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Cordial Labeling in Context of Ring Sum of Graphs

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Abstract:- A function f from vertex set V of a graph G to the set {0, 1} is called cordial labeling if the edge labels produced by absolute difference of the labels of end vertices of the respective edges in such a way that the number of edges with label 0 and 1 differ by atmost 1 and similarly the number of vertices with label 0 and 1 differ by atmost 1. A graph which admits cordial labeling is called cordial graph. In this paper we have derived cordial labeling of ringsum of different graphs.

Key words: *- cordial labeling, ring Sum. AMS Subject classification number: 05C78.*

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I. INTRODUCTION

Throughout this paper, a graph G = (V, E) is a undirected, finite, connected, and simple graph with vertex set V and edge set E. For different notations and terminology we follow Gross and Yellen[2].

For a graph G = (V, E), a function f from vertex set V to the set {0, 1} with an induced function f* from edge set E to the set {0, 1} given by f*(e = uv) = |f (u) − f (v)| is cordial labeling if the number of vertices with label 0 and 1 differ by atmost 1 and similarly the number of edges with label 0 and 1 differ by atmost 1. $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph which admits cordial labeling is called cordial graph. In this paper we have derived cordial labeling of ringsum of different graphs.

Definition 1. Ring sum of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, denoted by $G_1 \oplus G_2$, is the graph, $G_1 \oplus G_2 = ((V_1 \cup V_2), ((E_1 \cup E_2) - (E_1 \cap E_2)))$.

Cahit[3] introduced cordial labeling of graphs and derived various results on cordial graphs. Ghodasara and Sonchhatra[4] proved that the the graph obtained by joining two copies of fan graph by a path of arbitrary length is 3-equitable. They also prove similar results for wheel, helm, gear and cycle with one pendant edge. Bapat and Limaye[5] proved that Helms Hn, n ≥ 4 are cordial. Youssef[6] proved that $Wn = Cn + K_1$ is cordial for all n ≥ 4. Vaidya et al.[7] proved that the graphs obtained by joining apex vertices of two shells to a new vertex is cordial and cordial. A survey on different graph labeling techniques is given by Gallian[1].

II. MAIN RESULTS

Theorem 1. $Cn \oplus K_{1,n}$ is cordial for all n.

Proof. Let $V(G) = V_1 \cup V_2$, where $V_1 = \{u_1, u_2, \ldots, u_n\}$ be the vertex set of Cn and $V_2 = \{v = u_1, v_1, v_2, \ldots, v_n\}$ be the vertex set of $K_{1,n}$. Here $v_1, v_2, \ldots, v_n$ are pendent vertices. Also |V (G)| = |E (G)| = 2n.
The labeling f: V (G) → {0, 1} is defined as per the following cases.

Case 1: n ≡ 0(mod4)
$f(u_i) = 0$; if i ≡ 0, 1(mod4)
= 1; if i ≡ 2, 3(mod4)
$f(v_i) = 0$; if i ≡ 0, 1(mod4)
= 1; if i ≡ 2, 3(mod4)

Case 2: n ≡ 0, 1(mod4)
$f(u_i) = 0$; if i ≡ 2, 3(mod4)
= 1; if i ≡ 3, 4(mod4)
$f(v_i) = 0$; if i ≡ 2, 3(mod4)
= 1; if i ≡ 0, 1(mod4)

Case 3: n ≡ 2(mod4)
$f(u_i) = 0$; if i ≡ 1, 2(mod4)
= 1; if i ≡ 0, 3(mod4)
$f(v_i) = 0$; if i ≡ 0, 3(mod4)
= 1; if i ≡ 1, 2(mod4)

The labeling pattern defined in above cases satisfies the condition $|v_f(0) - v_f(1)| \leq 1$ and
$|e_f(0) - e_f(1)| \leq 1$ in each case which is shown in Table 1.
Hence, $Cn \oplus K_{1,n}$ is cordial.

Let n = 4a + b, where n ∈ N.

Table 1: Table for Theorem 1

b	vertex conditions	edge conditions
0,1,2,3	$v_f(0) = v_f(1)$	$e_f(0) = e_f(1)$

Example 1. cordial labeling of the graph $C_7 \oplus K_{1,7}$ is shown in Fig. 1 as an illustration for the Theorem 1. It is the case related to n ≡ 1(mod4).

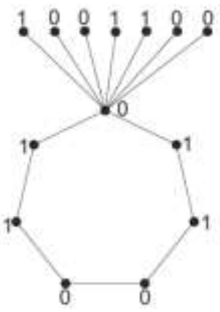

Fig. 1 : cordial labeling of the graph $C_7 \oplus K_{1,7}$.

Theorem 2. $G \oplus K_{1,n}$ is cordial, where G is cycle with one chord and chord forms a triangle with two edges of the cycle.

Proof. Let G be the cycle Cn with one chord and let $e = u_2u_n$ be the chord in G.
Let $V = V_1 \cup V_2$, where $V_1 = \{u_1, u_2, . . . ,u_n\}$ be the vertex set of G and
$V_2 = \{v = u_1, v_1, v_2, . . . ,v_n\}$ be the vertex set of $K_{1,n}$. Here $v_1, v_2, . . . ,v_n$ are pendent vertices.

The labeling $f : V \rightarrow \{0, 1\}$ is defined as per the following cases.

Case 1: $n \equiv 0(mod4)$
$f (u_i) = 0$; if $i \equiv 0, 1(mod4)$
$= 1$; if $i \equiv 2, 3(mod4)$
$f (v_i) = 0$; if $i \equiv 0, 1(mod4)$
$= 1$; if $i \equiv 2, 3(mod4)$

Case 2: $n \equiv 1, 3(mod4)$
$f (u_i) = 0$; if $i \equiv 0, 1(mod4)$
$= 1$; if $i \equiv 2, 3(mod4)$
$f (v_i) = 0$; if $i \equiv 2, 3(mod4)$
$= 1$; if $i \equiv 0, 1(mod4)$

Case 3: $n \equiv 2, 3(mod4)$
$f (u_i) = 0$; if $i \equiv 1, 2(mod4)$
$= 1$; if $i \equiv 0, 3(mod4)$
$f (v_i) = 0$; if $i \equiv 0, 3(mod4)$
$= 1$; if $i \equiv 1, 2(mod4)$

The labeling pattern defined in above cases satisfies the condition $|v_f (0) - v_f (1)| \leq 1$ and $|e_f (0) - e_f (1)| \leq 1$ in each case which is shown in Table 2.

Hence, $G \oplus K_{1,n}$ is cordial, where G is cycle with one chord.
Let $n = 4a + b$, where $n \in N$.

Table 2: Table for Theorem 2

b	vertex conditions	edge conditions
0,1	$v_f(0) = v_f(1)$	$e_f(0) + 1 = e_f(1)$
2,3	$v_f(0) = v_f(1)$	$e_f(0) = e_f(1) + 1$

Example 2. cordial labeling of ring sum of the graph the graph cycle C_8 with one chord and $K_{1,8}$ is shown in Fig. 2 as an illustration for the Theorem 2. It is the case related to $n \equiv 2(mod4)$.

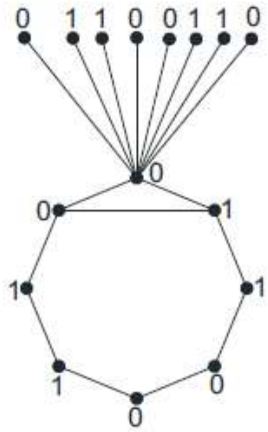

Fig. 2 : cordial labeling of ring sum of the graph cycle C_8 with one chord and $K_{1,8}$

Theorem 3. $G \oplus K_{1,n}$ is cordial, where G is cycle with triangle and chords forms three triangles and one cycle C_{n-5}, for all n.

Proof. Let G be the cycle Cn with triangle and let $e = u_2u_n$ and $e = u_3u_n$ be the chords in G.
Let $V = V_1 \cup V_2$, where $V_1 = \{u_1, u_2, . . . ,u_n\}$ be the vertex set of G and
$V_2 = \{v = u_1, v_1, v_2, . . . ,v_n\}$ be the vertex set of $K_{1,n}$. Here $v_1, v_2, . . . ,v_n$ are pendent vertices. Also $|V (G)| = |E (G)| = 2n$.

The labeling $f: V (G) \rightarrow \{0, 1\}$ is defined as per the following cases.

Case 1: $n \equiv 0(mod4)$
$f (u_i) = 0$; if $i \equiv 1, 2(mod4)$
$= 1$; if $i \equiv 0, 3(mod4)$

$f (v_i) = 0$; if $i \equiv 1, 2(mod4)$
$= 1$; if $i \equiv 0, 3(mod4)$

Case 2: $n \equiv 1, 2(mod4)$
$f (u_i) = 0$; if $i \equiv 1, 2(mod4)$
$= 1$; if $i \equiv 0, 3(mod4)$
$f (v_i) = 0$; if $i \equiv 0, 32, 5(mod4)$
$= 1$; if $i \equiv 1, 2(mod4)$

Case 3: $n \equiv 2(mod4)$
$f (u_i) = 0$; if $i \equiv 0, 1(mod4)$
$= 1$; if $i \equiv 2, 3(mod4)$
$f (v_i) = 0$; if $i \equiv 2, 3(mod4)$
$= 1$; if $i \equiv 0, 1(mod4)$

The labeling pattern defined in above cases satisfies the condition $|v_f (0) - v_f (1)| \leq 1$ and
$|e_f (0) - e_f (1)| \leq 1$ in each case which is shown in Table 1.
Hence, $G \oplus K_{1,n}$ is cordial.

Let n = 4a + b, where n ∈ N.

Table 3: Table for Theorem 3

b	vertex conditions	edge conditions
0,1,2,3	$v_f(0) = v_f(1)$	$e_f(0) + 1 = e_f(1)$

Example 3. cordial labeling of ring sum of the graph the graph cycle C_9 with twin chords and $K_{1,9}$ is shown in Fig. 3 as an illustration for the Theorem 3. It is the case related to n ≡ 3(mod4).

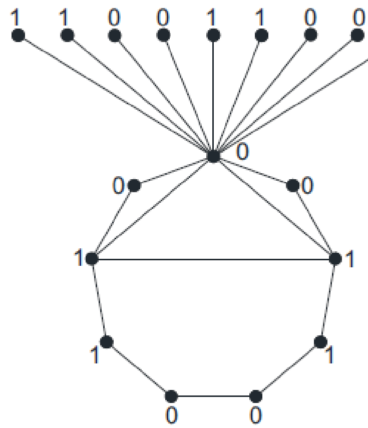

Fig. 3 : cordial labeling of ring sum of the graph cycle C_9 with triangle and $K_{1,9}$

Theorem 4. Wn ⊕ $K_{1,n}$ is cordial for all n.

Proof. Let V (G) = V_1 ∪ V_2, where $V_1 = \{u_0, u_1, u_2, . . . ,u_n\}$ with apex u_0 and $u_1, u_2, . . ., u_n$ are other vertices of Wn, and $V_2 = \{v = u_1, v_1, v_2, . . . , v_n\}$ be the vertex set of $K_{1,n}$.
Here $v_1, v_2, . . . ,v_n$ are pendent vertices. Also |V (G)| = |E (G)| = 2n + 1.
The labeling f: V (G) → {0, 1} is defined as per the following cases.

Case 1: n ≡ 0(mod4)
f (u_0) = 0
f (u_i) = 0; if i ≡ 1, 2(mod4)
= 1; if i ≡ 0, 3(mod4)
f (v_i) = 0; if i ≡ 1, 2(mod4)
= 1; if i ≡ 0, 3(mod4)

Case 2: n ≡ 1(mod4)
f (u_0) = 0
f (u_i) = 0; if i ≡ 0, 3(mod4)
= 1; if i ≡ 1, 2(mod4)
f (v_i) = 0; if i ≡ 0, 3(mod4)
= 1; if i ≡ 1, 2(mod4)

Case 3: n ≡ 2(mod4)
f (u_0) = 1
f (u_i) = 0; if i ≡ 0, 1(mod4)
= 1; if i ≡ 2, 3(mod4)
f (v_i) = 0; if i ≡ 1, 2(mod4)
= 1; if i ≡ 0, 3(mod4)

Case 4: n ≡ 3(mod4)
f (u_0) = 0
f (u_i) = 0; if i ≡ 0, 1(mod4)
= 1; if i ≡ 2, 3(mod4)
f (v_i) = 0; if i ≡ 1, 2(mod4)
= 1; if i ≡ 0, 3(mod4)

The labeling pattern defined in above cases satisfies the condition $|v_f(0) - v_f(1)| \le 1$ and
$|e_f(0) - e_f(1)| \le 1$ in each case which is shown in Table 4.
Hence, Wn ⊕ $K_{1,n}$ is cordial.

Let n = 4a + b, where n ∈ N.

Table 4: Table for Theorem 4

b	vertex conditions	edge conditions
0,2	$v_f(0) = v_f(1) + 1$	$e_f(0) = e_f(1)$
1	$v_f(0) + 1 = v_f(1)$	$e_f(0) = e_f(1) + 1$
3	$v_f(0) = v_f(1) + 1$	$e_f(0) + 1 = e_f(1)$

Example 4. cordial labeling of the graph W_{10} ⊕ $K_{1,10}$ is shown in Fig. 4 as an illustration for the Theorem 4. It is the case related to n ≡ 1(mod4).

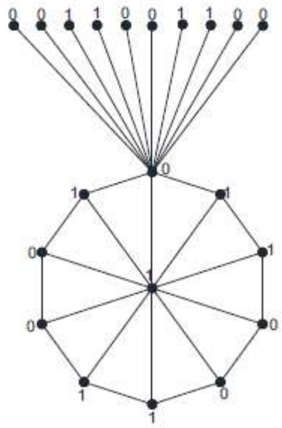

Fig. 4 : cordial labeling of the graph W_{10} ⊕ $K_{1,10}$

Theorem 5. Sn ⊕ $K_{1,n}$ is cordial for all n.

Proof. Let V (G) = V_1 ∪ V_2, where $V_1 = \{u_1, u_2, . . . ,u_n\}$ with apex u_1 and $u_2, u_3, . . .,u_n$ are other vertices of Sn and $V_2 = \{v = u_1, v_1, v_2, . . . ,v_n\}$ be the vertex set of $K_{1,n}$.
Here $v_1, v_2, . . . ,v_n$ are pendent vertices. Also |V (G)| = |E (G)| = 2n.
The labeling f: V (G) → {0, 1} is defined as per the following cases.

Case 1: n ≡ 0(mod4)
f (u_i) = 0; if i ≡ 1, 2(mod4)
= 1; if i ≡ 0, 3(mod4)
f (v_i) = 0; if i ≡ 1, 2(mod4)
= 1; if i ≡ 0, 3(mod4)

Case 2: n ≡ 1(mod4)
f (u_i) = 0; if i ≡ 0, 1(mod4)

= 1; if i ≡ 2, 3(mod4)
f (v_i) = 0; if i ≡ 0, 3(mod4)
= 1; if i ≡ 1, 2(mod4)

Case 3: n ≡ 2(mod4)
f (u_i) = 0; if i ≡ 0, 1(mod4)
= 1; if i ≡ 2, 3(mod4)
f (v_i) = 0; if i ≡ 0, 1(mod4)
= 1; if i ≡ 2, 3(mod4)

Case 4: n ≡ 3(mod4)
f (u_i) = 0; if i ≡ 0, 1(mod4)
= 1; if i ≡ 2, 3(mod4)
f (v_i) = 0; if i ≡ 2, 3(mod4)
= 1; if i ≡ 0, 1(mod4)

The labeling pattern defined in above cases satisfies the condition $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ in each case which is shown in Table 5. Hence, Sn ⊕ $K_{1,n}$ is cordial.

Let n = 4a + b, where n ∈ N.

Table 5: Table for Theorem 5

b	vertex conditions	edge conditions
0,2	$v_f(0) = v_f(1)$	$e_f(0) + 1 = e_f(1)$
1,3	$v_f(0) = v_f(1)$	$e_f(0) = e_f(1)$

Example 5. cordial labeling of the graph S_{10} ⊕ $K_{1,10}$ is shown in Fig. 5 as an illustration for the Theorem 5. It is the case related to n ≡ 4(mod4).

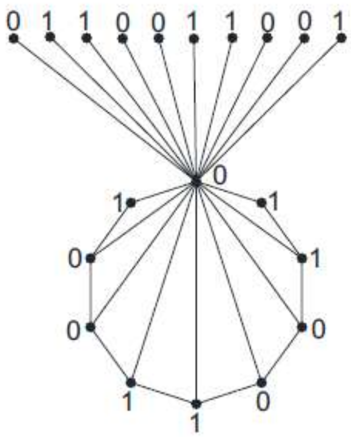

Fig. 5 : cordial labeling of the graph S_{10} ⊕ $K_{1,10}$

III. CONCLUSION

We have proved that the graph Cn ⊕ $K_{1,n}$, G ⊕ $K_{1,n}$, where G is cycle with one chord,
Cn,3 ⊕ $K_{1,n}$, Wn ⊕$K_{1,n}$, Sn ⊕ $K_{1,n}$ are cordial.

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