Naïve Properties on Rough Connectives under Fuzziness

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Abstract-In recent era, since implementing fuzzy concepts into the existing conventional models improve the efficiency, several hybridized models are being derived by the researchers. In this line, in 2005, G.Ganesan et. al., have introduced rough approximations on fuzzy sets using thresholds. Later, in 2008, G.Ganesan et.al., introduced an innovative way of approximating the connectives in fuzzy predicate calculus through rough sets. In this paper, additional properties of the connectives thus derived have been derived.

Keywords: fuzzy sets, rough sets, fuzzy predicate calculus,

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1. INTRODUCTION

In 2005, G.Ganesan et. al., [2] discussed the concept of thresholds in rough fuzzy computing. Later, in 2008, they introduced rough connectives for fuzzy predicates using [2]. In 1982, Z. Pawlak developed a mathematical model namely Rough Sets [8]. This model has been currently used by the researchers in the areas such as Knowledge acquisition, Knowledge Discovery, information retrieval etc to improve the efficiency.

Since the work of Atanassov's intuitionistic fuzzy sets [1] which is the generalization of fuzzy model [7] of Zadeh, in 2013, G.Ganesan et.al., developed the concept of rough connectives [5] on intuitionistic fuzzy predicates.

As these papers had focused only on the basic axioms of these connectives, in this paper, an initiative is made to derive additional properties on these connectives. This paper is organized into four sections. In the second section, we discussed the basic definition of Rough Sets and fuzzy predicates. In third section, we derived a few of the the properties of rough connectives in fuzzy predicates. Fourth section is devoted for the concluding remarks.

2. ROUGH SETS AND FUZZY PREDICATE CALCULUS

2.1. Rough Sets

According to Pawlak [8], for a given concept A, the lower approximation and upper approximations are given by $A_{-} = \{x \in U / [x]_E \subseteq A\}$ and $A^{-} = \{x \in U / [x]_E \cap A \neq \phi\}$ respectively, where E is an equivalence relation defined on U .

2.2. Fuzzy Predicates

The predicates, which do not have precise logical value, are called fuzzy predicates [4]. For any two fuzzy predicates p(x) and q(x), the fuzzy conjunction ($\wedge$), fuzzy disjunction ($\vee$), fuzzy negation (neg), fuzzy implication ($\rightarrow$) and fuzzy bi implication ($\leftrightarrow$) are defined as follows.

For any two fuzzy predicates p(a) and q(b),

Fuzzy Conjunction ($\wedge$) : $\mu_{(p(a)\wedge q(b))} = \min(\mu_{p(a)}, \mu_{q(b)})$

Fuzzy Disjunction ($\vee$) : $\mu_{(p(a)\vee q(b))} = \max(\mu_{p(a)}, \mu_{q(b)})$

Fuzzy Negation (neg) : $\mu_{(neg(p(a)))} = 1 - \mu_{p(a)}$

Fuzzy implication (→) : $\mu_{(p(a)\to q(b))} = \max(1-\mu_{p(a)}, \mu_{q(b)})$

Fuzzy bi-implication (↔): $\mu_{(p(a)\leftrightarrow q(b))} = \min(\mu_{(p(a)\to q(b))}, \mu_{(q(b)\to p(a))})$

In [5], we have established the approximated connectives under fuzziness and intuitionistic fuzziness proposed by Atanasov [1]. However to make the paper to be self explanatory, we describe the concepts of rough approximations on these predicates as given in [3, 5].

3. ROUGH CONNECTIVES OF FUZZY PREDICATES

For any fuzzy predicate P with an argument x, according to [3, 5] P{x} [may also be written as $\mu_{P(x)}$] denotes the grade of membership of P(x).

For any collection of fuzzy predicates {P_1,P_2,…,P_k} and the arguments {x_1,x_2,…,x_n}, let X be any partition defined on the collection of all arguments using some equivalence relation. Then P_i can be denoted as P_i=(P_i{x_1}, P_i{x_2},…, P_i{x_n}). The complement of P_i is given by P_i^c =(1-P_i{x_1}, 1-P_i{x_2},…, 1-P_i{x_n}). From this, it can be observed that the grades of membership of the elements of P_i^c are merely the grades of membership of the negations of P_i(x).

As in [3, 5], define the set M={s/s=P_i{x_j} or s=1- P_i{x_j}; i=1,2,…,k; j=1,2,…,n}.

Let $\alpha \in (0,1)$-M

For eachα, define P[α]={x: P{x}>α}.

The lower and upper rough approximations are defined by

$P_\alpha = \cup\ [x] \in X : [x] \subseteq P[\alpha]$ and

$P^\alpha = \cup\ [x] \in X : [x] \cap P[\alpha] \neq \Phi$

respectively.

3.1 Results:

a) P_i^c [α]={P_i[1-α]}^{c}

b) $(P_i^c)_\alpha = (P_i^{1-\alpha})^c$

c) $(P_i^c)^\alpha = (P_{i,1-\alpha})^c$

Hence, the corresponding membership values can be written as

$(negP_i(x))_\alpha = \tau(P_i^{1-\alpha}(x))$

$(negP_i(x))^\alpha = \tau(P_{i,1-\alpha}(x))$ Where τ represents negation in usual predicate calculus.

3.2. Rough Connectives on Fuzzy Predicates

In this section, the connectives are introduced similar to the connectives used in the predicate calculus.

For the given fuzzy predicates P_i(x) and P_j(y):

- **Rough conjunction** ($\underset{\alpha}{\wedge}$, $\overset{\alpha}{\wedge}$) is defined as P_i(x) $\underset{\alpha}{\wedge}$ P_j(y) = $P_{i,\alpha}$(x) $\wedge$ $P_{j,\alpha}$(y) and P_i(x) $\overset{\alpha}{\wedge}$ P_j(y) = P_i^α (x) $\wedge$ P_j^α (y) respectively.

- **Rough disjunction** ($\underset{\alpha}{\vee}, \overset{\alpha}{\vee}$) is defined as $P_i(x) \underset{\alpha}{\vee} P_j(y) = P_{i,\alpha}(x) \vee P_{j,\alpha}(y)$ and $P_i(x) \overset{\alpha}{\vee} P_j(y) = P_i^{\alpha}(x) \vee P_j^{\alpha}(y)$ respectively.
- **Rough implication** ($\underset{\alpha}{\longrightarrow}, \overset{\alpha}{\longrightarrow}$) is defined as $P_i(x) \underset{\alpha}{\longrightarrow} P_j(y) = P_i^{1-\alpha}(x) \rightarrow P_{j,\alpha}(y)$ and $P_i(x) \overset{\alpha}{\longrightarrow} P_j(y) = P_{i,1-\alpha}(x) \rightarrow P_j^{\alpha}(y)$ respectively.
- **Rough bi-implication** ($\underset{\alpha}{\longleftrightarrow}, \overset{\alpha}{\longleftrightarrow}$) is defined as $P_i(x) \underset{\alpha}{\longleftrightarrow} P_j(y) = [P_i(x) \underset{\alpha}{\longrightarrow} P_j(y)] \wedge [P_j(y) \underset{\alpha}{\longrightarrow} P_i(x)]$ and $P_i(x) \overset{\alpha}{\longleftrightarrow} P_j(y) = [P_i(x) \overset{\alpha}{\longrightarrow} P_j(y)] \wedge [P_j(y) \overset{\alpha}{\longrightarrow} P_i(x)]$ respectively.
- **Rough negation** ($\tau_{\alpha}, \tau^{\alpha}$) is defined as $\tau_{\alpha} P_i(x) = (negP_i(x))_{\alpha} = \tau(P_i^{1-\alpha}(x))$ and $\tau^{\alpha} P_i(x) = (negP_i(x))^{\alpha} = \tau(P_{i,1-\alpha}(x))$ respectively.

The Truth values of the above connectives are shown in the following table by using following example.

Consider the universe of discourse U={a,b,c,d,e,f} with the partition Ψ={{a,c,e},{b,f},{d}}. Consider the fuzzy predicates P and Q defined on U which are given by P= (0.2, 0.6, 0.5, 0.4, 0.3, 0.7) and Q= (0.4, 0.6, 0.3, 0.6, 0.5, 0.8).

Let α=0.45. Then P[α]={b,c,f} and Q[α]={b,d,e,f}. Hence, P_{α}={b,f}, P^{α}={a,b,c,e,f}, Q_{α}={b,d,f} and Q^{α}=Ψ.

x_1	a	b	d	a	b	d	a	b	d
x_2	a	a	a	b	b	b	d	d	d
$P(x_1) \underset{\alpha}{\wedge} Q(x_2)$	0	0	0	0	1	0	0	1	0
$P(x_1) \overset{\alpha}{\wedge} Q(x_2)$	1	1	0	1	1	0	1	1	0
$P(x_1) \underset{\alpha}{\vee} Q(x_2)$	0	1	0	1	1	1	1	1	1
$P(x_1) \overset{\alpha}{\vee} Q(x_2)$	1	1	1	1	1	1	1	1	1
$P(x_1) \underset{\alpha}{\rightarrow} Q(x_2)$	1	0	1	1	1	1	1	1	1
$P(x_1) \overset{\alpha}{\rightarrow} Q(x_2)$	1	1	1	1	1	1	1	1	1
$P(x_1) \underset{\alpha}{\leftrightarrow} Q(x_2)$	1	0	1	0	1	0	0	1	0
$P(x_1) \overset{\alpha}{\leftrightarrow} Q(x_2)$	1	1	1	1	1	0	1	1	0
$\tau_{\alpha} P(x_1)$	1	0	1	1	0	1	1	0	1
$\tau^{\alpha} P(x_1)$	1	0	1	1	0	1	1	0	1

The above rough connectives satisfy the following properties.

Property 3.2.1: $P(x) \underset{\alpha}{\longrightarrow} Q(y) = ((negQ(y)) \underset{\alpha}{\longrightarrow} (negP(x)))$

Proof: LHS ≡ $P(x) \underset{\alpha}{\longrightarrow} Q(y)$

$= (negP(x) \underset{\alpha}{\vee} Q(y))$

$= (Q(y) \underset{\alpha}{\vee} negP(x))$

$= (neg(negQ(y)) \underset{\alpha}{\vee} negP(x))$

$= ((negQ(y)) \underset{\alpha}{\longrightarrow} (negP(x)))$

≡ RHS

Property3.2.2: $P(x) \xrightarrow{\alpha} Q(y) = ((negQ(y)) \xrightarrow{\alpha} (negP(x)))$

Proof: LHS $\equiv P(x) \xrightarrow{\alpha} Q(y)$

$$= ((negP(x)) \overset{\alpha}{\vee} Q(y))$$

$$= (Q(y) \overset{\alpha}{\vee} (negP(x)))$$

$$= (neg(negQ(y)) \overset{\alpha}{\vee} negP(x))$$

$$= ((negQ(y)) \xrightarrow{\alpha} (negP(x)))$$

$\equiv$ RHS

Property3.2.3: $P(x) \xrightarrow[\alpha]{} (Q(y) \xrightarrow[\alpha]{} R(z)) = (P(x) \overset{1-\alpha}{\wedge} Q(y)) \xrightarrow[\alpha]{} R(z)$

Proof: LHS $\equiv P(x) \xrightarrow[\alpha]{} (Q(y) \xrightarrow[\alpha]{} R(z)$

$$= (negP(x)) \underset{\alpha}{\vee} (Q(y) \xrightarrow[\alpha]{} R(z))$$

$$= (negP(x)) \underset{\alpha}{\vee} (negQ(y) \underset{\alpha}{\vee} R(z))$$

$$= (((negP(x)) \underset{\alpha}{\vee} (negQ(y))) \underset{\alpha}{\vee} R(z)$$

$$= \tau(P(x) \overset{1-\alpha}{\wedge} Q(y)) \underset{\alpha}{\vee} R(z)$$

$$= (P(x) \overset{1-\alpha}{\wedge} Q(y)) \xrightarrow[\alpha]{} R(z)$$

$\equiv$ RHS

Property 3.2.4: $P(x) \xrightarrow{\alpha} (Q(y) \xrightarrow{\alpha} R(z)) = (P(x) \underset{1-\alpha}{\wedge} Q(y)) \xrightarrow{\alpha} R(z)$

Proof: LHS $\equiv P(x) \xrightarrow{\alpha} (Q(y) \xrightarrow{\alpha} R(z))$

$$= (negP(x)) \overset{\alpha}{\vee} (Q(y) \xrightarrow{\alpha} R(z))$$

$$= (negP(x)) \overset{\alpha}{\vee} (negQ(y) \overset{\alpha}{\vee} R(z))$$

$$= ((negP(x)) \overset{\alpha}{\vee} (negQ(y)) \overset{\alpha}{\vee} R(z)$$

$$= \tau(P(x) \underset{1-\alpha}{\wedge} Q(y)) \overset{\alpha}{\vee} R(z)$$

$$= (P(x) \underset{1-\alpha}{\wedge} Q(y)) \xrightarrow{\alpha} R(z)$$

$\equiv$ RHS

Property 3.2.5: $P(x) \xleftrightarrow[\alpha]{} negQ(y) = \tau(P(x) \xleftrightarrow{1-\alpha} Q(y))$

Proof: LHS $\equiv P(x) \xleftrightarrow[\alpha]{} negQ(y)$

$$= (P(x) \xrightarrow[\alpha]{} negQ(y)) \wedge (negQ(y) \xrightarrow[\alpha]{} P(x))$$

$$= (negP(x) \underset{\alpha}{\vee} negQ(y)) \wedge (Q(y) \underset{\alpha}{\vee} P(x))$$

$$= [((negP(x))_\alpha \vee (negQ(y))_\alpha) \wedge Q_\alpha(y)] \vee [((negP(x))_\alpha \vee (negQ(y))_\alpha) \wedge P_\alpha(x)]$$

$$= ((negP(x))_\alpha \wedge Q_\alpha(y)) \vee ((negQ(y))_\alpha \wedge Q_\alpha(y)) \vee ((negP(x))_\alpha \wedge P_\alpha(x)) \vee ((negQ(y))_\alpha \wedge P_\alpha(x))$$

$$= ((negP(x))_\alpha \wedge Q_\alpha(y)) \vee .f. \vee .f. \vee ((negQ(y))_\alpha \wedge P_\alpha(x))$$

$$= ((negP(x))_\alpha \wedge Q_\alpha(y)) \vee ((negQ(y))_\alpha \wedge P_\alpha(x))$$

$$= ((negP(x) \underset{\alpha}{\wedge} Q(y)) \vee ((negQ(y) \underset{\alpha}{\wedge} P(x))$$

$= [(negP(x) \underset{\alpha}{\wedge} (neg(negQ(y)))] \vee [(negQ(y) \underset{\alpha}{\wedge} (neg(negP(x)))]$

$= \tau(P(x) \overset{1-\alpha}{\vee} negQ(y)) \vee \tau(Q(y) \overset{1-\alpha}{\vee} negP(x))$

$= \tau(negQ(y) \overset{1-\alpha}{\vee} P(x)) \vee \tau(negP(x) \overset{1-\alpha}{\vee} Q(y))$

$= \tau(Q(y) \xrightarrow{1-\alpha} P(x)) \vee \tau(P(x) \xrightarrow{1-\alpha} Q(y))$

$= \tau(P(x) \xrightarrow{1-\alpha} Q(y)) \vee \tau(Q(y) \xrightarrow{1-\alpha} P(x))$

$= \tau[(P(x) \xrightarrow{1-\alpha} Q(y)) \wedge (Q(y) \xrightarrow{1-\alpha} P(x))]$

$= \tau(P(x) \xleftrightarrow{1-\alpha} Q(y)) \equiv$ RHS

Property3.2.6: $P(x) \xleftrightarrow{\alpha} negQ(y) = \tau(P(x) \xleftrightarrow[1-\alpha]{} Q(y))$

Proof: LHS $\equiv P(x) \xleftrightarrow{\alpha} negQ(y)$

$= (P(x) \xrightarrow{\alpha} negQ(y)) \wedge (negQ(y) \xrightarrow{\alpha} P(x))$

$= (negP(x) \overset{\alpha}{\vee} negQ(y)) \wedge (Q(y) \overset{\alpha}{\vee} P(x))$

$= [(negP(x))^{\alpha} \vee (negQ(y))^{\alpha}] \wedge [Q^{\alpha}(y) \vee P^{\alpha}(x)]$

$= [((negP(x))^{\alpha} \vee (negQ(y))^{\alpha}) \wedge Q^{\alpha}(y)] \vee [((negP(x))^{\alpha} \vee (negQ(y))^{\alpha}) \wedge P^{\alpha}(x)]$

$= ((negP(x))^{\alpha} \wedge Q^{\alpha}(y)) \vee ((negQ(y))^{\alpha} \wedge Q^{\alpha}(y)) \; ((negP(x))^{\alpha} \wedge P^{\alpha}(x)) \vee ((negQ(y))^{\alpha} \wedge P^{\alpha}(x))$

$= ((negP(x))^{\alpha} \wedge Q^{\alpha}(y)) \vee .f. \vee .f. \vee ((negQ(y))^{\alpha} \wedge P^{\alpha}(x))$

$= ((negP(x))^{\alpha} \wedge Q^{\alpha}(y)) \vee ((negQ(y))^{\alpha} \wedge P^{\alpha}(x))$

$= (negP(x) \overset{\alpha}{\wedge} Q(y)) \vee ((negQ(y) \overset{\alpha}{\wedge} P(x))$

$= [(negP(x)) \overset{\alpha}{\wedge} (neg(negQ(y)))] \vee [(negQ(y) \overset{\alpha}{\wedge} (neg(negP(x)))]$

$= \tau(P(x) \underset{1-\alpha}{\vee} negQ(y)) \vee \tau(Q(y) \underset{1-\alpha}{\vee} negP(x))$

$= \tau(negQ(y) \underset{1-\alpha}{\vee} P(x)) \vee \tau(negP(x) \underset{1-\alpha}{\vee} Q(y))$

$= \tau(Q(y) \xrightarrow[1-\alpha]{} P(x)) \vee \tau(P(x) \xrightarrow[1-\alpha]{} Q(y))$

$= \tau(P(x) \xrightarrow[1-\alpha]{} Q(y)) \vee \tau(Q(y) \xrightarrow[1-\alpha]{} P(x))$

$= \tau[(P(x) \xrightarrow[1-\alpha]{} Q(y)) \wedge (Q(y) \xrightarrow[1-\alpha]{} P(x))]$

$= \tau(P(x) \xleftrightarrow[1-\alpha]{} Q(y)) \equiv$ RHS

Property 3.2.7: $P(x) \xleftrightarrow[\alpha]{} Q(y) = (P(x) \underset{\alpha}{\wedge} Q(y)) \vee (negP(x) \underset{\alpha}{\wedge} negQ(y))$

Proof: LHS $\equiv P(x) \xleftrightarrow[\alpha]{} Q(y)$

$= (P(x) \xrightarrow[\alpha]{} Q(y)) \wedge (Q(y) \xrightarrow[\alpha]{} P(x))$

$= (negP(x) \underset{\alpha}{\vee} Q(y)) \wedge (negQ(y) \underset{\alpha}{\vee} P(x))$

$= ((negP(x))_{\alpha} \vee Q_{\alpha}(y)) \wedge ((negQ(y))_{\alpha} \vee P_{\alpha}(x))$

$= [((negP(x))_{\alpha} \vee Q_{\alpha}(y)) \wedge (negQ(y))_{\alpha}] \vee [((negP(x))_{\alpha} \vee Q_{\alpha}(y)) \wedge P_{\alpha}(x)]$

$= [(negP(x))_{\alpha} \wedge (negQ(y))_{\alpha}] \vee [Q_{\alpha}(y) \wedge (negQ(y))_{\alpha}] \vee [(negP(x))_{\alpha} \wedge P_{\alpha}(x)] \vee [Q_{\alpha}(y) \wedge P_{\alpha}(x)]$

$= [(negP(x))_{\alpha} \wedge (negQ(y))_{\alpha}] \vee .f. \vee .f. \vee [Q_{\alpha}(y) \wedge P_{\alpha}(x)]$

$= [(negP(x))_{\alpha} \wedge (negQ(y))_{\alpha}] \vee [Q_{\alpha}(y) \wedge P_{\alpha}(x)]$

$= [Q_{\alpha}(y) \wedge P_{\alpha}(x)] \vee [(negP(x))_{\alpha} \wedge (negQ(y))_{\alpha}]$

$$= [P_\alpha(x) \wedge Q_\alpha(y)] \vee [(negP(x))_\alpha \wedge (negQ(y))_\alpha]$$
$$= (P(x) \underset{\alpha}{\wedge} Q(y)) \vee (negP(x) \underset{\alpha}{\wedge} negQ(y)) \equiv \text{RHS}$$

Property 3.2.8: $P(x) \xleftrightarrow{\alpha} Q(y) = (P(x) \overset{\alpha}{\wedge} Q(y)) \vee (negP(x)) \overset{\alpha}{\wedge} negQ(y))$

Proof: LHS $\equiv P(x) \xleftrightarrow{\alpha} Q(y)$

$$= (P(x) \xrightarrow{\alpha} Q(y)) \wedge (Q(y) \xrightarrow{\alpha} P(x))$$
$$= (negP(x) \overset{\alpha}{\vee} Q(y)) \wedge (negQ(y)) \overset{\alpha}{\vee} P(x))$$
$$= ((negP(x))^\alpha \vee Q^\alpha(y)) \wedge ((negQ(y))^\alpha \vee P^\alpha(x))$$
$$= [((negP(x))^\alpha \vee Q^\alpha(y)) \wedge (negQ(y))^\alpha] \vee [((negP(x))^\alpha \vee Q^\alpha(y)) \wedge P^\alpha(x)]$$
$$= [((negP(x))^\alpha \wedge ((negQ(y))^\alpha] \vee [Q^\alpha(y) \wedge ((negQ(y))^\alpha] \vee [((negP(x))^\alpha \wedge P^\alpha(x)] \vee [Q^\alpha(y) \wedge P^\alpha(x)]$$
$$= [((negP(x))^\alpha \wedge ((negQ(y))^\alpha] \vee .f. \vee .f. \vee [Q^\alpha(y) \wedge P^\alpha(x)]$$
$$= [((negP(x))^\alpha \wedge ((negQ(y))^\alpha] \vee [Q^\alpha(y) \wedge P^\alpha(x)]$$
$$= [Q^\alpha(y) \wedge P^\alpha(x)] \vee [((negP(x))^\alpha \wedge ((negQ(y))^\alpha]$$
$$= [P^\alpha(x) \wedge Q^\alpha(y)] \vee [((negP(x))^\alpha \wedge ((negQ(y))^\alpha]$$
$$= (P(x) \overset{\alpha}{\wedge} Q(y)) \vee (negP(x)) \overset{\alpha}{\wedge} negQ(y)) \equiv \text{RHS}$$

4. CONCLUSION

In this paper, have discussed the naïve properties of the rough connectives defined for fuzzy predicates. In future, it is further aimed to define the rough approximated quantifiers on fuzzy and rough connectives and also aimed to derive various properties such as Implications, Modus Ponens, and Modus Tollens etc.

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