

Mathematical Modelling of Buck Converter

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Abstract—A dc converter is a lossless dc transformer that must supply regulated outputted voltage under varying load and input voltage condition. Also the converter values changes with time, temperature and pressure. Hence to control the output voltage, detailed understanding of the dynamic behaviour, transient response and small signal characteristics of the converter is required, which can't be understood by the physical model of the converter. So the mathematical modelling of the dc-dc converter is a way to model them as time independent systems, defined by a unified set of differential equations, capable of representing circuit waveforms. Therefore, it can be a convenient approach for designing controllers to be applied to switched converters. This paper derives the mathematical modelling of a pulse width modulated (PWM) open loop buck converter operating in continuous conduction mode (CCM) and presents its verification by simulation in SIMULINK. The parasitic resistance of reactive elements is included in the circuit model in order to avoid the problem of inconsistent initial condition. (**Abstract**)

Keywords-state space modelling, buck converter, Simulink, Differential Equations

I. INTRODUCTION

The dc-dc converters can be viewed as dc transformer that delivers a dc voltage or current at a different level than the input source. Electronic switching performs this dc transformation as in conventional transformers and not by electromagnetic means.[1] The application of dc-dc converters cover wide spectrum from power supply in hand held calculator, through a variety of spacecraft systems including solar array and battery power conditioning, to industrial process control and electric utility bulk power.[2] This could be viewed as fusion of control, power and electronics disciplines. The topology of DC-DC converters consists of two linear (resistor, inductor and capacitor) and nonlinear (diode and active switch) parts. Because of the switching properties of the power elements, the operation of these converters varies by time. [1]

A. Buck converter

The buck converter is the most widely used dc-dc converter topology in power management and microprocessor voltage-regulator (VRM) applications. Those applications require fast load and line transient responses and high efficiency over a wide load current range.[3] They can convert a voltage source into a lower regulated voltage. [2]

If the buck converter operates in Continuous Conduction Mode (CCM), the relationship between the input voltage (V_i) and the output voltage (V_o) is: [2]

$$V_o = V_i * D \quad (1)$$

The most common strategy for controlling the power transmitted to the load is the Pulse Width Modulation (PWM).[4] A control voltage V_m is compared to a triangular voltage V_t . The triangular voltage V_t determines the switching frequency f_t . The switch Q1 is controlled according to the difference $V_m - V_t$. Three operating phases are:

- T state-on and D state-off
- T state-off and D state-on
- T and D state-off

II. MATHEMATICAL MODELLING [6][7]

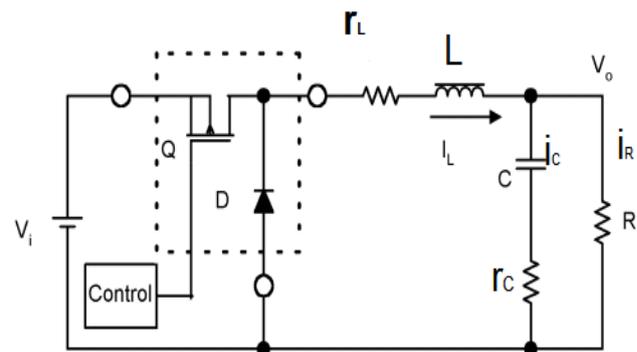


Figure 1: Buck Converter

The differential equations related to state variables are:

$$V_i(t) = V_o(t) + \frac{L di_L(t)}{dt} + r_L i_L(t) \quad (2)$$

$$i_L(t) = \frac{1}{L} \int (V_i(t) - V_o(t) - r_L i_L(t)) \quad (3)$$

$$V_o(t) = \frac{1}{C} \int i_C(t) dt + i_C(t) r_C \quad (4)$$

Applying KCL and KVL in the figure 1 we can get

$$i_C(t) = i_L(t) - i_R(t) \quad (5)$$

$$i_C(t) = \frac{CdV_C(t)}{dt} \quad (6)$$

$$V_C(t) = V_o(t) - r_C i_C(t) \quad (7)$$

$$V_o(t) = \frac{1}{C} \int i_L(t) dt - \frac{1}{C} \int \frac{V_o(t)}{R} dt + r_C i_C(t) \quad (8)$$

The state equations of the buck converter considering the parasitic resistances of filter inductance and capacitor given by

equation (3) and (8). [2][5] Since the buck converter is a switching device having two states of operation while working in CCM. These two states are taken into account using a logical variable F, can have two values 0 and 1. The description of two state operations of buck converter and logical variable F is given in table 1.

TABLE 1: PWM Control Logical Variables

T	D	F
On	Off	1
Off	On	0
Off	Off	0

$$\text{Sign}(i_L)=1, \text{ when } i_L>0 \quad (9)$$

$$\text{Sign}(i_L)=0, \text{ when } i_L<0 \quad (10)$$

Now,

$$V_L(t) = (V_i(t) - V_o(t) - r_L i_L(t)) * F - V_o(t) * \bar{F} * \text{sign}(i_L(t)) \quad (11)$$

This is the state equation of buck converter represents the mathematical model of the buck converter which is simulated in Simulink.

III. SIMULATION

The mathematical model and physical model of buck converter shown in figure 1 are simulated in Simulink. Parameters of this converter are tabulated in table 2.[8]

TABLE 2: Parameters of buck converter

Input voltage	25V
Duty cycle	.5
Switching frequency	1000Hz
Inductor(L)	15Mh
Capacitor(C)	25μF
ESR(r_c)	80mΩ
Dc resistance of inductor(r_L)	124mΩ
Load resistance(R)	57Ω

A. Physical Model

Simulink physical model of buck converter shown in figure 1 with parameters given in table 1 is presented here. Output current and voltage waveforms obtained from the physical model are shown in figure 3 and figure 4.

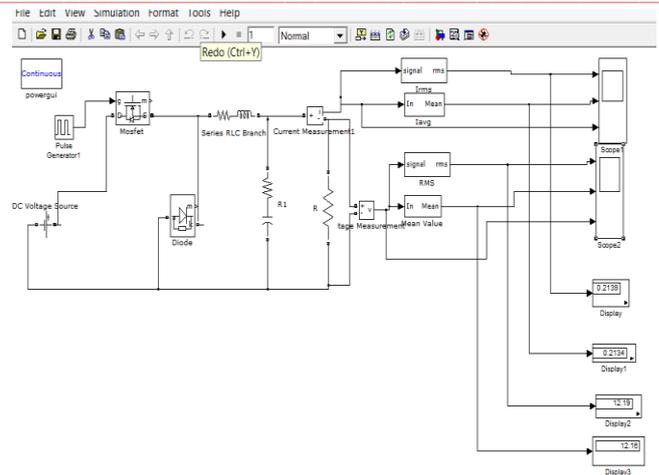


Figure 2: Physical Model of Buck Converter

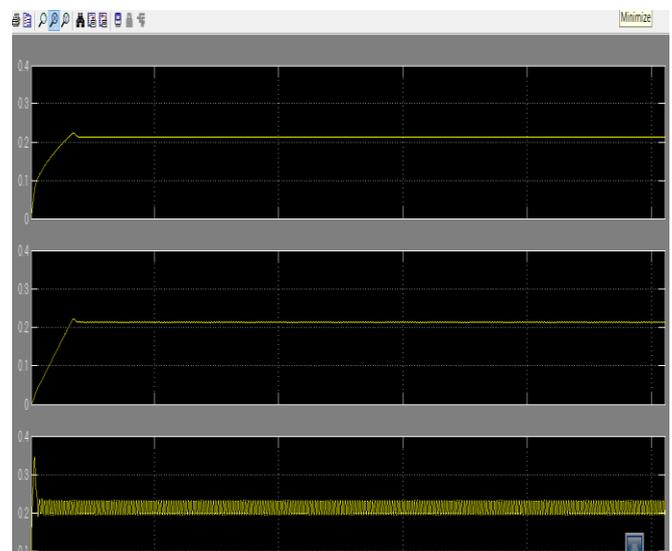


Figure 3: Output Current Waveform
 Waveform 1 is rms value of output current
 Waveform 2 is average value of output current
 Waveform 3 is instantaneous value of output current

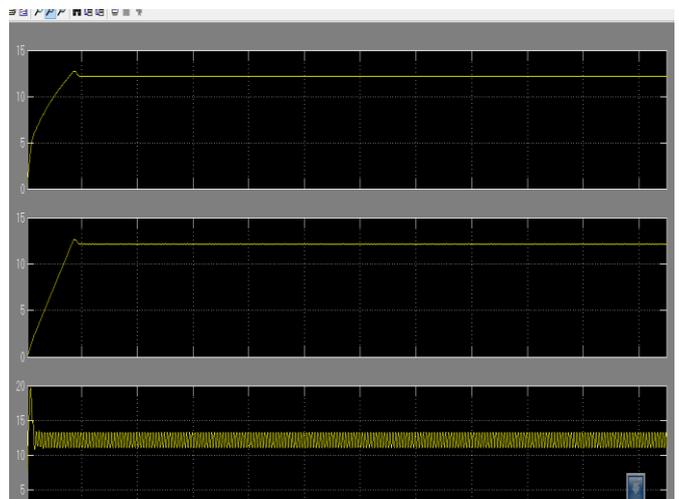


Figure 4: Output Voltage Waveform
 Waveform 1 is rms value of output voltage
 Waveform 2 is average value of output voltage
 Waveform 3 is instantaneous value of output voltage.

B. Mathematical Model

Simulink model developed for buck converter circuit shown in figure 1 with parameters given in table 1. Figure shows the mathematical model of buck converter based on equation (11). Output current and voltage waveforms are shown in figure 9-12.

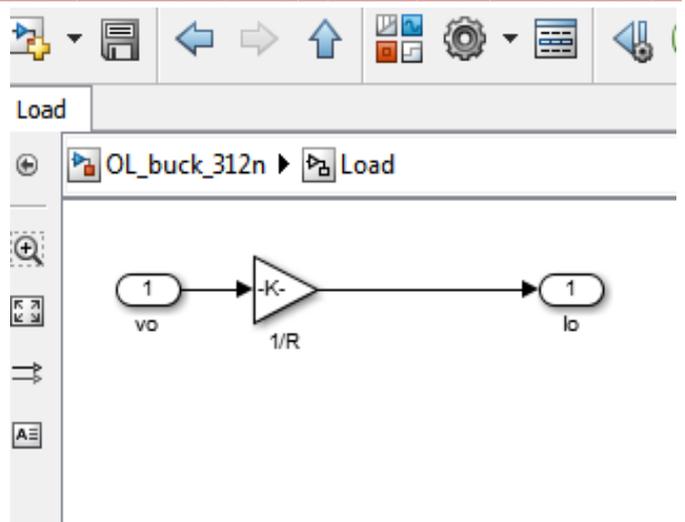


Figure 7: The Load Block of simulink buck model

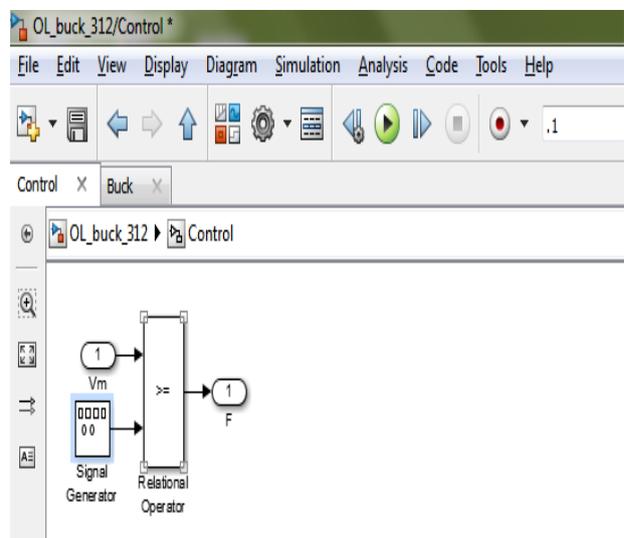


Figure 8: PWM Block of Simulink Buck Model.

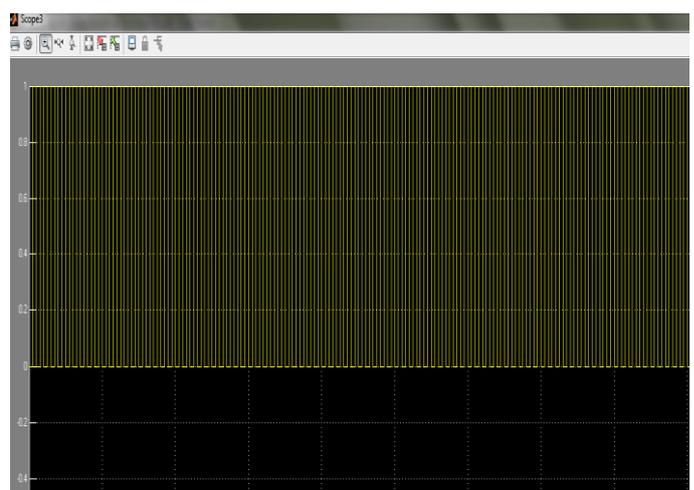


Figure 9: Waveform of Duty Cycle

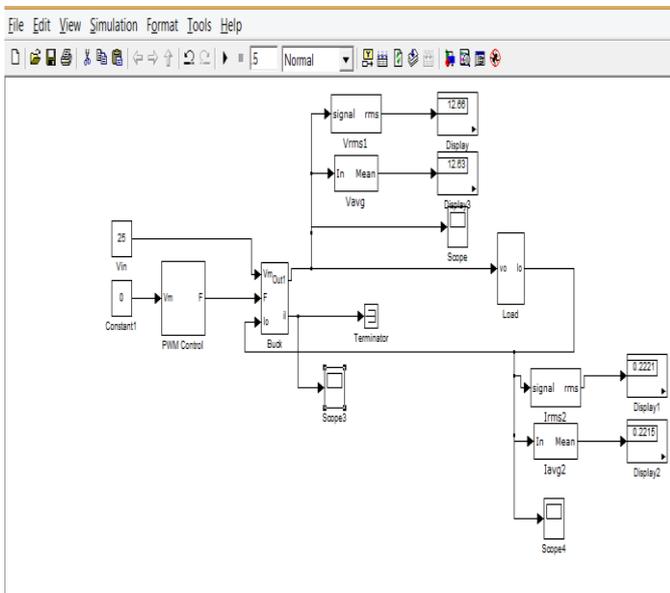


Figure 5: Simulink Model of complete Buck Converter Circuit.

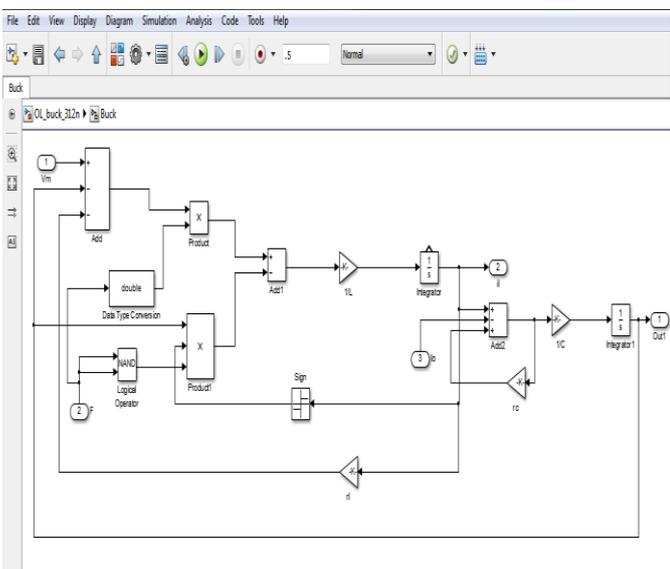


Figure 6: Mathematical Model of Buck Converter in Simulink

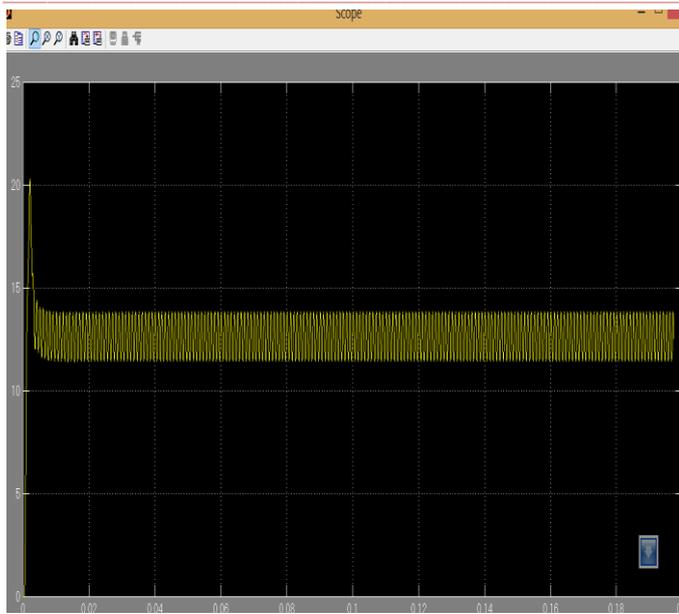


Figure 10: Waveform of Output Voltage

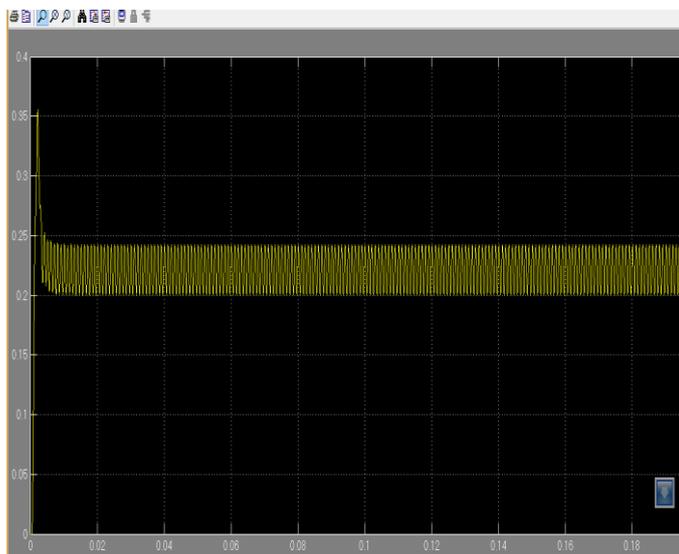


Figure 11: Waveform of Load Current

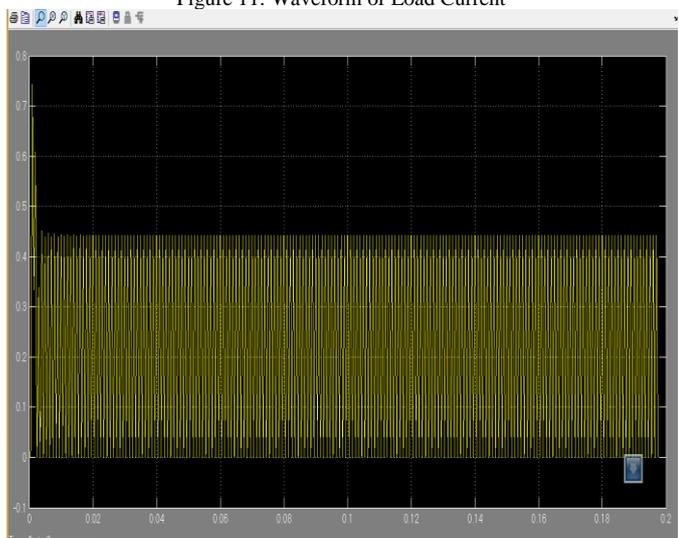


Figure 12: Waveform of Inductor Current

IV. RESULTS

After simulating the physical model and mathematical model of buck converter, waveforms so obtained are in acceptable agreement. Also comparison of output voltage and current for both the models is shown in table 3.

Table 3. Comparison of physical and mathematical model.

	Physical Model	Mathematical Model
Average Output Voltage	12.16 V	12.63 V
Rms Output Voltage	12.19 V	12.66 V
Average Output Current	.2134 A	.2215 A
Rms Output Current	.2139 A	.2221 A
Ripple factor	4.96	4.87

V. CONCLUSION

The mathematical modelling of nonlinear, switched open loop PWM controlled buck converter has been presented. The simulation response results of physical and mathematical model were in excellent agreement, confirming the validity of the mathematical model derived. Due to their compact and portable representations, the state-space representations that were obtained are suitable for use in system dynamic behaviour, transient and small-signal stability analysis. This open loop modelling could be used for closed-voltage loop modelling and for the design of a controller for the converter.

VI. REFERENCES

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