# A New Method for Solving Generalized Trapezoidal Fuzzy Inventory Model with Shortage Under Space Constraint

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**Abstract:** In this paper, an EOQ(economic order quantity) model with shortage and space constraint has been analyzed in a fuzzy environment. The costs namely holding cost, shortage cost, setup cost, the ware house capacity and the objective are usually deterministic in nature. In this paper these have been considered to be generalized trapezoidal fuzzy numbers which are more realistic in nature. The ranking method proposed by Amit Kumar [1,2] has been used for ranking the fuzzy numbers. The fuzzy inventory problem has been transformed in to crisp inventory problem. For this EOQ and minimum total cost have been calculated. This method has been illustrated by means of numerical example

**Keywords:** Fuzzy inventory problem, EOQ (Economic Order Quantity), trapezoidal fuzzy number, Generalized trapezoidal fuzzy number, fuzzy ranking.

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## I. INTRODUCTION

In the real world, keeping an inventory for future sale or use is very common in business. Often uncertainties may occur in business. These uncertainties may be associated with demand or various relevant costs. After the publication of classical lot-size formula by Harris in 1915,many researches utilized EOQ model and currently these results are available in reference books and survey papers such as Hadley and within [4],Ha x and candea[5] and Waters[9].

In a realistic situation , total expenditure for an inventory model and the space available to store the inventory may be limited. The inventory costs, Holding, Shortage, Setup costs and the ware house space capacity may be flexible with some vagueness for their values. All these parameters in any inventory model are normally variable uncertain, imprecise and adoptable to the optimum decision making process and the determination of optimum order quality becomes a vague decision making process. The vagueness pertained in the above parameters induces to analyzed the inventory problem in a fuzzy environment.

The method is to rank the fuzzy costs by some ranking method for generalized trapezoidal fuzzy numbers. On the basis of this idea the Amit Kumar ranking method [1,2] for generalized fuzzy numbers has been adopted to transform the fuzzy inventory problem to a crisp one so that we can find the optimum solutions.

### Preliminaries

In this section some basic definitions, arithmetic operations and are reviewed.

**Basic definitions**:

**Definition 1:** The characteristic function $\mu_A$ of a crisp set A $\subseteq$ X assigns a value either 0 or 1 to each member in X. this function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range i.e. $\mu_{\tilde{A}}$ :X→[0,1]. The assigned value indicate the membership grade of the element in the set A.

The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A}$={(x, $\mu_{\tilde{A}}$(x));x ϵ X} defined by $\mu_{\tilde{A}}$(x) for each x ϵ X is called a fuzzy set.

**Definition 2 :** A fuzzy set $\tilde{A}$, defined on the universal set of real numbers R, is said to be a fuzzy number if its membership function has the following characteristics:

(i) $\mu_{\tilde{A}}$:R→[0,1] is continuous.

(ii) $\mu_{\tilde{A}}$(x) =0 for all $x \in (-\infty, a] \bigcup [d, \infty)$.

(iii) $\mu_{\tilde{A}}$(x) strictly increasing on [a,b] and strictly decreasing on [c,d].

(iv) $\mu_{\tilde{A}}$(x) =1 for all $x \in [b,c]$, where a < b < c < d.

**Definition 3**: A fuzzy number $\tilde{A}$=(a,b,c,d) is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{(x-a)}{(b-a)} ; a<x<b \\ 1 \quad ; b \le x \le c \\ \dfrac{(x-d)}{(c-d)} ; c < x < d \end{cases}$$

**Definition 4 :** A fuzzy set $\tilde{A}$, defined on the universal set of real numbers R, is said to be generalized fuzzy number if its membership function has the following characteristics:

(i) $\mu_{\tilde{A}}$ : R $\rightarrow$ [0, w] is continuous.

(ii) $\mu_{\tilde{A}}$ (x) = 0 for all $x \in (-\infty, a] \bigcup [d, \infty)$.

(iii) $\mu_{\tilde{A}}$ (x) strictly increasing on [a,b] and strictly decreasing on [c,d].

(iv) $\mu_{\tilde{A}}$ (x) = w for all $x \in [b, c]$, where 0<w≤1.

**Definition 5**: A fuzzy number $\tilde{A}$=(a,b,c,d,w) $_{LR}$ is said to be a L-R type generalized fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} wL\dfrac{(b-x)}{(b-a)} ; a<x<b \\ w \quad ; b \leq x \leq c \\ wR\dfrac{(x-c)}{(d-c)} ; c < x < d \end{cases}$$

**Definition 6**: A L-R type generalized fuzzy number $\tilde{A}$=(a, b, c, d, w) $_{LR}$ is said to be a generalized trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} w\dfrac{(x-a)}{(b-a)} ; a<x<b \\ w \quad ; b \leq x \leq c \\ w\dfrac{(x-d)}{(c-d)} ; c < x < d \end{cases}$$

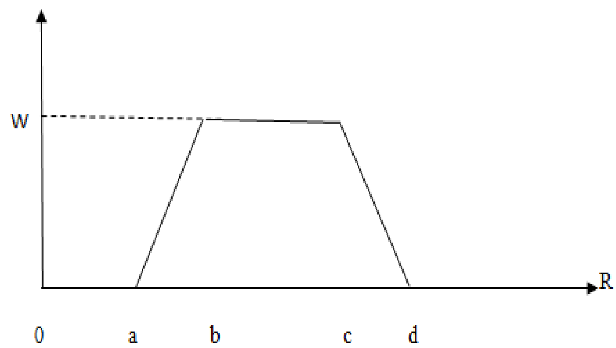


**Figure:1 A generalized trapezoidal fuzzy number**

**Arithmetic operations**:

In this subsection , arithmetic operations between two generalized trapezoidal fuzzy numbers, defined on universal set of real numbers R, are reviewed [16].

Let $\tilde{A}_1$ = ($a_1$, $b_1$, $c_1$, $d_1$; $w_1$) and $\tilde{A}_2$ = ($a_2$, $b_2$, $c_2$, $d_2$; $w_2$) be two generalized trapezoidal fuzzy numbers then

(i) $\tilde{A}_1 \oplus \tilde{A}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2; \min(w_1,$

(ii) $\tilde{A}_1 \, ! \, \tilde{A}_2 = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2; \min(w_1,$

(iii) $$\lambda\tilde{A}_1 = \begin{cases} (\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1; w_1) \lambda > 0 \\ (\lambda d_1, \lambda c_1, \lambda b_1, \lambda a_1; w_1) \lambda < 0 \end{cases}$$

**Ranking function**

The ranking function proposed by Amit kumar [1,2] for ranking function the generalized trapezoidal fuzzy number is defined by

$$R(\tilde{A}) = \frac{1}{2}\int_0^w \{L^{-1}(x) + R^{-1}(x)\} dx$$

**(1)**

Where $L^{-1}(x) = a_1 + \dfrac{(b_1 - a_1)}{w} x$,

$$R^{-1}(x) = c_1 + \frac{(b_1 - c_1)}{w} x$$

$$R(\tilde{A}) = \frac{w(a_1 + b_1 + c_1 + d_1)}{4}$$

for a generalized trapezoidal fuzzy number $\tilde{A} = (a_1, b_1, c_1, d_1; w_1)$.

**Model formulation**:

In this model, an inventory with shortage together with a space constraint is taken into account. The purpose of this EOQ model is to find out the optimum order quantity of inventory item subject to constraint by minimizing the total average cost. We discuss the model using the following notations and assumptions throughout the paper.

**Notations:**

$C_1$: Holding cost per unit time per unit quantity.

$C_2$: Shortage cost per unit time per unit quantity.

$C_3$: Setup cost per period

D: The total number of units produced per time period.

A-The space required by each unit(in sq.mt)

B-Maximum available ware house space (in sq.mt)

$Q_1$: The amount which goes into inventory

$Q_2$: The unfilled demand

Q: The lot size in each production run.

**Assumption:**

(i) Demand is known and uniform.

(ii) Production or supply of commodity is instantaneous.

(iii) Shortages are allowed.

(iv) Lead time is zero.

Let the amount of stock for the item be $Q_1$ at time t=0 in the interval $(0,t(=t_1+t_2))$, the inventory level gradually decrease to meet the demands. By this process the inventory level reaches zero level at time $t_1$ and then shortages are allowed to occur in the interval $(t_1,t)$. The cycle repeats itself.(Fig. 2)

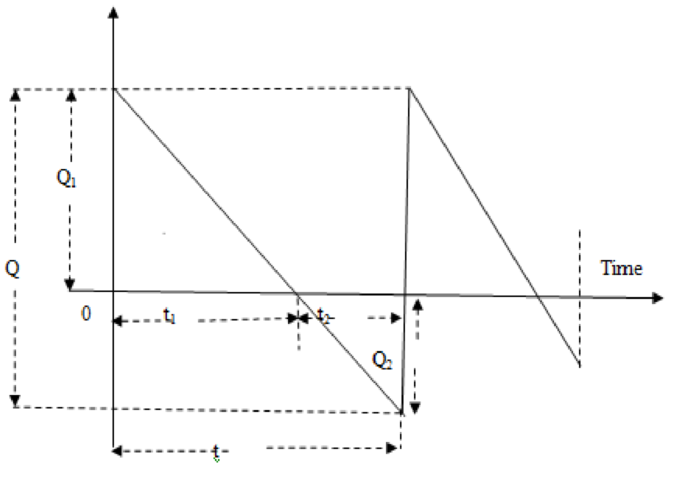


**Fig:2 Inventory level of the i[th] item**

The order level Q>0 which minimizes the average total cost (Q) per unit time subject to the space constraint is given by

$$\min C(Q)=\frac{1}{2}C_1\left(\frac{Q_1^{\,2}}{Q}\right)+\frac{1}{2}C_2\left(\frac{Q_2^{\,2}}{Q}\right)+C_3\left(\frac{D}{Q}\right)$$

Subject to : $AQ_1 \leq B$

**(2)**

Up to this stage, we are assuming that the demand, ordering cost, holding cost etc. as real numbers i.e .of fixed value. But in real life business situations all these components are not always fixed, rather these are different in different situations. To overcome these ambiguities we apply with fuzzy variables where the cost component are considered as generalized trapezoidal fuzzy numbers.

Let us assume the generalized fuzzy holding cost $\tilde{C}_1=(c_{11},c_{12},c_{13},c_{14},w_1)$, the generalized fuzzy shortage cost $\tilde{C}_2=(c_{21},c_{22},c_{23},c_{24},w_2)$, the generalized fuzzy ordering cost $\tilde{C}_3=(c_{31},c_{32},c_{33},c_{34},w_3)$ and the generalized fuzzy storage area $\tilde{B}=(B_1,B_2,B_3,B_4,w_4)$.

Replacing the real valued variables $C_1$,$C_2$, $C_3$, B by the generalized trapezoidal fuzzy variables $\tilde{C}_1,\tilde{C}_2,\tilde{C}_3,\tilde{B}$ we get,

min

$$\tilde{C}(Q)=\frac{1}{2}\tilde{C}_1\left(\frac{Q_1^{\,2}}{Q}\right)+\frac{1}{2}\tilde{C}_2\left(\frac{Q_2^{\,2}}{Q}\right)+\tilde{C}_3\left(\frac{D}{Q}\right)$$

S.to: $AQ_1 \leq \tilde{B}$.

**(3)**

Now we apply the ranking method [2] to defuzzify the fuzzy costs, the storage space and then we can get the minimum total cost $\tilde{C}^*(Q)$.

$$R\left(\tilde{C}^*(Q)\right)=\min C(Q)$$

$$=\frac{1}{2}R\left(\tilde{C}_1\right)\left(\frac{Q_1^{\,2}}{Q}\right)+\frac{1}{2}R\left(\tilde{C}_2\right)\left(\frac{Q_2^{\,2}}{Q}\right)+R\left(\tilde{C}_3\right)\left(\frac{D}{Q}\right)$$

Subject to: $AQ_1 \leq R(\tilde{B})$

**(4)**

Since $R(\tilde{C}_1), R(\tilde{C}_2), R(\tilde{C}_3)$ and $R(\tilde{B})$ are crisp values ,this problem is obviously the crisp inventory problem, for which we can find the optimum order quantity Q* and the optimum total cost C*(Q).

**Numerical example:**

Let us consider $C_1$=Rs.5,$C_2$=Rs.25, $C_3$=Rs.100, D=5000, A=0.5 sq.mt, B=150 sq.mt

Taking these as a generalized trapezoidal fuzzy numbers, $\tilde{C}_1$ =(3,4,5,7,$w_1$), $\tilde{C}_2$ =(21,23,25,31,$w_2$), $\tilde{C}_3$ =(85,98,103,109,$w_3$) and $\tilde{B}$ =(146,147,151,161,$w_4$).

**Case -1:** Take $w_1 = w_2 = w_3 = w_4 = 1$.

Here

$$R(\tilde{C}_1) = R(3,4,5,7,1) = 4.75$$

$$R(\tilde{C}_2) = R(21,23,25,31,1) = 25$$

$$R(\tilde{C}_3) = R(85,98,103,109,1) = 98.75$$

$$R(\tilde{B}) = R(146,147,151,161,1) = 151.25.$$

So,

$$R(\tilde{C}^*(Q)) = \min C(Q)$$

$$= \frac{1}{2} \times 4.75\left(\frac{Q_1^{\ 2}}{Q}\right) + \frac{1}{2} \times 25\left(\frac{Q_2^{\ 2}}{Q}\right) + 98.75 \times \left(\frac{5000}{Q}\right)$$

Subject to: 0.5 $Q_1 \leq$ 151.25 **(5)**

**Case -2:** Take equal values to $w_1$, $w_2$, $w_3$, $w_4$ other than 1.

$w_1 = w_2 = w_3 = w_4 = 0.7$

Here

$$R(\tilde{C}_1) = R(3,4,5,7,0.7) = 3.325$$

$$R(\tilde{C}_2) = R(21,23,25,31,0.7) = 17.5$$

$$R(\tilde{C}_3) = R(85,98,103,109,0.7) = 69.125$$

$$R(\tilde{B}) = R(146,147,151,161,0.7) = 105.87$$

So,

$$R(\tilde{C}^*(Q)) = \min C(Q)$$

$$= \frac{1}{2} \times 3.325\left(\frac{Q_1^{\ 2}}{Q}\right) + \frac{1}{2} \times 17.5\left(\frac{Q_2^{\ 2}}{Q}\right) + 69.125 \times \left(\frac{5000}{Q}\right)$$

Subject to: 0.5 $Q_1 \leq$ 105.87 **(6)**

**Case -3:** Give different values to $w_1$, $w_2$, $w_3$, $w_4$.

$w_1 = 0.6$, $w_2 = 0.35$, $w_3 = 0.8$, $w_4 = 1.0$

Here

$$R(\tilde{C}_1) = R(3,4,5,7,0.6) = 2.85$$

$$R(\tilde{C}_2) = R(21,23,25,31,0.35) = 8.75$$

$$R(\tilde{C}_3) = R(85,98,103,109,0.8) = 79$$

$$R(\tilde{B}) = R(146,147,151,161,1) = 151.25.$$

So,

$$R(\tilde{C}^*(Q)) = \min C(Q)$$

$$= \frac{1}{2} \times 2.85\left(\frac{Q_1^{\ 2}}{Q}\right) + \frac{1}{2} \times 8.75\left(\frac{Q_2^{\ 2}}{Q}\right) + 79 \times \left(\frac{5000}{Q}\right)$$

Subject to: 0.5 $Q_1 \leq$ 151.25

**(7)**

**Results and discussion**:

The solutions obtained from (5), (6) and (7) are given in table 1,2,3.

**Table 1: optimum results for case I**

| Q* | $Q_1$* | Min C(Q) |
|---|---|---|
| 385.22 | 302.5 | 2067.94 |

**Table 2**: **optimum results for case II**

| Q* | $Q_1$* | Min C(Q) |
|---|---|---|
| 304.72 | 211.75 | 1627.06 |

**Table 3:optimum results for case III**

| Q* | $Q_1$* | Min C(Q) |
|---|---|---|
| 459.99 | 302.5 | 1378.095 |

**Comparison table:**

| Model | $C_1$ | $C_2$ | $C_3$ | B | Q* | $Q_1$* | Min C(Q) |
|---|---|---|---|---|---|---|---|
| Crisp | 5 | 25 | 100 | 150 | 384.70 | 300 | 2117.69 |
| Crisp | 3 | 20 | 100 | 150 | 391.79 | 300 | 1835.82 |
| Crisp | 5 | 25 | 95 | 130 | 345.14 | 260 | 2128.44 |
| Fuzzy (Case 1) | 4.75 | 25 | 98.75 | 151.25 | 385.22 | 302.5 | 2067.94 |
| Fuzzy (Case 2) | 3.325 | 17.5 | 69.125 | 105.875 | 304.72 | 211.75 | 1627.06 |
| Fuzzy (Case 3) | 2.85 | 8.75 | 79 | 151.25 | 459.99 | 302.5 | 1378.095 |

## II. CONCLUSION:

In this paper, the costs are considered as imprecise numbers described by generalized trapezoidal fuzzy numbers which are more realistic and general in nature. Moreover, the fuzzy inventory model has been transformed in to crisp inventory problem using Amit Kumar ranking indices[1,2].Numerical example shows that by this method we can have the optimal total cost. By using Amit kumar [1,2] ranking method we have shown that the total cost obtained is optimal. Moreover, one can conclude that the solution of fuzzy inventory problems can be obtained by ranking method effectively.

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