Performance Evaluation of MIMO OFDM System by using Adaptive MMSE Equalizer

Giligittha Swetha
M.Tech Final Year Student, ETM dept.
G. Narayanaamma Institute of Technology & Science (For Women), Shaikpet, Hyderabad, India
e-mail: swetha47g@gmail.com

Abstract—In wireless communication system, MIMO-OFDM has been proposed as an efficient technique for providing high data rates, low power consuming and very efficient in 3G and 4G technologies. Multiple-Input Multiple-Output (MIMO) & Orthogonal Frequency-Division Multiplexing (OFDM) systems have a assures ability to increase the capacity and performance with acceptable Bit Error Rate (BER) proportionally with the increased number of antennas and due to this, the system has grabbed the attention of wireless academic community in industry. AMMSE MIMO-OFDM is one of the prominent equalizer techniques which NULLify the noise by using the adaptive minimum mean square error detection. The speed of the adaptive detectors can be improved by using a partially filtered LMS (Adaptive) algorithm. The performance parameters of AMMSE MIMO-OFDM in terms of BER & Channel capacity is determined and it is analyzed for three different detector schemes such as ZF, MMSE and Adaptive MMSE MIMO OFDM System.

Keywords— Adaptive filter, Adaptive MMSE, LMS algorithm, ZF, MMSE.

I. INTRODUCTION

Wireless local area networks employ OFDM; it offers High Spectral efficiency, and superior tolerance to multipath fading. In OFDM computationally efficient Fast Fourier Transform is used to transmit data in parallel of orthogonal sub Carriers which is maintained even in frequency selective fading.

Multiple Input Multiple Output (MIMO) system leads to the high achievements of high data rate transmission without increasing the Bandwidth. The capacity of MIMO wireless channel can be approached by the use of spatial diversity. Bit Error Rate and capacity can be improved when the multiple antennas are applied at transmitter and receiver side and also with the help of Equalizer techniques at the receiver side.

Conventional approaches implements an equalizer to remove ISI (Inter-Symbol-Interference) detection. Equalizer needs arises from the channel which has phase dispersion and amplitude which results in the interface of transmitted signals with one another. It works when SNR (Signal to Noise Ratio) is high and BER (Bit Error Rate) is low. Equalizer gives the inverse of the channel to the received signal and flat frequency response. Negative effects of the channel can be reduced with the help of equalizer.

There are two types of equalizers:

- Linear Equalizer
- Non-Linear Equalizer

Zero Forcing Equalizer and Minimum Mean Square Error (MMSE) is a linear equalization techniques used in communication. Zero Forcing Equalizer inverse the frequency response of the channel. Minimum Mean Square Error (MMSE) minimizes the mean square error. The goal of MMSE is total power of noise can be minimized and output in the ISI (Inter Symbol Interference) components.

To overcome from the drawbacks of above two techniques this Adaptive MMSE linear equalization Technique is introduced. Adaptive MMSE (Minimum Mean Square Error) is an equalization filter. It automatically adapts the time varying properties of the common channel. It uses LMS (Least Mean Square) algorithm.

Section ii Contains Existing techniques, Section iii Contains Adaptive MMSE MIMO OFDM Equalizer technique & Section iv contains Result analysis & Section v contains Conclusion.

II. EXISTING TECHNIQUES

Zero Forcing & MMSE (Minimum Mean Square Error):

1) Zero forcing Equalizer (ZF):
ZF Equalizer is introduced by Robert Lucky which correlates to decrease the ISI to zero in a noise free case. It is used when ISI is compared to noise [9]. A channel with frequency response F(f) the zero forcing equalizer C(f) is formulated in a way that C(f) = 1 / F(f).

Flow Chart of Zero Forcing Equalizer:

\[ Y = HX + N \]
\[ YH^{-1} = X + H^{-1}N \]
\[ (Y - RX)^2 = Y - RX = Y - RXY - RX \]
\[ -2H^2Y + 2H^2HX = 0 \]
\[ X = (H^2H)^{-1}H^2Y \]

Fig: Flow Chart of Zero Forcing Equalizer
Where G is equalization matrix and \((H^H H)^{-1}\) is Pseudo inverse of the matrix.

As the Zero forcing Equalizer suffers from the problem of noise enhancement due to the least square estimation, another alternative Equalizer technique is concentrated to avoid the problem of noise enhancement i.e. known as MMSE equalizer technique.

2) MMSE (Minimum Mean Square Error) Equalizer:

The MMSE receiver suppresses noise and interference components, but whereas Zero Forcing receiver only removes the Inter Symbol Interference (ISI) component[14]. Here the keyword itself refers to minimizes Mean Square Error.

**Flow Chart of Minimum Mean square error**

![Flow Chart of Minimum Mean square error](image)

Where G is an Equalization matrix and \(H^H H\) is a Pseudo inverse of the signal.

**Advantages:**
- It minimizes the noise enhancement.
- It improves the BER performance

**Disadvantages:**
- It does not usually eliminate the ISI component.
- At high SNR, the MMSE equalizer reduces to Zero Forcing Equalizer.

To overcome from the disadvantage of above two techniques one of the Equalizer is introduced to minimize the Bit Error Rate performance and the channel capacity. It is known as Adaptive Minimum Mean Square Error (AMMSE) MIMO OFDM Equalizer.

**III. ADAPTIVE MINIMUM MEAN SQUARE ERROR(AMMSE) MIMO OFDM EQUALIZER**

In adaptive filter parameter change continuously due to received training sequence from transmitter. It informs receiver to adjust parameters of the filter to match a desired signal[5].

![Block Diagram of Adaptive MMSE MIMO OFDM](image)

It is one of the linear equalization technique. Adaptive filter uses the LMS algorithm. It was proposed by Prof.Windrow. In adaptive filters parameters are changed due to received sequence from transmitter. It informs receiver to adjust parameter of filter of desired signal. To approach adaptively update linear coefficients LMS algorithm is used. Adaptive algorithm is used to minimize adaptive MMSE filter, and to minimize the mean square error by steepest decent method is used.

AMMSE MIMO OFDM system model with \(N_t\) and \(N_r\) transmit and receive antennas with \(K\) sub carriers. At time \(t\) data block \(b[n, k]\) : \(K = 0,1,\ldots,n\) transformed into different signals \(x_1[n, k]\) : \(k = 0,\ldots, K-1\) and \(i=1,2,\ldots,n\) and are numbers of sub channels of OFDM System. The Adaptive MIMO OFDM Contains the Blocks. These are BPSK Modulation, IFFT, Channel estimator, FFT & Demodulation and Adaptive MMSE Blocks[17].

Consider the received signals symbols \(y_1[n, k]\), \(y_2[n, k]\) and \(y_3[n, k]\) and let their general form for any node and any path in the network is \(y_0[n, k] = r_s[m]\). Where \(n\) is the specific number assigned to the signals at nodes. \(y_1[n, k]\), \(y_2[n, k]\) and \(y_3[n, k]\) received digital output symbol block from adaptive filters[2] is \(b'[n, k]\).

![Adaptive Filter](image)

Independently information transmitted by transmitter in multi-cellular environment. The digital output of the \(n^{th}\) filter for the \(m^{th}\) symbol period on target is given by

\[
r_n[m] = RH x_n[m] + v_n[m]
\]

Where H is matrix of respective channels and \(v_n[m]\) is noise.

**LMS Algorithm:**

The Least Mean Square (LMS) algorithm is introduced by Prof.Widrow and Hoff in 1959. It is an adaptive filter
algorithm[6] which uses a gradient based method of steepest descent algorithm. LMS algorithm uses the conclusions of the gradient vector from the available data and it contains an iterative operation that makes successive corrections to the weight vector in the direction of the negative of the gradient vector which ultimately leads to the minimum mean square error. LMS algorithm is relatively simple compared to other algorithms[11]; it neither requires correlation function calculation nor matrix inversion.

**Flow Chart of LMS algorithm:**

The error between the reference signal and the output of adaptive filter is

\[ \varepsilon = \langle x[n] - \hat{x}_n(n) \rangle \]

\[ \varepsilon = \langle x_n[m] - a_n^H[m]r_n[m] \rangle \]

\( a_n^H[m] \) is M dimensional complex valued weight vector at \( m \)th symbol time when the variable filter evaluates the desired weight vector coefficients by convolving the input signal with the impulse response. \( a_n[m] = [a_1, a_2, \ldots, a_M] \) are random coefficients of filter and \( r_n[m] = [r_1, r_2, \ldots, r_M] \): The weight parameters are varied and modified in a way that mean square error \( J_{a_n} \) is minimized in \( m \)th symbol time during the adaptation mode. For ease of description \( m \) is with every term but we are not mentioning it here

\[ J_{a_n} = \varepsilon[e_n \varepsilon_n^*] \]

The adaptive process search for the minimum point at which the weight vector is optimal which differentiates the mean squared error function \( J_{a_n} \) with respect to each coefficient of the weight vector and yields the gradient \( a_n \)[17].

The optimal weight vector \( a_{opt} \), can be identified by setting the gradient \( \nabla_n \) equal to zero,

\[ \nabla_n = -2Z + 2Ra_{opt} \]

where 0 is an M x 1 null vector at the minimum point of the error surface. The adaptive MMSE is optimum in the mean squared error sense, and equation can be simplified in the form

\[ Ra_{opt} = Z \]

which is Weiner-Hopf equation or the normal equation. One possible solution of this equation is matrix inversion,

\[ a_{opt} = R^{-1}Z \]

**Flow Chart of Steepest Descent Method:**

Let \( a_n \) and \( \nabla_n \) represents the values of the weight vector and the gradient vector at time \( m \) respectively then succeeding values of the weight vector are retrieved by the recursive relation. After each symbol period \( m \) the weight of the filter updated till optimum coefficient will get the best cross correlation value and then filter can go to decision directed mode where coefficients continuously change with the variation of channel. If we express \( \nabla_n \) in terms of instantaneous conclusions and then the equation can be simplified as

\[ \alpha_n[m + 1] = \alpha_n[m] + 2\mu_r[r_n[m]]x_n^*[m] - \nu_r[r_n[m]]a_n[m] \]

Where \( \mu \) is step size constant that controls stability and the rate of adaptation which can be expressed in terms of \( \epsilon_n^*[m] \) as,

\[ \alpha_n[m + 1] = \alpha_n[m] + 2\mu r_n[m]x_n^*[m] \]

Here \( 2\mu r_n[m]x_n^*[m] \) is correction factor, where \( m = 0, 1, 2, \ldots, \), till \( a_{opt} \) achieved[17]. The equation explains that the
updated weight vector is evaluated from the current weight vector by adding the input vector scaled by the complex conjugate value of the error and by \( \mu \). The iteration of the equation produces the value of the Mean Square Error at which the vector tends to its optimal value \( a_{opt} \).

The weight vector must be updated at a rate fast enough to record the channel variations to achieve proper adaptation. The method of steepest descent can be viewed as feedback model which may become unstable. The eigenvalues of \( R \) are all real and positive; the condition for convergence and stability of the steepest descent algorithm depends on the step size parameter \( \mu \) and the auto correlation matrix \( R \).

IV. RESULT ANALYSIS

I. Channel capacity

MIMO systems consist of transmit and receive antennas, it is considered a network with transmission paths connecting each input to output. From [8], the expression for MIMO capacity is given by:

\[
C = \log_2\left|I_m + \left(\frac{2}{n}\right)HH^H\right| \text{ b/s/Hz}
\]

where \( H \) is complex transpose channel correlation matrix. Transmitter \( i \) to receiver \( j \) at the \( k \)th tone of the OFDM. The capacity formula for the proposed scheme, based on the assumption that the channel matrix which consists of independent and identically distributed Rayleigh ading coefficients and the sum rate capacity figure of the developed scheme which is very close to MIMO capacity.

The capacity formulae for adaptive detection by Predrag as follows:

\[
C = \log\left(\frac{1}{\sigma^2}\right)
\]

where \( \sigma^2 \) is the noise variance of the signal at the receiver.

Channel Capacity Simulation Results:

The analysis of the Equalizer techniques has been carried out using MATLAB 7.0. To provide a fair comparison, some parameters were held constant during simulation for 4x4 MIMO OFDM and Adaptive MIMO OFDM Systems.

II. Bit Error Rate

The definition of bit error rate can be translated into a simple formula:

\[
\text{Bit Error Rate, BER} = \frac{\text{Number of Errors}}{\text{Total number of Bits Sent}}
\]

If the medium between the transmitter and receiver is good and the signal to noise ratio is high, then the bit error rate will be very small - possibly insignificant and having no noticeable effect on the overall system. However if noise can be detected [3], then there is chance that the bit error rate will need to be considered.

The achievable bit error rate of the following two cases was compared: MIMO OFDM ZF 4x4 antenna and MIMO OFDM MMSE 4x4 antenna systems. Perfect channel state information was assumed, BPSK Modulation and flat fading channel model were employed in the simulation. The configurations considered for OFDM system for antenna configuration \( N_t = N_r = 4 \).

Fig: Comparison plots for 4x4 MMIO OFDM ZF & MIMO OFDM MMSE

The Fig. shows the graph plotted for BER and SNR for different values. This graph mainly compares the MIMO OFDM ZF Equalizer with MIMO OFDM MMSE Equalizer for 4x4 MIMO System. For the average SNR 10dB, BER of MIMO OFDM ZF is \( 10^{-3} \) and BER of MIMO OFDM MMSE is approximate \( 10^{-4} \). This shows that MIMO OFDM MMSE have better performance compared to MIMO OFDM ZF.
The Fig. shows the graph plotted for BER and SNR for different values. This graph mainly compares the MIMO OFDM MMSE equalizer with Adaptive MMSE MIMO OFDM Equalizer for 4x4 MIMO System. For the average SNR 10dB, BER of MIMO OFDM MMSE is 10^{-3} and BER of AMMSE MIMO OFDM is approximate 10^{-4}. This shows that AMMSE MIMO OFDM have better performance compared to MIMO OFDM MMSE.

V. CONCLUSION

MIMO-OFDM is a major technology upgrade enabling demanding new applications with huge market potential and facilitating significant growth in existing applications. This technology is a combination of multiple-input multiple-output (MIMO) wireless technology with orthogonal frequency division multiplexing (OFDM) that has been recognized as important technique in the field of wireless communication system. MIMO-OFDM technology is more than the latest technical improvement for wireless networks.

Adaptive Minimum Mean Square Error Multiuser Detection (AMMSE) provides robustness and mobility in a time variable frequency selective multipath fading channel; it improves the bit error rate performance and therefore enhances channel capacity of a multi-cellular environment. MIMO OFDM mitigates multiple access interference and increases capacity.

REFERENCES


