentiable at $t \in I$ if there exists a $F'(t) \in E^n$ such that the limits $\lim_{h \to 0^+} \frac{F(t+h) - F(t)}{h}$ and $\lim_{h \to 0^+} \frac{F(t) - F(t-h)}{h}$ exist and are equal to $F'(t)$. Here the limits are taken in the metric space (E^n, d).

Definition 2.5: If $F : I \to E^n$ is continuous, then it is integrable and $\int_a^b F = \int_a^c F + \int_c^b F$. The following properties of the integral are valid. If $F, G : I \to E^n$ are integrable, $\lambda \in \Re$, the following properties hold:

$$\int (F+G) = \int F + \int G;$$

$$\int \lambda F = \lambda \int F, \lambda \in \Re$$

$$d[F,G] \text{ is integrable;}$$

$$d\left[\int F, \int G\right] \le d[F,G]$$

Finally, let $F : I \to E^n$ be continuous. Then the integral $G(t) = \int_a^t F$ is differentiable and $G'(t) = F(t)$, Furthermore, $F(t) - F(a) = \int_a^t F'(t)$. Refer Kaleva [12,13], Nieto [11], Lakshmikantham [10] for details.

Definition 2.6: The nonlinear function $F : [0, \infty) \times E^n \to E^n$ is a continuous function and satisfies the Lipschitz condition

$$d([F(t, x(\cdot))]^\alpha, [F(t, y(\cdot))]^\alpha) \le h\, d([x(\cdot)]^\alpha, [y(\cdot)]^\alpha)$$

for all $x(\cdot), y(\cdot) \in E^n$, h is a positive constant.

Definition 2.7: A dynamical system is said to be completely controllable, if for any initial and final states X_0 and X_T in the state space X, there exist control U that will steer the system from $X(t_0) = X_0$ to $X(T) = X_T$.

Definition 2.8: A dynamical system is said to be approximately controllable, if for any initial and final states X_0 and X_T in the state space X, there exist control U that will steer the system from $X(t_0) = X_0$ to $X(T) = X_T$.

Definition 2.9: Consider a linear time invariant system in R^n

$$\dot{x}(t) = Ax(t) + Bu(t),$$
$$x(t_0) = X_0$$

A necessary and sufficient condition for the pair (A,B) to be controllable is

$$\text{rank}(W) = \text{rank}(B \mid AB \mid A^2B \mid \ldots \mid A^{n-1}B) = n.$$

here, $W_{n \times nm}$ is called Kalman's controllability matrix.

III. CONTROLLABILITY OF CRISP SEMI-LINEAR SYSTEM

The crisp or ordinary time invariant semi-linear system given by

$$\frac{dX}{dt} = AX + BU + F(X,t) \tag{5}$$
$$X(t_0) = X_0$$

where,

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix};\ U = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix};\ A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \\ a_{n1} & a_{21} & & a_{nn} \end{bmatrix};$$

$$B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1m} \\ b_{21} & b_{22} & \cdots & b_{2m} \\ \vdots & \vdots & & \\ b_{n1} & b_{21} & & b_{nm} \end{bmatrix};\ F(X) = \begin{bmatrix} f_1(X) \\ f_2(X) \\ \vdots \\ f_n(X) \end{bmatrix};$$

where, A is nonsingular and the nonlinearity in the system (5) is bounded or Lipschitz.

Linearization of system (5) about the equilibrium state, X_e gives the system

$$\frac{dX}{dt} = A_L X + B_L U \tag{6}$$
$$X(t_0) = X_0$$

where,

$$A_L = \left[a_{ij} + \left(\frac{\partial f_i}{\partial x_j} \right)_{X = X_e} \right];\ B_L = \left[b_{ij} + \left(\frac{\partial f_i}{\partial u_j} \right)_{X = X_e} \right];$$

For system (6) if the controllability condition is satisfied then the minimum norm steering controller can be defined as,

$$U(t) = -B_L{}^T(t) e^{(t_0 - t)A_L{}^T} W^{-1}(t_0, T)[X_0 - e^{A_L(t_0 - T)} X_T] \tag{7}$$

$W(t_0, T)$, in equation (7) is called controllability grammian given by

$$W(t_0, T) = \int_{t_0}^{t_1} e^{(t_0 - t)A_L} B_L B_L{}^T e^{(t_0 - t)A_L{}^T} dt$$

The controller given by (7) steers the system from the initial state to the desired final state, and the state trajectory at time t is given by,

$$X(t) = e^{(t - t_0)A_L} X_0 + \int_{t_0}^{t} e^{(t - \tau)A_L} B_L U(\tau) d\tau \tag{8}$$

Now we state our results for system (1) for which the initial condition is represented by fuzzy number.

IV. Main Result

We come back to our system

$$\frac{dX}{dt} = AX + BU + F(X,t) \tag{9}$$
$$X(t_0) = \tilde{X}_0$$

The only difference between system (5) and (9) is that in system (9) the initial condition is represented by fuzzy number.

Since $F(X,t)$ is Lipschitz, system (9) can be linearized similar to (5) around the equilibrium, giving us

$$\frac{dX}{dt} = A_L X + B_L U \tag{10}$$
$$X(t_0) = \tilde{X}_0$$

where, $A_L = \left[a_{ij} + \left(\frac{\partial f_i}{\partial x_j} \right)_{X = X_e} \right]$; $B_L = [b_{ij}]$;

For the sake of convenience the evolution and control matrix will be denoted by A and B respectively even for the linearized system.

Theorem 4.1 If A is non-singular and the transfer functions f_i's are Lipschitz continuous then system (9) is controllable form the initial condition $X(0) = \tilde{X}_0$ to the final state $X(T) = \tilde{X}_T$, by the controller given by

$$\tilde{U}(t) = -B^T(t) e^{(t_0 - t)A^T} W^{-1}(t_0, T)[\tilde{X}_0 - e^{A(t_0 - T)} \tilde{X}_T]$$

This controller steers the system from the initial state $X(0) = \tilde{X}_0$ to the final state $X(T) = \tilde{X}_T$, giving the state trajectory for $t > 0$, as

$$\tilde{X}(t) = e^{(t - t_0)A} \tilde{X}_0 + \int_{t_0}^{t} e^{(t - \tau)A} B \tilde{U}(\tau) d\tau \tag{11}$$

Proof: Since $X(0) = \tilde{X}_0$, is a triangular fuzzy number, we can take its level cuts. For $\alpha = 0$ we get, ${}^{\alpha}\tilde{X}(0) = [\underline{X}(0), \overline{X}(0)] = [\underline{X}_0, \overline{X}_0]$, where, $\underline{X}_0, \overline{X}_0$ are real numbers and ${}^{\alpha}\tilde{X}(T) = [\underline{X}(T), \overline{X}(T)] = [\underline{X}_T, \overline{X}_T]$, where, $\underline{X}_T, \overline{X}_T$ are real numbers. Also taking α -cut for $\tilde{U}(t)$, we get ${}^{\alpha}\tilde{U}(t) = [\underline{U}(t), \overline{U}(t)]$ given as

$$[\underline{U}, \overline{U}] = -B^T(t) e^{(t_0 - t)A^T} W^{-1}(t_0, T) \left[[\underline{X}_0, \overline{X}_0] - e^{A(t_0 - T)} [\underline{X}_T, \overline{X}_T] \right]$$

Comparing the lower and upper of the interval we get,

$$\underline{U}(t) = -B^T(t) e^{(t_0 - t)A^T} W^{-1}(t_0, T)[\underline{X}_0 - e^{A(t_0 - T)} \underline{X}_T] \tag{12}$$
$$\overline{U}(t) = -B^T(t) e^{(t_0 - t)A^T} W^{-1}(t_0, T)[\overline{X}_0 - e^{A(t_0 - T)} \overline{X}_T] \tag{13}$$

Since $\underline{U}(t)$ and $\overline{U}(t)$ exists for all $\alpha > 0$, system (10) has at-least weak fuzzy controller, such a weak solution would be strong fuzzy controller if

(i) $\underline{U}(t) \le \overline{U}(t)$ for all $\alpha \in (0,1]$.

(ii) For $\alpha, \beta \in (0,1]$ $\alpha < \beta$ ${}^{\alpha}\underline{U}(t) \le {}^{\beta}\underline{U}(t) \le {}^{\beta}\overline{U}(t) \le {}^{\alpha}\overline{U}(t)$

The fuzzy controller can be constructed as shown in the next lemma.

Using (12) we can compute

$$\underline{X}(t) = e^{(t - t_0)A} \underline{X}_0 + \int_{t_0}^{t} e^{(t - \tau)A} B \underline{U}(\tau) d\tau \tag{14}$$

And (13) allows us to compute

$$\overline{X}(t) = e^{(t - t_0)A} \overline{X}_0 + \int_{t_0}^{t} e^{(t - \tau)A} B \overline{U}(\tau) d\tau \tag{15}$$

Here also if $\underline{X}(t)$ and $\overline{X}(t)$ exists for all $\alpha > 0$, system (10) has at-least weak fuzzy state vector, and we would have strong fuzzy state trajectory if

(i) $\underline{X}(t) \le \overline{X}(t)$ for all $\alpha \in (0,1]$.

(ii) For $\alpha, \beta \in (0,1]$ $\alpha < \beta$ ${}^{\alpha}\underline{X}(t) \le {}^{\beta}\underline{X}(t) \le {}^{\beta}\overline{X}(t) \le {}^{\alpha}\overline{X}(t)$

The fuzzy state vector for all t can be constructed as shown in the next lemma.

Lemma: *The ith component of the fuzzy state transition vector, $\tilde{X}$ of the dynamical system (9) with fuzzy initial condition $\tilde{X}_0$ can be reconstructed form the ith components of ${}^{\alpha}X_l$ and ${}^{\alpha}X_u$ defined in the Theorem 1. And the and ith component is given as*

$$\tilde{x}_i = \bigcup_{\alpha \in [0,1]} {}_{\alpha}\tilde{x}_i$$

where, ${}_{\alpha}\tilde{x}_i = \alpha \cdot {}^{\alpha}\tilde{x}_i$ and ${}^{\alpha}\tilde{x}_i = \left[{}^{\alpha}x_{li}, {}^{\alpha}x_{ui}\right]$.

Proof: At some time $t > 0$, Consider the ith component $\tilde{x}_i$ of the fuzzy state transition vector $\tilde{X}$. For each particular $y \in R$, let $a = \tilde{x}_i(y)$. Then

$$\left(\bigcup_{\alpha \in [0,1]} {}_{\alpha}\tilde{x}_i\right)(y) = \sup_{\alpha \in [0,1]} {}_{\alpha}\tilde{x}_i(y)$$

$$= \max\left[\sup_{\alpha \in [0,a]} {}_{\alpha}\tilde{x}_i(y), \sup_{\alpha \in (a,1]} {}_{\alpha}\tilde{x}_i(y)\right]$$

For each $\alpha \in (a, 1]$, we have $\tilde{x}_i(y) = a < \alpha$ and, therefore ${}_{\alpha}\tilde{x}_i(y) = 0$. On the other hand for each $\alpha \in [0, a]$, we have $\tilde{x}_i(y) = a \geq \alpha$, therefore ${}_{\alpha}\tilde{x}_i(y) = \alpha$.

Hence,

$$\left(\bigcup_{\alpha \in [0,1]} {}_{\alpha}\tilde{x}_i\right)(y) = \sup_{\alpha \in [0,1]} \alpha = a = {}_{\alpha}\tilde{x}_i(y) \qquad \#$$

Thus each component of the state vector can be constructed form the corresponding α - levels obtained for $\tilde{X}$ of system (9).

V. Illustrative Example:

Consider the system

$$\dot{X} = AX + Bu + f(X)$$

where, $A = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ and $f(X) = \begin{pmatrix} x_1^2 \\ 0 \end{pmatrix}$

with fuzzy condition $X(0) = \tilde{X}_0 = \begin{pmatrix} (0,1,2) \\ (2,3,4) \end{pmatrix}$ and $X(1) = \tilde{X}_1 = \begin{pmatrix} (1,2,3) \\ (3,4,5) \end{pmatrix}$. Since f(x) is Lipschitz in [0, 1] we can linearize the given system.

Linearizing we get,

$$\dot{X} = A_L X + B_L u$$

with $A_L = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $B_L = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$.

Therefore the fundamental matrix is $\psi(t) = \begin{pmatrix} e^t & 0 \\ 0 & e^t \end{pmatrix}$ and the state transition matrix is given by $\phi(t, t_0) = \begin{pmatrix} e^t & 0 \\ 0 & e^t \end{pmatrix}$, so $\phi(t_0, t) = \begin{pmatrix} e^{-t} & 0 \\ 0 & e^{-t} \end{pmatrix}$

The controllability Grammian matrix is given by

$$W(0,t) = \int_0^t \phi(t_0, t) B B^T \phi^T(t_0, t)\, dt$$

$$= \begin{pmatrix} -2(e^{-2} - 1) & 0 \\ 0 & -2(e^{-2} - 1) \end{pmatrix}$$

$$W(0,1) = \begin{pmatrix} 1.7293 & 0 \\ 0 & 1.7293 \end{pmatrix}$$

Since, $|W(0,1)| = 2.9905 \neq 0$ the system is controllable.

Now the alpha-cut of ${}^{\alpha}X_0 = \begin{pmatrix} [\alpha, 2 - \alpha] \\ [\alpha + 2, 4 - \alpha] \end{pmatrix}$ and ${}^{\alpha}X_1 = \begin{pmatrix} [\alpha + 1, 3 - \alpha] \\ [\alpha + 3, 5 - \alpha] \end{pmatrix}$.

Now considering them we get systems to be solved as:
System [L]: with the initial state vector $I_0 = \begin{pmatrix} \alpha \\ \alpha + 2 \end{pmatrix}$ and final state vector $I_f = \begin{pmatrix} \alpha + 1 \\ \alpha + 3 \end{pmatrix}$, with respect to the lower cuts of state vector at $t = 0$ and $t = 1$.
System [U] : with the initial state vector $I_0 = \begin{pmatrix} 2 - \alpha \\ 4 - \alpha \end{pmatrix}$ and final state vector $I_f = \begin{pmatrix} 3 - \alpha \\ 5 - \alpha \end{pmatrix}$, with respect to the upper cuts of state vector at $t = 0$ and $t = 1$.

We separately solve systems (L) and (U) which gives the lower cut and upper cut of the components of u.

For system [L] we can find the controller $u_A(t)$ as follows

$$u_L(t) = -B^T \emptyset(t_0, t) W^{-1}[I_0 - \emptyset(t_0, t_1) I_f]$$

$$u_L(t) = \begin{pmatrix} -0.7311e^{-t}\alpha + 0.4255e^{-t} \\ -0.7311e^{-t}\alpha - 1.0367e^{-t} \end{pmatrix}$$

Similarly, for system [U] we can find the controller $u_M(t)$

$$u_U(t) = -B^T \emptyset(t_0, t) W^{-1}[I_0 - \emptyset(t_0, t_1) I_f]$$

$$u_U(t) = \begin{pmatrix} 0.7311e^{-t}\alpha - 1.0367e^{-t} \\ 0.7311e^{-t}\alpha - 2.4988e^{-t} \end{pmatrix}$$

The lower cut and upper cut of components of state vector X for (L) and (U) can be computed as follows

$$X_L(t) = \emptyset(t, t_0)\left[I_0 + \int_0^t \emptyset(t_0, \tau) B u_L\, d\tau\right]$$

$$X_L(t) = \begin{pmatrix} 0.7311e^{-t}\alpha - 0.4255e^{-t} + 0.2689e^{t}\alpha + 0.4255e^{t} \\ 0.7311e^{-t}\alpha + 1.0367e^{-t} + 0.2689e^{t}\alpha + 0.9633e^{t} \end{pmatrix}$$

Therefore fuzzy $X_L(t)$ is

$${}^{\alpha}X_L(t) = \begin{matrix}[0.7311e^{-t}\alpha - 0.4255e^{-t} + 0.2689e^{t}\alpha + 0.4255e^{t}, \\ 0.7311e^{-t}\alpha + 1.0367e^{-t} + 0.2689e^{t}\alpha + 0.9633e^{t}]\end{matrix}$$

For system [U] we can find the solution $X_U(t)$ as follows

$$X_U(t) = \begin{pmatrix} -0.7311e^{-t}\alpha + 1.0367e^{-t} - 0.2689e^{t}\alpha + 0.9633e^{t} \\ -0.7311e^{-t}\alpha + 2.4988e^{-t} - 0.2689e^{t}\alpha + 1.5012e^{t} \end{pmatrix}$$

Therefore fuzzy $X_U(t)$ is

$${}^{\alpha}X_U(t) = \begin{matrix}[-0.7311e^{-t}\alpha + 1.0367e^{-t} - 0.2689e^{t}\alpha + 0.9633e^{t}, \\ -0.7311e^{-t}\alpha + 2.4988e^{-t} - 0.2689e^{t}\alpha + 1.5012e^{t}]\end{matrix}$$

Using lemma we construct back the fuzzy state vector.

The transition of state for each components are shown in Fig. 1 (a) and (b).

VI. Conclusion

We have established results for the approximate controllability of semilinear dynamical system with fuzzy initial condition. The result for the existence and computation for the fuzzy controller and fuzzy state transition vector is given. The results are substantiated by illustrative examples.

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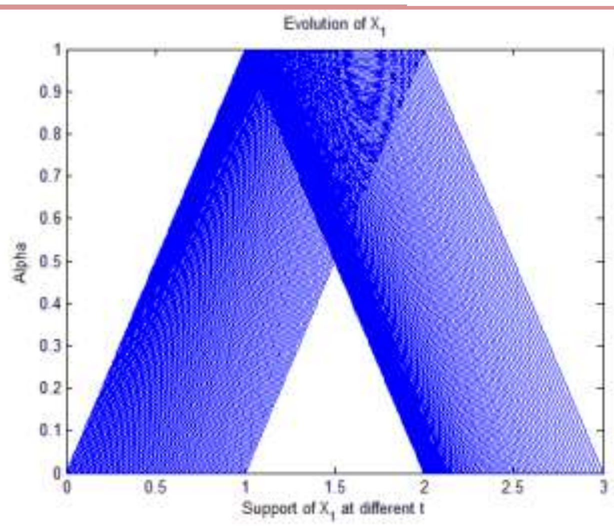

Figure 1. (a)

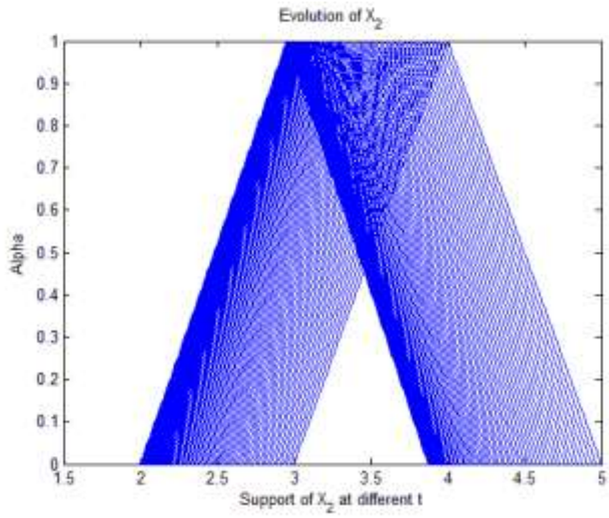

Figure 1. (b)