

# Study of Thermal Radiation and Ohmic Heating for Steady Magnetohydrodynamic Natural Convection Boundary Layer Flow in a Saturated Porous Regime

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**Abstract**— An analysis is performed to study the thermal radiation and Ohmic heating effects on coupled heat and mass transfer by steady magnetohydrodynamic natural convective laminar boundary-layer flow of a viscous incompressible electrically conducting Newtonian fluid past a vertical permeable surface embedded in a Darcian porous medium. The heat equation includes the terms involving the radiative heat flux, Ohmic dissipation, viscous dissipation and the internal absorption whereas the mass transfer equation includes the effects of chemically reactive species of first-order. The non-linear coupled differential equations are solved analytically by perturbation technique. The numerical results are benchmarked with previously published studies and found to be in excellent agreement. Finally, the effects of the pertinent parameters which are of physical and engineering interest on the flow and heat transfer characteristics are presented graphically and in tabulated form. It is observed that the effect of heat absorption is to decrease the velocity and temperature profiles in the boundary layer.

**Keywords**- Laminar boundary layer flow; Darcian drag force; Ohmic heating; viscous dissipation; Natural Convection; Heat absorption.

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## I. INTRODUCTION

The study of fluid flow problems associated with heat transfer is of widespread interest in almost all the fields of engineering as well as in astrophysics, biology, biomedicine, meteorology, physical chemistry, plasma physics, geophysics, oceanography and scores of other disciplines. Hydromagnetic flows and heat transfer in porous media have been considered extensively in recent years due to their occurrence in several engineering processes such as compact heat exchangers, metallurgy, casting, filtration of liquid metals, cooling of nuclear reactors high speed aerodynamics and magnetic braking technologies with fusion control [1-5]. Merkin [6] investigated a mixed convective boundary-layer flow on a semi-infinite vertical flat plate, when the buoyancy forces aid in the development of the boundary layer or prevent it. Watanabe [7] presented the effects of the surface mass transfer on a mixed convective flow on a permeable vertical surface. With the combined effect of heat transfer many challenging flow problems have been studied in magnetohydrodynamic convection flows with different suitable configurations. The effects of transversely applied magnetic field, on the flow of an electrically conducting fluid past an impulsively started infinite isothermal vertical plate studied by Soundalgekar *et al.* [8]. MHD effects on impulsively started vertical plate with variable temperature in the presence of transverse magnetic field were considered by Soundalgekar *et al.* [9]. Free convection flows in a porous media with chemical reaction have wide applications in geothermal and oil reservoir engineering as well as in chemical reactors of porous structure. Many transport processes exist in industrial applications in which the simultaneous heat and mass transfer occur as a result of combined buoyancy effects of diffusion of chemical species. Moreover, considerable interest has been evinced in

radiation interaction with convection and chemical reaction for heat and mass transfer in fluids. This is due to the significant role of thermal radiation in the surface heat transfer when convection heat transfer is small, particularly, in free convection problems involving absorbing-emitting fluids. Khair and Bejan [10] studied heat and mass on flows past an isothermal flat plate. Lin and Wu [11] analyzed combined heat and mass transfer by laminar natural convection from a vertical plate. Yin [12] studied numerically the force convection effect on magnetohydrodynamics heat and mass transfer of a continuously moving permeable surface. Acharya *et al.* [13] have studied heat and mass transfer over an accelerating surface with heat source in the presence of suction and blowing. Muthucumaraswamy and Janakiraman [14] studied MHD and radiation effects on moving isothermal vertical plate with variable mass diffusion. Hossain *et al.* [15] investigated radiation effects on the free convection flow of an optically incompressible fluid along a uniformly heated vertical infinite plate with a constant suction. Orhan and Kaya [16] examined MHD mixed convective heat transfer along a permeable vertical infinite plate in the presence of radiation and solutions are derived using Kellar box scheme and accurate finite-difference scheme. Ahmed and Liu [17] examined the effects of mass transfer on a mixed convection three dimensional heat transfer flow of a viscous incompressible fluid past an infinite vertical porous plate in the presence of transverse periodic suction velocity. The problem of combined heat and mass transfer of an electrically conducting fluid in MHD natural convection adjacent to a vertical surface is analyzed by Chen [18] by taking into account the effects of Ohmic heating and viscous dissipation but neglected chemical reaction of the species. Chaudhury *et al.* [19] have analyzed the effect of radiation on heat transfer

in MHD mixed convection flow with simultaneous thermal and mass diffusion from an infinite vertical plate with viscous dissipation and Ohmic heating. The classical model introduced by Cogley *et al.* [20] is used for the radiation effect as it has the merit of simplicity and enables us to introduce linear term in temperature in the analysis for optically thin media. Ahmed and Zueco [21] studied the effect of the transverse magnetic field on a steady mixed convective heat and mass transfer flow of an incompressible viscous electrically conducting fluid past an infinite vertical isothermal porous plate taking into account the induced magnetic field, viscous and magnetic dissipations of energy in presence of chemical reaction of first order and heat generation/absorption, and the non-linear coupled equations are solved by network simulation technique. The thermal radiation and Darcian drag force MHD unsteady thermal-convection flow past a semi-infinite vertical plate immersed in a semi-infinite saturated porous regime with variable surface temperature in the presence of transversal uniform magnetic field have been discussed by Ahmed *et al.* [21]. A numerical analysis of conduction-radiation, porosity and chemical reaction on unsteady hydromagnetic free convection flow past an impulsively-started semi-infinite vertical plate embedded in a porous medium in presence of thermal radiation is presented by Ahmed [22].

In this paper, it is proposed to study the effects of viscous dissipation and Ohmic dissipation on steady two dimensional magnetohydrodynamic natural convection heat and mass transfer flow of a Newtonian, electrically conducting and viscous incompressible radiative fluid over a porous vertical plate embedded in a porous medium taking into the account of combined effects of buoyancy force and first-order chemical reaction. The present study may have useful applications in several transport processes as well as in processing magnetic materials. The analytical results for some particular cases are compared with those from [19] and are found to be in excellent agreement. The governing equations for this investigation are formulated and solved by using perturbation technique.

## II. MATHEMATICAL FORMULATION

A two-dimensional laminar boundary layer flow of a viscous incompressible electrically conducting and heat absorbing fluid past a semi-infinite vertical permeable plate embedded in a uniform porous medium which is subject to thermal and concentration buoyancy effects has been presented. As shown in Fig. 1,  $x^*$ -axis is along the plate and  $y^*$  is perpendicular to the plate. The wall is maintained at a constant temperature  $T_w$  and concentration  $C_w$  higher than the ambient temperature  $T_\infty$  and concentration  $C_\infty$ , respectively. Also, it is assumed that there exists a homogeneous chemical reaction of first-order with constant rate  $R$  between the diffusing species and the fluid. Under these assumptions, the governing equations of the Newtonian flow model of electrically conducting radiative and chemically reacting fluid through porous medium in presence of magnetic field with heat generation and viscous dissipative heat are

$$\frac{dv^*}{dy^*} = 0 \Rightarrow v^* = -v_0 \text{ (Constant)}, \quad (1)$$

$$\frac{dp^*}{dy^*} = 0 \Rightarrow p^* \text{ is independent of } y^*, \quad (2)$$

$$\rho v^* \frac{du^*}{dy^*} = \left[ \begin{array}{l} \mu \frac{d^2 u^*}{dy^{*2}} - \left( \sigma B_0^2 + \frac{\mu}{K^*} \right) u^* \\ \rho g \beta_T (T^* - T_\infty) + \rho g \beta_C (C^* - C_\infty) \end{array} \right], \quad (3)$$

$$\rho C_p v^* \frac{dT^*}{dy^*} = \alpha \frac{d^2 T^*}{dy^{*2}} + \left[ \begin{array}{l} \mu \left( \frac{du^*}{dy^*} \right)^2 - \frac{\partial q^*}{\partial y^*} \\ + \sigma B_0^2 u^{*2} - Q_0 (T^* - T_\infty) \end{array} \right], \quad (4)$$

$$v^* \frac{dC^*}{dy^*} = D \frac{d^2 C^*}{dy^{*2}} - R(C^* - C_\infty), \quad (5)$$

The second and third terms on RHS of the momentum equation (3) denote the thermal and concentration buoyancy effects, respectively. Also second and fourth terms on the RHS of energy equation (4) represent the viscous dissipation and Ohmic dissipation, respectively. The third and fifth term on the RHS of equation (4) denote the inclusion of the effect of thermal radiation and heat absorption effects, respectively.

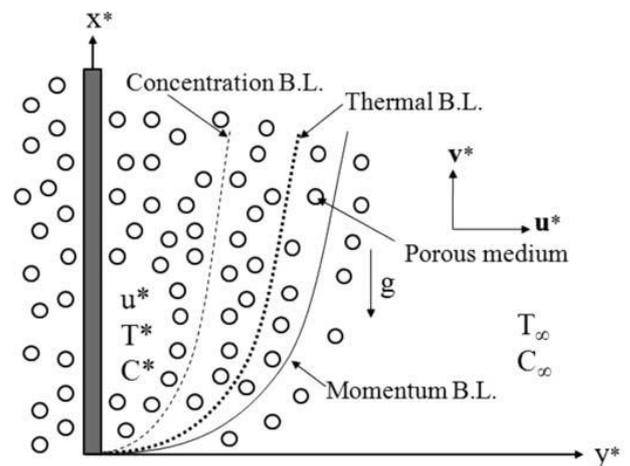


Fig. 1: Flow model of the problem

For the radiative heat flux using the Cogley model [20] is given

$$\frac{\partial q^*}{\partial y^*} = 4(T^* - T_\infty)I^*, \quad (6)$$

$$\text{where } I^* = \int_0^\infty K_{\lambda w} \frac{\partial e_{b\lambda}}{\partial T^*} d\lambda,$$

$K_{\lambda w}$  is the absorption coefficient at the wall and  $e_{b\lambda}$  is Planck's function.

The appropriate boundary conditions for velocity, temperature and concentration fields are

$$y^* = 0: \quad u^* = 0, \quad T^* = T_w, \quad C^* = C_w, \quad (7)$$

$$y^* \rightarrow \infty: \quad u^* \rightarrow 0, \quad T^* \rightarrow T_\infty, \quad C^* \rightarrow C_\infty, \quad (8)$$

Introducing the following non-dimensional quantities:

$$\left[ \begin{array}{l} y = \frac{v_0 y^*}{\nu}, \quad u = \frac{u^*}{v_0}, \quad M^2 = \frac{\sigma B_0^2 v^2}{\mu v_0^2}, \\ \theta = \frac{T^* - T_\infty}{T_w - T_\infty}, \quad \phi = \frac{C^* - C_\infty}{C_w - C_\infty}, \quad \gamma = \frac{Rv}{v_0^2}, \\ Gr = \frac{\rho g \beta_T (T_w - T_\infty)}{\mu v_0^3}, \quad Gm = \frac{\rho g \beta_C (C_w - C_\infty)}{\mu v_0^3}, \\ Sc = \frac{\nu}{D}, \quad \psi = \frac{Q_0 \nu}{\rho C_p v_0^2}, \quad F = \frac{4\nu I'}{\rho C_p v_0^2}, \\ Pr = \frac{\mu C_p}{\alpha}, \quad Ec = \frac{v_0^2}{C_p (T_w - T_\infty)}, \quad K = \frac{K^* v_0^2}{\nu^2} \end{array} \right] \quad (9)$$

On using (6) and (9), the equations (3)–(5) reduce to the following non-dimensional equations:

$$\frac{d^2 u}{dy^2} + \frac{du}{dy} - (M^2 + K^{-1})u = Gr \theta - Gm \phi, \quad (10)$$

$$\frac{d^2 \theta}{dy^2} + Pr \frac{d\theta}{dy} + Pr E \left( \frac{du}{dy} \right)^2 = \left[ Pr(E + \psi)\theta \right], \quad (11)$$

$$\frac{d^2 \phi}{dy^2} + Sc \frac{d\phi}{dy} - Sc \gamma \phi = 0, \quad (12)$$

The dimensionless form of the boundary conditions (7) and (8) are

$$y = 0: \quad u = 0, \quad \theta = 1, \quad \phi = 1 \quad (13)$$

$$y \rightarrow \infty: \quad u \rightarrow 0, \quad \theta \rightarrow 0, \quad \phi \rightarrow 0 \quad (14)$$

### III. METHOD OF SOLUTION

Equations (10)–(12) represent a set of partial differential equations that cannot be solved in closed-form. However, these equations can be solved analytically after reducing them to a set of ordinary differential equations in dimensionless form. Thus we can represent the velocity  $u$ , temperature  $\theta$  and concentration  $\phi$  in terms of power of Eckert number  $Ec$  as in the flow of an incompressible fluid Eckert number is always less than unity since the flow due to the Joules dissipation is super imposed on the main flow. Hence, we can assume

$$\left[ \begin{array}{l} u(y) = u_0(y) + Ec u_1(y) + o(Ec^2) \\ \theta(y) = \theta_0(y) + Ec \theta_1(y) + o(Ec^2) \\ \phi(y) = \phi_0(y) + Ec \phi_1(y) + o(Ec^2) \end{array} \right] \quad (15)$$

Substituting (15) in equations (10)–(12) and equating the coefficient of zeroth powers of  $Ec$  (i.e.  $O(Ec^0)$ ), we get the following set of equations:

$$u_0'' + u_0' - Nu_0 = -Gr \theta_0 - Gm \phi_0, \quad (16)$$

$$\theta_0'' + Pr \theta_0' - Pr(F + \psi) \theta_0 = 0, \quad (17)$$

$$\phi_0'' + Sc \phi_0' - Sc \gamma \phi_0 = 0, \quad (18)$$

The coefficients of first-order of  $Ec$  (i.e.  $O(Ec^1)$ ), we obtain

$$u_1'' + u_1' - Nu_1 = -Gr \theta_1 - Gm \phi_1, \quad (19)$$

$$\theta_1'' + Pr \theta_1' - Pr(F + \psi) \theta_1 + Pr u_0' + Pr M^2 u_0^2 = 0, \quad (20)$$

$$\phi_1'' + Sc \phi_1' - Sc \gamma \phi_1 = 0, \quad (21)$$

where  $N = M^2 + K^{-1}$ .

$$y = 0: \quad \left\{ \begin{array}{l} u_0 = 0, \quad u_1 = 0, \quad \theta_0 = 1, \\ \theta_0 = 0, \quad \phi_0 = 1, \quad \phi_0 = 0 \end{array} \right\} \quad (20)$$

$$y \rightarrow \infty: \quad \left\{ \begin{array}{l} u_0 \rightarrow 0, \quad u_1 \rightarrow 0, \quad \theta_0 \rightarrow 0, \\ \theta_1 \rightarrow 0, \quad \phi_0 \rightarrow 0, \quad \phi_1 \rightarrow 0 \end{array} \right\} \quad (21)$$

The solution of velocity, temperature and concentration fields have restricted up to  $O(Ec)$  and neglected the higher order of  $O(Ec^2)$  as the value of  $Ec \ll 1$ . The solutions of equations (16)–(21) with the help of boundary conditions (22) and (23) are obtained as follows:

$$u_0 = A_5(e^{-A_4 y} - e^{-A_1 y}) + A_6(e^{-A_4 y} - e^{-m_1 y}), \quad (24)$$

$$\theta_0 = e^{-A_1 y}, \quad (25)$$

$$\phi_0 = e^{-m_1 y}, \quad (26)$$

$$u_1 = \left( \begin{array}{l} A_{17} e^{-A_4 y} - B_{10} e^{-A_1 y} + B_{11} e^{-2A_1 y} \\ + B_{12} e^{-2A_4 y} - B_{13} e^{-A_{10} y} + B_{14} e^{-2m_1 y} \\ - B_{15} e^{-B_1 y} + B_{16} e^{-B_2 y} \end{array} \right), \quad (27)$$

$$\theta_1 = \left( \begin{array}{l} B_9 e^{-A_1 y} - B_3 e^{-2A_1 y} - B_4 e^{-2A_4 y} + B_5 e^{-A_{10} y} \\ - B_6 e^{-2m_1 y} + B_7 e^{-B_1 y} - B_8 e^{-B_2 y} \end{array} \right), \quad (28)$$

$$\phi_1 = 0 \quad (29)$$

The physical quantities of interest are the wall shear stress  $\tau_w$  is given by

$$\tau_w = \mu \left. \frac{\partial u^*}{\partial y^*} \right|_{y^*=0}, \quad (30)$$

$$C_{fx} = \frac{\tau_w}{\rho v_0^2} = u'(0). \quad (31)$$

Using (24), (27) and (30) in (31), we get

$$C_{fx} = \left[ \begin{array}{l} A_6(m_1 - A_4) + A_5(A_1 - A_4) \\ -Ec \left( \begin{array}{l} B_{17} A_4 - B_{10} A_1 + 2B_{11} A_1 + 2B_{12} A_4 \\ -B_{13} A_{10} + 2B_{14} m_1 - B_{15} B_1 + B_{16} B_2 \end{array} \right) \end{array} \right] \quad (32)$$

The local surface heat flux is given by

$$q_w = -\kappa \left. \frac{\partial T^*}{\partial y^*} \right|_{y^*=0} \quad (33)$$

The Local Nusselt number

$$Nu_x = x q_w / \kappa (T_w - T_\infty) \quad (34)$$

Using (25), (28) and (33) in (34), then the Local Nusselt number can be written as

$$\frac{Nu_x}{Re_x} = \theta'(0) = \left[ \begin{array}{l} -A_1(1 + Ec B_9) \\ + Ec \left( \begin{array}{l} 2B_3 A_1 + 2B_4 A_4 - B_5 A_{10} \\ + 2B_6 m_1 - B_1 B_7 + B_2 B_8 \end{array} \right) \end{array} \right] \quad (35)$$

where  $Re_x = v_0 x / \nu$  is the local Reynolds number.

#### IV. VALIDATION

Validation of the analysis has been performed by comparing the present results with those available in the open literature [19] and a very good agreement has been established, when  $K=\infty$ ,  $\psi=0.0$ ,  $\gamma=0.0$ . In order to verify the accuracy of the present results, we have considered the analytical solutions obtained by Chaudhary *et al.* [19] and computed these solutions for various physical parameters for skin-friction coefficient and local Nusselt number.

Table 1: Comparison of present results with those of Chaudhary *et al.* [19] with different values of  $F$  for  $C_{fx}$  and  $Nu_x/Re_x$ ; at  $Pr = 0.71$ ,  $Sc = 0.78$ ,  $M = 5.0$ ,  $Gr = 5.0$ ,  $Gm = 5.0$ ,  $Ec = 0.05$ .

Chaudhary <i>et al.</i> [19]			Present results		
$F$	$C_{fx}$	$Nu_x/Re_x$	$F$	$C_{fx}$	$Nu$
1.0	1.82701	1.52710	1.0	1.82981	1.52803
2.0	1.77315	1.82047	2.0	1.77918	1.82209
3.0	1.73182	2.17602	3.0	1.73423	2.17803
4.0	1.60718	2.32635	4.0	1.60817	2.32725
5.0	1.52541	2.50981	5.0	1.52598	2.51083

#### V. RESULTS AND DISCUSSION

To get a physical insight into the problem the numerical evaluation of the analytical results reported in the previous section was performed and a set of results is reported graphically in Figures 2-7 for the cases cooling  $Gr>0$  of the plate i.e. free convection currents convey heat away from the plate into the boundary layer. During the numerical calculations the physical parameters are considered as  $Pr=0.71$  (diffusing air),  $Gr=5$  (thermal buoyancy forces are dominant over the viscous hydrodynamic forces in the boundary layer),  $F=5>1$  (thermal radiation is dominant over the thermal conduction),  $Ec=0.05\leq 1$  (Enthalpy difference is dominant over the kinetic energy).

Figs. 2 and 3 illustrate the influence of the heat absorption and porosity parameters  $\psi$  and  $K$ , respectively on the flow velocity. The effect is observed on velocity profile by increasing the value of the heat absorption parameter  $\psi$ , and the boundary layer thickness decreases with increase in the absorption parameter as shown in Fig. 2, which is expected. The opposite trend is observed in Fig. 3 for the case when the value of the porous permeability is increased. As depicted in this figure, the effect of increasing the value of porous permeability is to increase the value of the velocity component in the boundary layer due to the fact that drag is reduced by increasing the value of the porous permeability on the fluid flow which results in increased velocity. Fig. 4 depicts the effect of radiation on the flow velocity. We note from this figure that there is decrease in the value of flow velocity with increase in radiation parameter  $F$  which shows the fact that increase in radiation parameter decrease the velocity in the boundary layer due to decrease in the boundary layer thickness. The effect of chemical reaction parameter  $\gamma$  is highlighted in Fig. 5 which shows that the velocity decreases with increasing the rate of chemical reaction  $\gamma$ . Hence increase in the chemical reaction rate parameter leads to a fall in the momentum boundary layer. The trend of the velocity profile in this figure is same as shown in Fig. 4. The effect of absorption parameter ( $\psi$ ) on fluid temperature ( $\theta$ ) is presented in Fig. 6.

This is due to the fact that the thermal boundary layer absorbs energy which causes the temperature fall considerably with increasing the value of internal heat absorption parameter. The effect the reaction rate parameter ( $\gamma$ ) on the species concentration profiles ( $\phi$ ) for generative chemical reaction is shown in Fig. 7. It is noticed from the graphs that there is a decreasing effect on concentration distribution with increasing the value of the chemical reaction rate parameter in the boundary layer.

In Table 1, it has been observed that the skin friction coefficient decreases and local Nusselt number raises sharply due to the increase of radiation parameter  $F$ .

#### VI. CONCLUSIONS

A theoretical analysis of the steady magnetohydrodynamic flow and natural convection heat and mass transfer in a viscous, incompressible, electrically-conducting fluid along a semi-infinite vertical plate immersed in a porous medium with thermal radiation has been conducted. The flow model has been setup for homogeneous chemical reaction of first-order in the presence of Ohmic heating and viscous dissipation. The nonlinear and coupled governing equations are solved analytically by perturbation technique. Analytical solutions using the method of complex variables have been derived. Above investigation reveals the following facts:

- It is seen that the velocity starts from minimum value of zero at the surface and increases till it attains the peak value and then starts decreasing until it reaches the minimum value at the end of the boundary layer.
- Increasing *heat absorption* acts to decelerate the flow velocity in the boundary layer.
- Flow velocity is accelerated with increasing porosity parameter in the porous regime.
- It is seen that with an increase in *heat absorption* of the steady motion, the temperatures are decreased

For the steady state case, there is a strong reduction in the concentration distribution for the effect of *generative chemical reaction*.

#### NOMENCLATURE

$B_0$	Uniform magnetic field
$C^*$	Species concentration ( $\text{Kg. m}^{-3}$ )
$C_p$	Specific heat at constant pressure ( $\text{J. kg}^{-1}. \text{K}$ )
$C_\infty$	Species concentration in the free stream ( $\text{Kg.m}^{-3}$ )
$C_w$	Species concentration at the surface ( $\text{Kg.m}^{-3}$ )
$D$	Chemical molecular diffusivity ( $\text{m}^2.\text{s}^{-1}$ )
$Ec$	Eckert number/dissipative heat
$F$	radiation parameter
$g$	Acceleration due to gravity ( $\text{m.s}^{-2}$ )
$Gr$	Thermal Grashof number
$Gm$	Mass Grashof number
$\gamma$	Chemical reaction parameter
$K$	porosity parameter
$M$	Hartmann number/Magnetic parameter
$Nu_x$	Local Nusselt number
$Pr$	Prandtl number
$q^*$	Heat flux per unit area
$Re_x$	Local Reynolds number
$Sc$	Schmidt number
$T^*$	Temperature (K)
$T_\infty$	Fluid temperature at the surface (K)
$T_\infty$	Fluid temperature in the free stream (K)

$u$	Dimensionless velocity component in x-direction (m. s <sup>-1</sup> )
$u^*$	dimensional velocity along x* direction
$v^*$	dimensional velocity along y* direction
$v_0$	Dimensionless suction velocity (m. s <sup>-1</sup> )

#### Greek symbols

$\alpha$	fluid thermal diffusivity
$\beta_T$	coeff. of volume expansion for heat transfer (K <sup>-1</sup> ),
$\beta_C$	coeff. of volume expansion for mass transfer (K <sup>-1</sup> )
$\gamma$	Chemical reaction parameter
$\theta$	Dimensionless fluid temperature (K),
$\kappa$	Thermal conductivity (W. m <sup>-1</sup> . K <sup>-1</sup> ),
$\mu$	Coefficient of viscosity (kg. m <sup>-3</sup> )
$\nu$	Kinematic viscosity (m <sup>2</sup> .s <sup>-1</sup> ),
$\sigma$	Electrical conductivity (VA <sup>-1</sup> . m <sup>-1</sup> ),
$\tau_w$	wall shearing stress (N. m <sup>-2</sup> )
$\phi$	Dimensionless species concentration (Kg.m <sup>-3</sup> )
$\psi$	heat source parameter

#### Subscripts

w	conditions on the wall
$\infty$	conditions at the free stream

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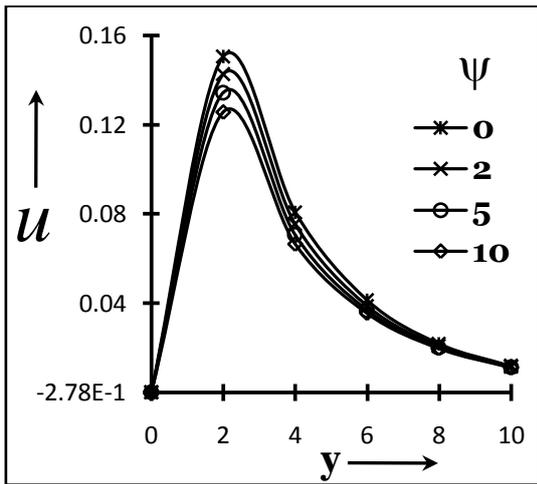
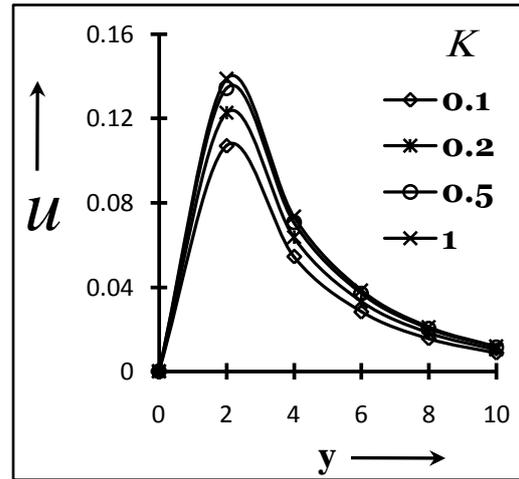


Fig. 2: Velocity distribution for heat absorption ( $\psi$ )



3: Velocity distribution for porosity ( $K$ )

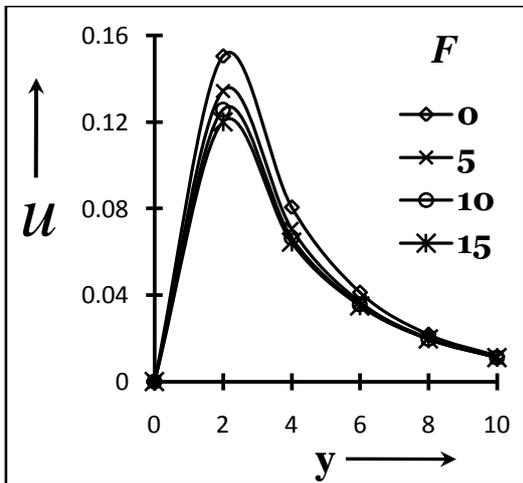


Fig. 4: Velocity distribution for radiation ( $F$ )

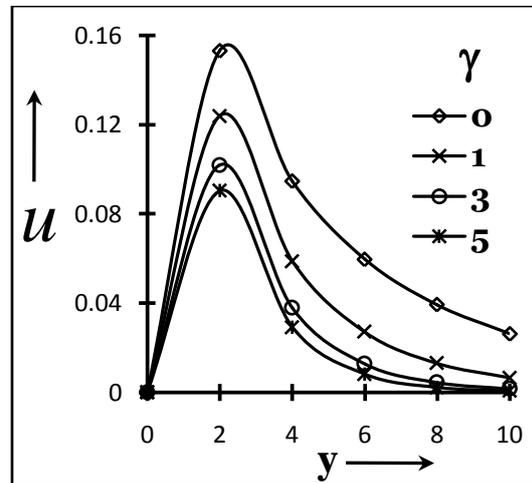


Fig. 5: Velocity distribution for chemical reaction ( $\gamma$ )

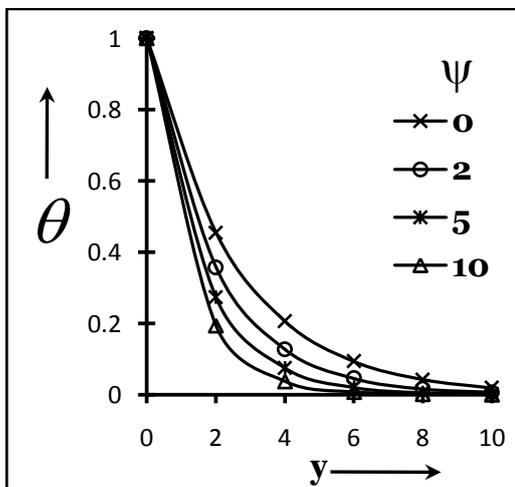


Fig. 6: Temperature for heat absorption ( $\psi$ )

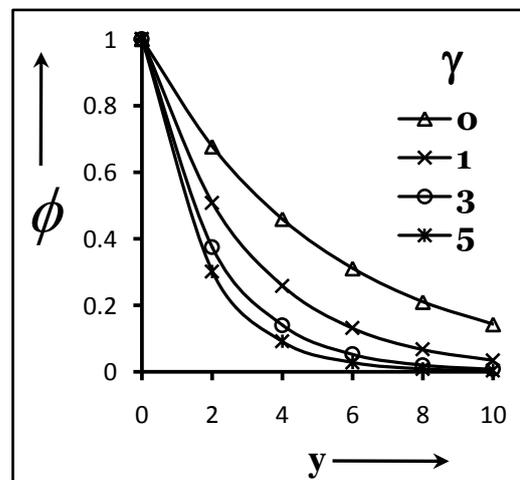


Fig. 7: Temperature for chemical reaction ( $\gamma$ )