International Journal on Recent and Innovation Trends in Computing and Communication ISSN: 2321-8169
Volume: 4 Issue: 4 194 - 204

Investigation of the Movement of a Mass Loaded Spring on Insertion in an Acoustic Medium

Institute of Mathematics and Mechanics of the National Academy of Sciences of Azerbaijan,
Seyfullayev Alizade Imamali Oqli
Candidate of Physics and Mathematics Sciences, docent
Mamedova Guldasta Akif Kizi
Candidate of Physics and Mathematics Sciences, docent
Email: a.seyfullayev@yahoo.com
Email: gular-gulshan@rambler.ru

Abstract: Using an oscillator, the unsteady motion of a cylindrical inclusion, in an acoustic medium, after the passage of the wave, has been investigated. The time dependence of the boundary functions and potentials has been calculated and illustrated. Free oscillations of the suspended mass contained cylindrical inclusion, have also been investigated in the acoustic medium. Eigen-frequencies of oscillations have been described for several parameters of the system.

Key words:

Oscillations, vibrations, wave, frequencies, density, inclusion, pressure, potential.

*****

Introduction:

At present the investigations of the motion of constructions, which interact with their environment, are very important. Modern engineering and construction industries require the calculation of the effect of structural elements of constructions on the motion of waves in the medium surrounding the body. As the dynamical system of constructions is very complex, they can be modelled as a system with one grade of free motion. Different types of constructions such as ground, underground, cylindrical or spherical constructions and containers, which are in contact with seismic and blast waves, can be modelled as rigid inclusions containing elastically fastened mass, which is in interaction with the continuous solid medium. For instance such constructions include oil refining constructions, high-rise buildings, cylindrical and spherical containers etc.

Regarding the above mentioned, in order to simplify problems the simplified models are considered. The facilities are presented in the form of discrete vibrational system and the surrounding medium is considered in two dimensions. The impacts to the facilities can have different nature and therefore problem statements should be different.

The study of the joined motion of the continuous medium and the discrete systems has a great practical significance. For example, in the sensor displays of measurement systems, for wave processes the interference of their eigen-frequencies are included or the interaction with seismic waves of the facility they can be considered as discrete systems.

From the practical point of view, the investigation of the behavior of- shell systems of constructions, where the complementary masses are fastened are of great interest. The focus in these problems is as how to estimate the damping ability of the construction, under the action of dynamical impacts. If the rigidity of the shell is larger than the rigidity of shock absorbers springs, the shell deformation can be ignored in some cases.

The effect of loads located in the cylindrical shell, on the behavior of the shell, due to the interaction with the spherical wave of pressure was studied by in Huang et.al. (1974) In this work, the system of loads, concentrated masses, is attached to the inner surface of the shell with the help of elastic springs. Numerical calculations were carried out for a steel shell immersed in water, under the effect of the fall of spherical exponentially profiled wave on it.

Gorshkov and Tarlakovskii (1990) have reported the results of diffraction of elastic waves in a totally rigid medium with internal elements. Limarchenko and Tkachenko (2014) have investigated the non-linear oscillations of a totally solid cylindrical tank filled with a liquid and having a free surface, which is also fastened to the platform by a spring, at the action of the harmonic loading. Seyfullayev and Agayeva (1998) have investigated the unsteady problem, i.e. the motion of a spherical inclusion with an oscillator in the acoustic medium after the passage of the wave.

In this paper the unsteady motion of a cylindrical inclusion with an oscillator is investigated in an acoustic medium, after the passage of the wave. The free oscillations of the above mentioned system are also studied since the investigation of eigen frequencies is important in practice, especially when the inclusion seems to be a construction containing sources of vibrations, which may cause resonance. The system can thus appear to be in resonance by action of external vibration sources.

Statement of the problem:

The motion of a rigid cylindrical inclusion, which has an elastically suspended mass in its inner is studied. It is supposed that the cylinder moves continuously in the medium. See fig 1.

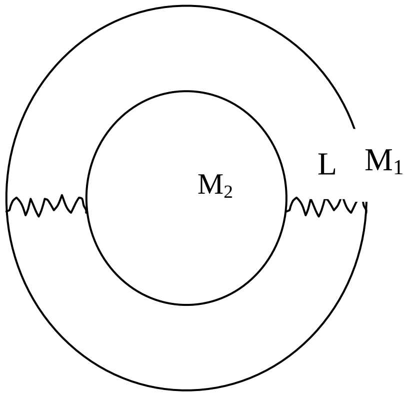

Fig.1

The considered problem is flat. The irrotational motion of the medium in the acoustic statement is described by the equation:

$$\Delta\varphi = \frac{1}{a^2}\frac{\partial^2\varphi}{\partial t^2}, \tag{1}$$

where a is velocity of sound distribution, φ - is velocity potential $\vec{\vartheta} = grad\varphi$,

$\vec{\vartheta}$ -velocity , Δ - Laplace's operator. After the passage of the wave, if it is possible to ignore some transient phenomena, diffraction, the unmoving inclusion appears at the initial time to be surrounded by the medium that moves in the same direction with the certain velocity. According to the principle of relativity we can consider the medium as unmoving one and the inclusion has the velocity of the fluid. According to the principle of relativity we can consider that the medium is an unmoving one and the inclusion has the velocity of the fluid.

The inclusion moves by the law

$$\begin{cases} M_1\dfrac{d^2x_1}{dt^2} = P + L(x_2 - x_1) \\ M_2\dfrac{d^2x_2}{dt^2} = -L(x_2 - x_1) \end{cases} \tag{2}$$

M_1 – cage mass, M_2 – the mass of the spring body , x_1 –replacement of the cage, x_2 – replacement of the spring body, L – rigidity of the spring , P –power of the fluid's act to the inclusion.

For the cylindrical inclusion with radius r_0

$$P = r_0\int_0^{2\pi} p\cos\theta d\theta, \qquad \text{where} \qquad p = -\rho\frac{\partial\varphi}{\partial t}, \tag{3}$$

$\rho -$ density, θ - polar angle.

The condition of equality of normal components of the velocity to the cage surface which are the components of the velocity of the fluid and the cage, has this view:

$$\frac{\partial \varphi}{\partial r} = \frac{dx_1}{dt} \cos\theta \tag{4}$$

The reformed, the problem for equation (1) with boundary conditions (2) is considered taking into account (3) and (4) and initial conditions

$$\varphi\big|_{t=0} = 0 \qquad \frac{\partial \varphi}{\partial t}\bigg|_{t=0} = 0 \tag{5}$$

Equation (1) in cylindrical coordinates has this view:

$$\frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial \varphi}{\partial r} + \frac{1}{r^2} \cdot \frac{\partial^2 \varphi}{\partial \theta^2} = \frac{1}{a^2} \frac{\partial^2 \varphi}{\partial t^2} \tag{6}$$

Solution of equation (6) can be given in the form:

$$\varphi(\mathrm{r}, \theta, \mathrm{t}) = \varphi_1(\mathrm{r}, \mathrm{t}) \cos\theta \tag{7}$$

Taking into account the expression for Laplace's operator the correlation (6) takes the form:

$$\frac{\partial^2 \varphi_1}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial \varphi_1}{\partial r} - \frac{1}{r^2} \cdot \varphi_1 = \frac{1}{a^2} \cdot \frac{\partial^2 \varphi_1}{\partial t^2} \tag{8}$$

After Laplace-Carson transformation equation (8) will be

$$\overline{\varphi}_1^{//} + \frac{1}{r} \overline{\varphi}_1^{/} - \left(\frac{p^2}{a^2} + \frac{1}{r^2} \right) \overline{\varphi}_1 = 0 \tag{9}$$

Solution of equation (9) at the condition of restriction of infinity has the form

$$\overline{\varphi}_1 = CK_1\left(\frac{pr}{a} \right) \tag{10}$$

r-*d*istance from the cage center, K_1- Macdonald's function of the first order. Taking into account (7), it follows from (3) that

$$p = -\rho \frac{\partial \varphi_1}{\partial t} \cdot \cos\theta$$

$$P = -\rho\, r_0 \frac{\partial \varphi_1}{\partial t} \cdot \pi \tag{11}$$

Taking into account (11), (4) and(7) equations (2) will take the form:

$$\begin{cases} M_1 \dfrac{\partial^2 \varphi_1}{\partial r \partial t} + \rho\, r_0 \pi \dfrac{\partial \varphi_1}{\partial t} = L(x_2 - x_1) \\ M_2 \dfrac{\partial^2 x_2}{\partial t^2} + L(x_2 - x_1) = 0 \end{cases} \tag{12}$$

The later ones after Laplace- Carson transformation have the form:

$$\begin{cases} pM_1\left(\overline{\varphi_1}' - \dot{x}_0\right) + \rho\, r_0 \pi\; p\, \overline{\varphi_1} = L\left(\overline{x_2} - \overline{x_1}\right) \\ M_2\left(p^2 \overline{x_2} - p\dot{x}_0\right) + L\left(\overline{x_2} - \overline{x_1}\right) = 0 \end{cases} \tag{13}$$

$\dot{x}_0$ - initial velocity of the inclusion. Taking into account (10) from (13) we get

$$C = \frac{-\left(M_1 M_2 p^2 + M_1 L + M_2 L\right)\cdot \dot{x}_0}{\left[\left(M_1 M_2 p^2 + M_1 L + M_2 L\right)\cdot\left(\frac{p}{a}K_0 + \frac{1}{r}K_1\right) - \rho\, r_0 \pi\;\; K_1\left(M_2 p^2 + L\right)\right]} \tag{14}$$

K_0-Macdonald's function of the zero order.

We rewrite expression (14) in the form:

$$C = \frac{-r_0 \cdot \dot{x}_0}{\frac{pr_0}{a}K_0 + K_1 + \left(1 + \frac{M_2 L}{M_1 M_2 p^2 + M_1 L + M_2 L}\right)\frac{\rho\, r_0^2 \pi}{M_1}\;\; K_1} \tag{15}$$

To find the original denominator in (15) we need the following originals

$$PK_0\left(\frac{pr}{a}\right) \to \frac{a}{r\sqrt{\left(\frac{at}{r}\right)^2 - 1}}; \qquad PK_1\left(\frac{pr}{a}\right) \to \sqrt{\left(\frac{at}{r}\right)^2 - 1};$$

$$\frac{p}{p^2 + e^2} \to \frac{\sin et}{e}, \qquad t > \frac{r}{a}, \qquad e^2 = \left(\frac{1}{M_1} + \frac{1}{M_2}\right)L$$

Marking the original of all the denominator by z, and $\theta_1 = \frac{at}{r_0}$ we have

$$-z = \frac{1}{\sqrt{\theta_1^2 - 1}} + \left(1 - \frac{\rho r_0^2 \pi}{M_1}\right)\sqrt{\theta_1^2 - 1} + \frac{L\rho\pi r_0^2}{M_1^2 e}\int_0^t \sin(t-\eta)\, e \cdot \sqrt{\left(\frac{a\eta}{r_0^2}\right)^2 - 1}\, d\eta$$

By introducing $M_1 = \rho_*\, r_0^2 \pi, \qquad L = M_2 \omega^2, \qquad e = \omega\sqrt{\frac{M_2}{M_1} + 1}$ we have:

$$-z = \frac{1}{\sqrt{\theta_1^2 - 1}} + \left(1 - \frac{\rho}{\rho_*}\right)\sqrt{\theta_1^2 - 1} + \frac{\rho}{\rho_*}\cdot\frac{\omega\, r_0}{a\sqrt{\frac{M_1}{M_2} + \left(\frac{M_1}{M_2}\right)^2}}\cdot\int_1^{\theta_1}\sqrt{\theta_*^2 - 1}\;\sin\frac{r_0 e}{a}\left(\theta_1 - \theta_*\right)\, d\theta_* \tag{15^a}$$

In particular cases for some values of parameters of the problem the boundary functions are determined.

We take $M_1 = 0$. For this case expression (14) takes the form:

$$C = \frac{1}{p} \cdot \frac{-\dot{x}_0}{\frac{1}{a} K_0 - pK_1 \cdot \frac{\rho \pi r_0}{L} + \frac{K_1}{p}\left(-\frac{\rho \pi r_0}{M_2} + \frac{1}{r_0}\right)} \quad (16)$$

In (16) by denoting the denominator originals as *z*, we get:

$$z = \frac{1}{a} \cdot Arcch \frac{at}{r} - \frac{\rho \pi r_0}{L} \cdot \frac{a}{r_0} \cdot \frac{t}{\sqrt{t^2 - \frac{r_0{}^2}{a^2}}} + \left(-\frac{\rho \pi r_0}{M_2} + \frac{1}{r_0}\right)\left(\frac{at}{2r_0}\sqrt{t^2 - \frac{r_0{}^2}{a^2}} - \frac{r_0}{2a} Arcch \frac{at}{r_0}\right)$$

or

$$z = \frac{1}{a} \cdot Arcch\,\theta_1 - \frac{\rho \pi r_0}{L} \cdot \frac{\theta_1}{\sqrt{\theta_1{}^2 - 1}} + \left(-\frac{\rho \pi r_0{}^2}{M_2} + 1\right)\left(\frac{\theta_1}{2a}\sqrt{\theta_1{}^2 - 1} - \frac{1}{2a} Arcch\,\theta_1\right)$$

where $\theta_1 = \frac{at}{r_0}$

Let's denote $\frac{pC(p)}{\dot{x}_0} \to S(t)$

Taking into account $\frac{C \cdot p}{\dot{x}_0} \cdot \bar{z} = 1$, then by Borel's theorem:

$$\frac{d}{dt}\int_0^t S(t-\tau)\, z(\tau) d\tau = 1.$$

Henceweget :

$$\int_0^t S(t-\tau)\, z(\tau) d\tau = t + c_1$$

It is known that for $t \prec \frac{r_0}{a}$, $S = 0, \quad z = 0$

$$\int_{\frac{r_0}{a}}^t S(t-\tau)\, z(\tau) d\tau = t + c_1 \quad (17)$$

Taking into account $t = \frac{r_0}{a}$ the interval is equal to zero. In this case C_1 is determined as:

$$c_1 = -\frac{r_0}{a}$$

We introduce the following notations:

$$t = \frac{r_0 \theta_1}{a}$$

and find the originals of function C(p) using the integral equation of the first kind by Volterr

$$\int_1^{\theta_1} S(\theta_1 - \tau)\, z(\tau) d\tau = \theta_1 - 1$$

Let's consider the particular case. For $\rho = \rho_*, \qquad M_1 = M_2, \qquad e = \omega\sqrt{2}$

Equation (15[a]) takes the form:

$$-z=\frac{1}{\sqrt{\theta_1^2-1}}+2\sqrt{\theta_1^2-1}+\frac{\omega\, r_0}{a\sqrt{2}}\int_1^{\theta_1}\sqrt{\theta_*^2-1}\sin\frac{r_0\omega\sqrt{2}}{a}(\theta_1-\theta_*)\,d\theta_* \tag{18}$$

For example we take

$\frac{\omega\, r_0}{a}=\frac{1}{\sqrt{2}}$ where $\frac{a}{r_0}=\omega_1$ - eigen frequency of the medium in r_0, and then

$$-z_n=\frac{1}{\sqrt{\theta_1^2-1}}+2\sqrt{\theta_1^2-1}+\frac{1}{2}\int_1^{\theta_1}\sqrt{\theta_*^2-1}\sin(\theta_1-\theta_*)\,d\theta_* \tag{19}$$

For $\theta_1\approx 1$, $S\approx\frac{2}{\pi z}$ S -is found by the numerical method like in work [5] and potential φ is found by the formula

$$\frac{1}{a\dot{x}_0}\cdot\varphi_1=\int_{r/r_0}^{\theta_1}\frac{S(\theta_1-\theta_*)\theta_*}{\sqrt{\theta_*^{\,2}-\frac{r^2}{r_0^2}}}\,d\theta$$

The numerical analyze of the problem has been carried out.

Results:

In Fig. 2 and Fig. 3 the graphs of dependencies of the boundary function S and potential φ on the time for density values $\rho=1$ and 2 and the mass of the vibrating body $m=\frac{1}{M}=2;1;\frac{2}{3};0,5$

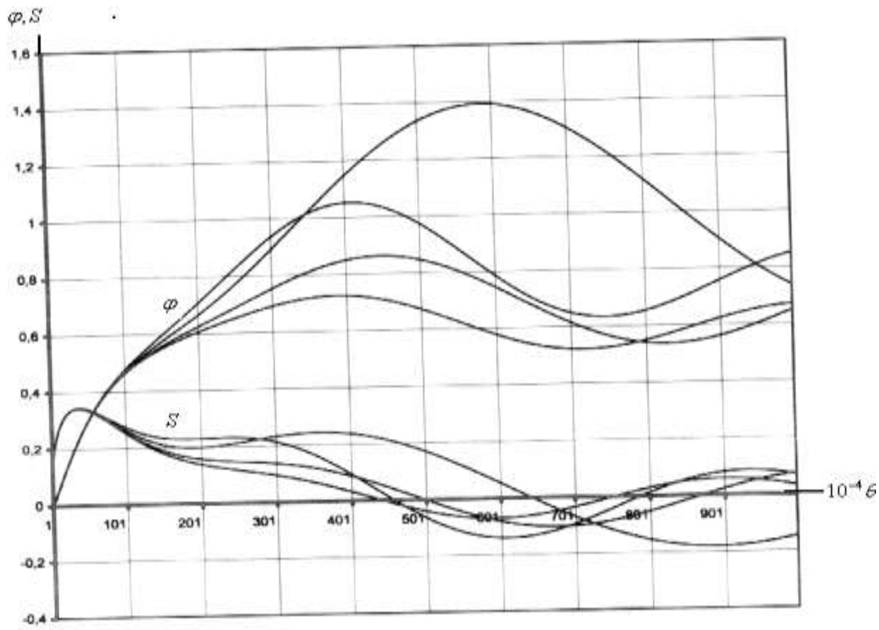

Fig. 2. Graphs of dependencies of the boundary function S and potential φ on the time for the density value $\rho=1$

 ISSN: 2321-8169
194 - 204

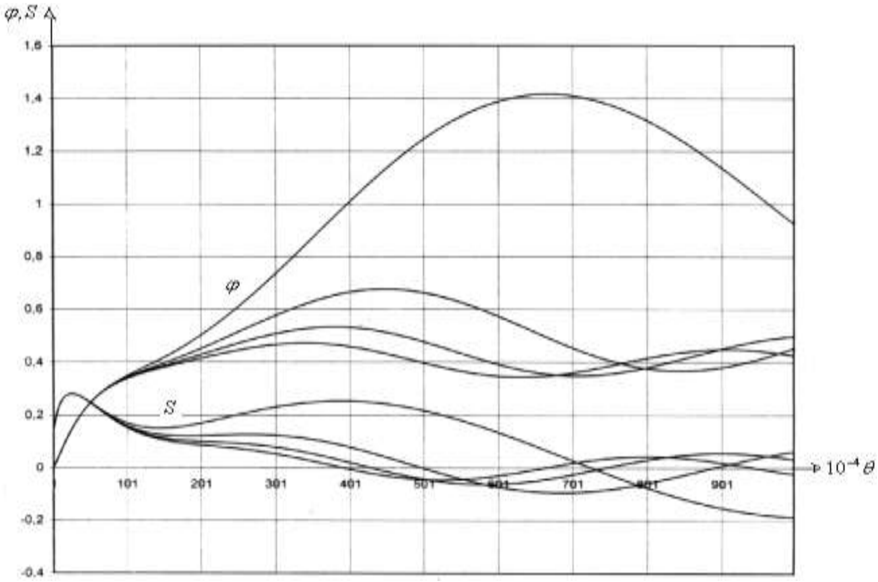

Fig. 3. Graphs of dependencies of the boundary function S and potential φ on the time for the density value $\rho = 2$

Discussion:

As is obvious from the graphs, function *S* vibrates after the initial deflection, with attenuation approaching to zero, but with vibrations potential φ approaches to the constant value, for each density ρ and at different relative internal masses $\frac{1}{M}$. At relative mass $\frac{1}{M}$ the functions vibrate with smaller amplitude.

Along with the study of unsteady (non-stationary) motion of an elastic medium, the investigation of eigen oscillations of the above described system is of interest. Due to that, in the present work, eigen oscillations of the above described system in an acoustic medium are also investigated.

The considered problem is flat; the motion of the fluid is potential. Equation of the motion of the fluid is a wave equation (1). In the area of contact of the fluid with the moving cylinders the normal components of the velocity of the fluid and the inclusion are equal. By considering the harmonic vibrations of the system the transcendental equation for frequencies is obtained. The problem is solved by the inverse method as reported by Mamedova et.al. (2013), Seyfullayev et.al. (2012; 2014) and Agalarova (1997), i.e. frequency of the system without any fluid (oscillator) is expressed analytically by the frequency of the main system, not solving the transcendental equations.

Sinyavskii et.al. (1980) have reported that for calculation of eigen frequencies and amplitudes of vibrations of the elastic element in the fluid, for example, of the cylindrical fuel element in nuclear reactor or the tube of heat exchanger device, it is necessary to know the measure of the fastened (added) mass and the damping force. Moreover, these properties depend on the location of unmoving borders surrounding the cylinder as reported by Ibragimovov et. al.(1975) and Chen et.al. (1976). That is why, supposing that the medium from the external side is restricted by unmoving surface $r = r_1$ or in the case of the unrestricted medium on the surface $r = r_1$ there is the node of standing wave, we will have the condition

$$\vartheta\big|_{r=r_1} = \frac{\partial \varphi}{\partial r}\bigg|_{r=r_1} = 0 \tag{20}$$

In order to find eigen frequencies of the system we use the method of separation of variables. In this way, we can illustrate the solution of equation (6) in the form:

$$\varphi(r,\theta,t) = R(r)e^{i\omega t}\cos\theta \tag{21}$$

$$x_1 = Be^{i\omega t}; \quad x_2 = Ce^{i\omega t}$$

ISSN: 2321-8169
194 - 204

where B, C are unknown which should be determined. Taking into account (21) in(6), equation (6) takes the form:

$$R'' + \frac{1}{r_1} R' + \left(1 - \frac{1}{r_1^{\,2}}\right) R = 0 \tag{22}$$

where $r = \frac{a}{\omega} r_1$

Solution of equation (22) has the following form as reported by Smirnov (1974) :

$$R = EJ_1\left(\frac{\omega r}{a}\right) + DN_1\left(\frac{\omega r}{a}\right) \tag{23}$$

where E, D are unknown which should be determined.

Here $J_1\left(\frac{\omega r}{a}\right), N_1\left(\frac{\omega r}{a}\right)$ are cylindrical function of Bessel and Neumann.

Putting (21) into (2) and in (4) and taking into account (3) we get the system of algebraic homogeneous equations in relation to constants E, B, C, D:

$$\begin{cases} \left(M_1\omega^2 - L\right)B + \rho r_0 \pi R i\omega + LC = 0 \\ LB + C\left(M_2\omega^2 - L\right) = 0 \\ R' - i\omega B = 0 \\ R' = 0 \end{cases} \tag{24}$$

For the existing of non-trivial solutions of system (24) we take the main determiner of the named system to zero

$$\begin{vmatrix} \rho r_0 \pi i\omega J_1\left(\frac{r_0\omega}{a}\right) & M_1\omega^2 - L & L & \rho r_0 \pi i\omega N_1\left(\frac{r_0\omega}{a}\right) \\ 0 & L & M_2\omega^2 - L & 0 \\ J_1'\left(\frac{r_0\omega}{a}\right) & -i\omega & 0 & N_1'\left(\frac{r_0\omega}{a}\right) \\ J_1'\left(\frac{r_1\omega}{a}\right) & 0 & 0 & N_1'\left(\frac{r_1\omega}{a}\right) \end{vmatrix} = 0 \tag{25}$$

As a result we get the frequency equation :

$$J_1'\left(\frac{\omega r_0}{a}\right)N_1'\left(\frac{\omega r_1}{a}\right)\left(M_1M_2\omega^4 - L_2\omega^2\left(M_1 + M_2\right)\right) - J_1'\left(\frac{\omega r_1}{a}\right)N_1'\left(\frac{\omega r_0}{a}\right)\left(M_1M_2\omega^4 - L_2\omega^2\left(M_1 + M_2\right)\right) -$$
$$- J_1'\left(\frac{\omega r_1}{a}\right)N_1'\left(\frac{\omega r_0}{a}\right)\rho\pi r_0\left(M_2\omega^2 - L\right)\omega^2 = 0 \tag{26}$$

We introduce the following denotations:

$$\frac{L}{M_1} = k_1^2, \quad \frac{L}{M_2} = k_2^2, \quad \frac{L}{\rho r_0^2 \pi} = k_0^2, \; m = \frac{\rho\pi r_0}{M_1}$$

Then equation (26) takes the following form:

$$\left(-J_1'\left(\frac{\omega r_1}{a}\right)N_1'\left(\frac{\omega r_0}{a}\right)\right)\left(\omega^4-\omega^2\left(-2k_3^2\right)+k_1^2k_2^2\right)-m\frac{1}{r_0}\left(\omega^4-\omega^2k_2^2\right)=0$$

$$J_1'\left(\frac{\omega r_0}{a}\right)N_1'\left(\frac{\omega r_1}{a}\right)\left(\omega^2-k_1^2-k_2^2\right)-J_1'\left(\frac{\omega r_1}{a}\right)N_1'\left(\frac{\omega r_0}{a}\right)\left(\omega^2-k_1^2-k_2^2\right)-J_1'\left(\frac{\omega r_1}{a}\right)N_1'\left(\frac{\omega r_0}{a}\right)m\frac{1}{r_0}\left(\omega^2-k_2^2\right)=0 \quad (27)$$

We begin to use dimensionless quantities and for that we introduce the following denotations:

$$\frac{\omega r_0}{a}=\overline{\omega},\quad \overline{k}_1=\frac{k_1r_0}{a},\quad \overline{k}_2=\frac{k_2r_0}{a},\quad \overline{k}_0=\frac{k_0r_0}{a},\frac{\omega r_1}{a}=\overline{\omega}_1$$

In the result equation (27) takes the form:

$$\overline{k}=\sqrt{\frac{F_1}{F_2}} \quad (28)$$

where

$$F_1=c\overline{\omega}^4\left(J_0(\overline{\omega})N_0(c\overline{\omega})-J_0(c\overline{\omega})N_0(\overline{\omega})\right)+\overline{\omega}^3\left(J_1(c\overline{\omega})N_0(\overline{\omega})-J_0(\overline{\omega})N_1(c\overline{\omega})\right)+$$
$$+c\overline{\omega}^3\left(J_0(c\overline{\omega})N_1(\overline{\omega})-J_1(\overline{\omega})N_0(c\overline{\omega})\right)+\left(J_1(\overline{\omega})N_1(c\overline{\omega})-J_1(c\overline{\omega})N_1(\overline{\omega})\right)+$$
$$+c\overline{\omega}^3mJ_0(c\overline{\omega})N_1(\overline{\omega})-\overline{\omega}^2J_1(c\overline{\omega})N_1(\overline{\omega})$$

$$F_2=-c\overline{\omega}\left(J_0(c\overline{\omega})N_0(\overline{\omega})-J_1(c\overline{\omega})N_1(\overline{\omega})\right)m-\left(1+b^2\right)\left(\overline{\omega}^3c\left(J_0(\overline{\omega})N_0(c\overline{\omega})-J_0(c\overline{\omega})N_0(\overline{\omega})\right)\right)+$$
$$+\overline{\omega}\left(J_1(c\overline{\omega})N_0(\overline{\omega})-J_0(\overline{\omega})N_1(c\overline{\omega})\right)+c\overline{\omega}\left(J_0(c\overline{\omega})N_1(\overline{\omega})-J_1(\overline{\omega})N_0(\overline{\omega})\right)+\left(J_1(\overline{\omega})N_1(c\overline{\omega})-J_1(c\overline{\omega})N_1(\overline{\omega})\right)$$

$$\overline{k}_1=b\overline{k}_2 \qquad \overline{k}_2=\overline{k} \qquad \overline{k}_1=b\overline{k} \qquad \overline{\omega}_1=c\overline{\omega}$$

Equation (28) connects the free frequency of the system with the free frequency of the inclusion without any fluid in it. The finding of the frequencies of eigen oscillations of the system is connected in the whole to the solution of transcendental equation (27).

For solving the transcendental equation authors use in general approximate methods, in particular to asymptotical ones. However, solution of the inverse problem allows to build the spectrum of graphs which simplify the investigation, including the determining of frequency.

For some values of parameters of the problem at the interval $0-250$ for $\overline{k}$ and $0-0{,}6$ for $\vec{\omega}$ the graphs $\overline{k}-\overline{\omega}$ (Fig. 4) are built where $\overline{\omega}=A$

$\overline{k}=X(A),\ b=2,\ m=1,\quad c=10,,$

$\overline{k}=Y(A),\ b=2,\ m=1,\quad c=20,$

$\overline{k}=Z(A),\ b=2,\ m=1,\quad c=30,$

$\overline{k}=N(A),\ b=2,\ m=1,\quad c=40,$

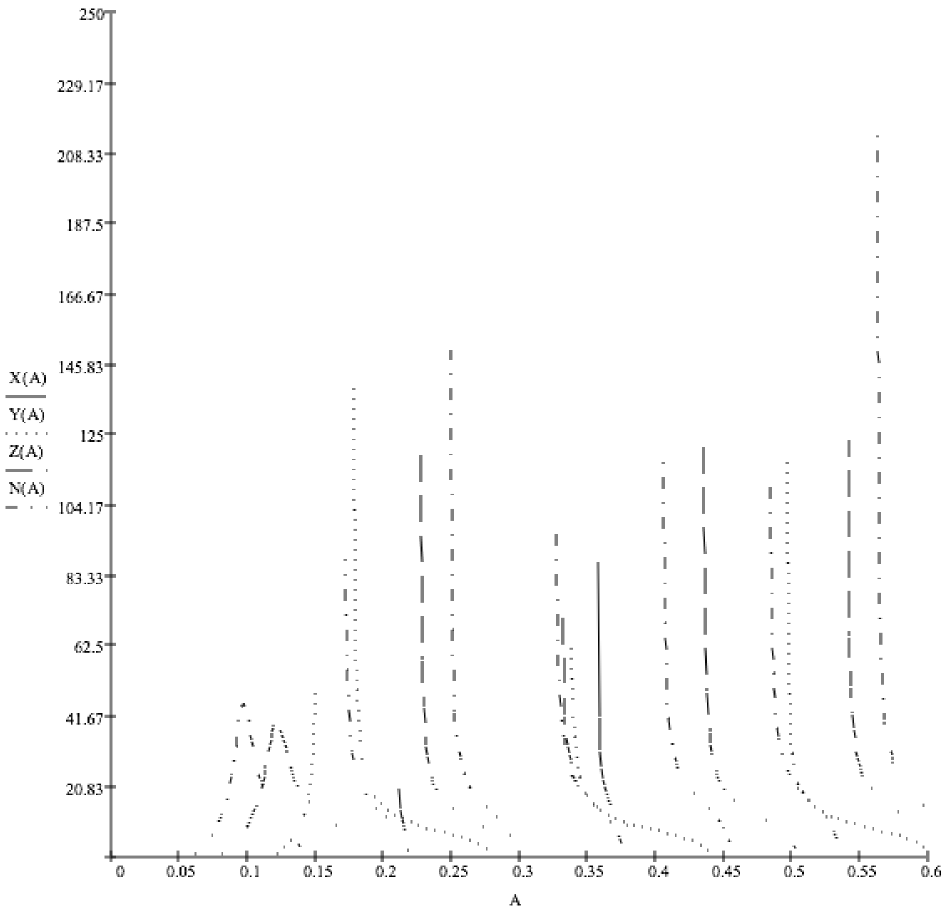

Fig. 4. Graph of dependence of the frequency of the oscillator $\bar{k}$ on the frequency of the system $\vec{\omega}$.

It is obvious from the picture that on the first mode by increasing of $\vec{\omega}$, $\bar{k}$ increases. In the following modes by increasing of $\vec{\omega}$, $\bar{k}$ decreases. That can be explained by the counter (oncoming) motion of the cylinder and the medium. The number of curves preceding the considered curve corresponds to number of wave nodes in the medium.

As it is obvious from Figure 4-the graphs of various vibration modes have vertical asymptote, the abscissas of which responds to the node surfaces of the standing wave. In order to determine frequencies of the system at the given frequency of vibrations of the oscillator $\bar{k}$ in the graph $\bar{k}-\overline{\omega}$ (Fig. 4) the horizontal straight line is held with ordinate $\bar{k}$ and by measuring the branches of the graph $\bar{k}-\overline{\omega}$, we obtain the quantities $\overline{\omega}_k$, by the consequence of which it is possible to calculate the values $\omega_k = \dfrac{\overline{\omega}_k a}{r_0}$ of the spectrum of frequencies of eigen oscillations of the system.

This investigation can be carried out not only graphically but also analytically, for example, by differentiating the expression (28) by $\vec{\omega}$ and defining the character of frequency changes of the system. The graphs $\bar{k}-\overline{\omega}$ lets us analyze the character of the dependence of the frequency on the rigidity of the spring constant of the oscillator.

Acknowledgement:

The research of A. I. Seyfullayev is supported by the grant of Presidium of Azerbaijan National Academy of Science 2015.

References:

[1] Huang H., Lu Y.P., Wang Y.FF. Transient interaction of spherical acoustic waves of a cylindrical elastic shell and its internal multi-degree-of –freedom mechanical systems. J. Acoust. Soc. America 1974 vol 56, №1. p. 4-10.
[2] Gorshkov A.G., Tarlakovskii D.V. Unsteady aero hydro elasticity of bodies with spherical form. Moscow. Nauka. Mailedit. Phys. andMaтh. Met. 1990, pp. 264. ISBN-5-20-14006-6.

[3] Limarchenko O.S., Tkachenko R.V. Effect of a spring fastened body to the dynamics of the cylindrical fluid reservoir located in the moving platform. International Scientific Journal. AppliedMechanics, ISSN 0032-8243, volume50, №3, 2014, pp. 69-79
[4] SeyfullayevA.I., Agayeva N.A. Solution of the problem on motion of the spherical inclusion with the spring body in the acoustic medium. Transactions of the National Academy of Sciences of Azerbaijan, Phys.–tech-and math.sciences,1998. Vol. 18 №2,pp. 133-135.
[5] Mamedova G.A, Rustamova M. A., Agasiyev S.P. Investigation of free oscillations of the spherical shell with fluid by inverse method. Eastern European Journal ofadvancedtechnologies, 6/7 (66), 2013,pp.16-20.
[6] SeyfullayevA. I., Mamedova G.A.,RustamovaM. A., YuzbashiyevaA.O. Analyzesoff reeoscialltions of thin-walled cylindrical shells containing a compressible liquid. Engeneering-Physical Journal, Volume.85, №6, 2012. Minsk
[7] Seyfullayev A. I, Rustamova M. A., Agasiev S.R. Free oscillations of two concentrically located cylindrical shells with a fluid between them. International Journal of Engineering and Innovative Technology (IJEIT), volume 3, Issue 10, April 2014, p.33-37
[8] Agalarova T.D. Interaction of acoustic wave with the oscillator. Collection of scinetific works on Mechanics,№7, Baku,1997,pp. 181-184
[9] Sinyavskii V.F, Phedotovskii V.S., Kukhtin A.B. On vibrations of the cylinder in a viscous liquid. Applied Mechanics, 1980, Vol.XVI,№1, pp.62-67.
[10] Ibragimovov M. Kh., Sinyavskii V. F., Belyakov A. G. et.al. Investigation of the added bodes and damping coefficients at vibrations of the cylinder in viscous fluids. Previous print ФЭИ-585.Obninsk,1975, pp. 23.
[11] Chen S.S., Wambsganss M. W., Jendrzejczyk J.A. Added mass and damping of a vibration rod in confined viscous fluid.-Trans. ASME. J. Appl. Mech.1976, 43,№2, p.325-329.
[12] Smirnov V. N. Course of Higher Mathematics. Vol.3, part.2, Moscow, Nauka, 1974, pp. 672.