Imbibition in Double Phase Flow Through Porous Media

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***Abstract*—** In this paper, the phenomenon of Imbibition in two immiscible phase flow through porous media is discussed. The Successive over Relaxation (S.O.R.) method is applied to solve the governing partial differential equation and the numerical results have been represented using graphs.

Keywords- *Imbibition, Porous Media, S.O.R.*

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I. Introduction

It is well known physical fact that when a porous medium is filled with some fluid which preferentially wets the medium then there is a spontaneous flow of the resident fluid from the medium. The phenomenon arising due to the difference in the wetting abilities of the fluid is called counter-current imbibition.

This phenomenon has been formally discussed by Graham and Richardson [1], Scheidegger [3], Verma and Rama Mohan [4], Mehta and Verma [5] and some other who have either drawn interfaces from the governing differential system or obtained numerical solutions. Verma [8] has considered the presence of heterogeneity in the medium marginally. He has obtained an approximate solution to determine the saturation distribution for imbibition phenomenon.

II. Statement of the Problem

We consider here that a finite cylindrical piece of homogenous porous matrix of length L (=1) is fully saturated with a native liquid (N). It is completely surrounded by an impermeable surface except for one end is exposed to an adjacent formation of injected liquid (I). It is assumed that injected water is preferentially more wetting than that of native liquid (oil) and this arrangement give rise to the phenomenon of linear counter-current imbibition, that a spontaneous linear flow of water into the medium and a counter flow of the resident fluid (oil) from the medium.

The governing laws and governing equations to this phenomenon and basic assumptions give rise to the partial differential equation, which has been solved by Successive over Relaxation Method.

III. Mathematical Formulation of the Problem

A. *Fundamental Equation of the Problem*

Assuming the validity of Darcy's law, which is governing law , the equations of seepage velocity of flowing fluids may be written as:

$$V_w = -\frac{k_w}{\mu_w} k \frac{\partial P_w}{\partial x} \qquad (1)$$

$$V_o = -\frac{k_o}{\mu_o} k \frac{\partial P_o}{\partial x} \qquad (2)$$

Where V_w and V_o are seepage velocity of water and oil respectively, k is the permeability of the homogeneous medium, k_w and k_o are relative permabilities of water and oil respectively, P_w and P_o are the pressures and μ_o and μ_w are viscosities of water and oil respectively.

The equations of continuity for the flowing phase are:

$$\varphi \frac{\partial S_w}{\partial t} + \frac{\partial V_w}{\partial x} = 0 \qquad (3)$$

$$\varphi \frac{\partial S_o}{\partial t} + \frac{\partial V_o}{\partial x} = 0 \qquad (4)$$

Where φ is the porosity of the medium and S_w and S_o are water and oil saturation respectively.

An analytic condition, governing imbibition phenomenon is given by

$$V_w = -V_o \qquad (5)$$

$$P_c = P_o - P_w \qquad (6)$$

B. *Equation for Motion for Saturation*

Combining equations (1), (2) and (5), we get

$$\frac{k_o}{\mu_o}\frac{\partial P_o}{\partial x} + \frac{k_w}{\mu_w}\frac{\partial P_w}{\partial x} = 0 \qquad (7)$$

By using the condition for capillary pressure, equation (7) becomes,

$$\frac{k_o}{\mu_o}\left\{\frac{\partial P_c}{\partial x} + \frac{\partial P_w}{\partial x}\right\} + \frac{k_w}{\mu_w}\frac{\partial P_w}{\partial x} = 0$$

$$\therefore \frac{\partial P_w}{\partial x} = \frac{-k_o/\mu_o}{\left\{\frac{k_o}{\mu_o}+\frac{k_w}{\mu_w}\right\}}\frac{\partial P_c}{\partial x} \qquad (8)$$

Substituting (1) into (3), we get

$$\varphi \frac{\partial s_w}{\partial t} - \frac{\partial}{\partial x}\left\{\frac{k_w}{\mu_w} k \frac{\partial P_w}{\partial x}\right\} = 0 \qquad (9)$$

Combining equations (8) and (9), we get

$$\varphi \frac{\partial s_w}{\partial t} + \frac{\partial}{\partial x}\left\{k \frac{k_o k_w}{k_o \mu_w + k_w \mu_o}\left(\frac{dP_c}{dS_w}\right)\left(\frac{\partial S_w}{\partial x}\right)\right\} = 0 \qquad (10)$$

Letting $D(S_w) = \frac{k_0 k_w}{k_0\mu_w + k_w\mu_0}$

Now using the condition for capillary Pressure depending upon phase saturation as

$$P_c = -\beta S_w \quad (11)$$

Equation (10) can be written as

$$\varphi \frac{\partial s_w}{\partial t} - \frac{\partial}{\partial x}\left[kD(s_w)\beta \frac{\partial s_w}{\partial x}\right] = 0 \quad (12)$$

With $s_w(0,t) = s_{w_0}$, $s_w(L,t) = s_{w_l}$ (13)

$\frac{\partial}{\partial x} s_w(L,t) = 0, 0 \le x \le L$ (14)

Let $k = \frac{\varphi_{ef}^3}{c\tau^2 s^2}$ where $s = \frac{\varphi}{R_0(1-\varphi)}$

Where R_0 is the hydraulic radius,

c is the Kozeny constant

$\tau = \left(\frac{L_e}{L}\right)$ istortousity

L_e is the effective length of the path of the fluid

Now $\varphi_{ef} = \epsilon\varphi_T$, where ϵ is the connectivity or fraction of total porosity contained in pathways that are connected across a sample (ranges from 0 to 1) and φ_T is the total porosity of the sample.

Assume that $D(s_w)=\bar{D}(s_w)$ is a constant.

$$\frac{\partial s_w}{\partial t} - \frac{\beta\bar{D}(s_w)\epsilon^3 R_0^{\,2}(1-\varphi)^2}{c\tau^2}\frac{\partial^2 s_w}{\partial x^2} = 0 \quad (15)$$

Let $X = \frac{x}{L}$, $T = \frac{\beta\bar{D}(s_w)\epsilon^3 R_0^{\,2}(1-\varphi)^2}{c\tau^2}t$

$$\frac{\partial s_w}{\partial T} - \beta\frac{\partial^2 s_w}{\partial X^2} = 0 \quad (16)$$

With $(0,T) = s_{w_0}$, $s_w(1,T) = s_{w_l}$ (17)

$\frac{\partial}{\partial X} s_w(1,T) = 0, 0 \le X \le 1$ (18)

IV. MATHEMATICAL SOLUTION

Using S.O.R. method [9-11], we have

$$s_{w_{i,j+1}} = s_{w_{i,j}} + \frac{\beta k}{2h^2}\left(s_{w_{i+1,j}} - 2s_{w_{i,j}} + s_{w_{i-1,j}} + s_{w_{i+1,j+1}} - 2s_{w_{i,j+1}} + s_{w_{i-1,j+1}}\right)$$

Let $r = \frac{k}{h^2}$,

$$c_m = s_{w_{i,j}} + \frac{\beta r}{2}\left(s_{w_{i+1,j}} - 2s_{w_{i,j}} + s_{w_{i-1,j}}\right)$$

$$s_{w_{i,j+1}} = (1-\omega)s_{w_{i,j}} + \omega\left[\frac{\beta r}{2(1+\beta r)}\left(s_{w_{i+1,n}} + s_{w_{i-1,j+1}}\right) + \frac{c_m}{(1+\beta r)}\right]$$

Choose k = 0.1, h=0.1, β=0.05, $\omega = 1.5$, $s_{w_0} = 1$, $s_{w_l} = 0$

$$s_{w_{i,j+1}} = -0.5s_{w_{i,j}} + 1.5\left[0.1667\left(s_{w_{i+1,n}} + s_{w_{i-1,j+1}}\right) + \frac{c_m}{1.5}\right]$$

Where $c_m = 0.5s_{w_{i,j}} + 0.25\left(s_{w_{i+1,j}} + s_{w_{i-1,j}}\right)$

x	T=0.1	T=0.2	T=0.3	T=0.4
	s_w			
0	1	1	1	1
0.1	0.5005	0.562576	0.64078	0.676949
0.2	0.125038	0.281432	0.353763	0.41771
0.3	0.031266	0.105773	0.176469	0.2030967
0.4	0.008283	0.035301	0.076148	0.116648
0.5	0.002071	0.011157	0.029552	0.053543
0.6	0.000518	0.003372	0.010674	0.022599
0.7	0.00013	0.000989	0.003645	0.008916
0.8	0.000032	0.000265	0.001193	0.003323
0.9	0.000008	0.000067	0.000365	0.001129
1	0	0	0	0

V. GRAPHICAL REPRESENTATION

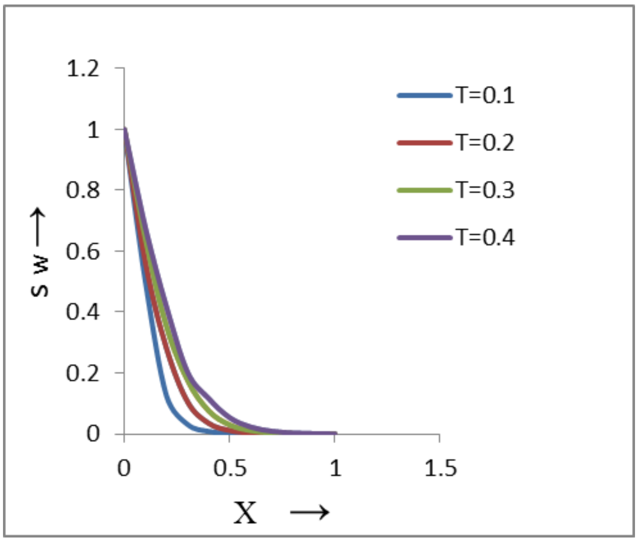

Figure-a

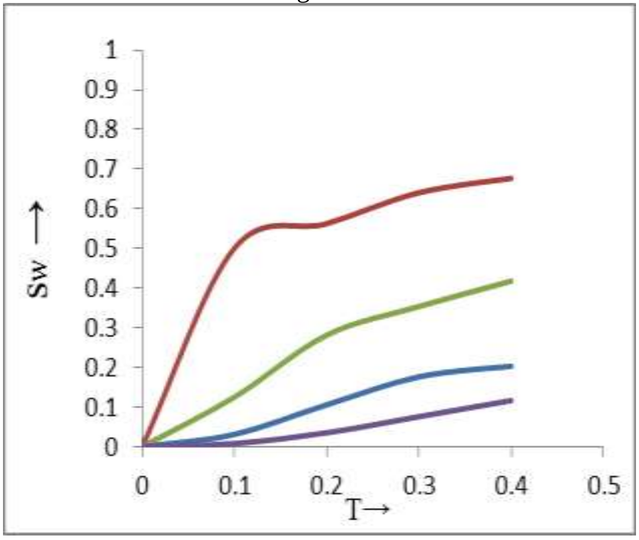

Figure-b

VI. INTERPRETATION

From figure-a, we can say that as x increases the saturation (S_w) decreases parabolically. Also the governing equation is parabolic. From figure -b, it is clear that as T increases saturation (S_w) increases.

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