Thermoelastic behavior of a Thin Circular Functionally Graded Material (FGM) Disk Subjected to Thermal Loads

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Abstract--In this paper, a general analysis of thermal stresses and strain in a thin rotating circular disk made of functionally graded material is developed. The temperature distribution is assumed to be a function of radius, inner surface of the disk is assumed to be fixed to a shaft so that isothermal conditions prevail on it and outer surface of the disk is free from any mechanical load and maintained at uniform temperature gradient. The material properties, except Poisson's ratio, are assumed to depend on variable the r and they are expressed as power functions of r. The direct method is used to solve the heat conduction and Navier equations.

Keywords: *Functionally graded materials; disc; Thermoelasticity; stress.*

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1. INTRODUCTION

The FGMs are microscopically non homogeneous materials where the composition of the constituents of materials is changed continuously. The mechanical benefits obtained by a material gradient may be significant, as can be seen by the excellent structure performance of some of these materials.

Functionally Graded Material has been recently developed as the ideal heat resistant material and has numerous potential applications in medium undergoing high temperature rise (such as heat shields for space vehicle). [1] Studied the case of a graded sphere under non-uniform temperature variations by using a numerical integration procedure. Obata and Noda [2] used a perturbation approach to study the thermal stresses in functionally graded hollow sphere that was uniformly heated. Lutz and Zimmerman [3] solved the problem of uniform heating of spherical body whose elastic modulus and thermal expansion coefficients each vary linearly with radial position. Eslami et al. [4] analytically solved the governing equation of a functionally graded spherical vessel and investigated the temperature, displacement and relevant thermal stresses due to the thermal and mechanical loads.

A work was also published by Horgan and Chan [5] where it was noted that increasing the positive exponent of the radial coordinate provided a stress shielding effect, whereas decreasing it created stress amplification. Nayebi and Abdi [6] developed a numerical program to investigate the steady-state behavior of thick-walled spherical and cylindrical pressure vessels subjected to cyclic pressure and/or

temperature using linear kinematic hardening in the plastic condition and a Norton power law in the creep condition. Geometric non-linearity and effect of
Coupling item for different thermal loading conditions were considered in the works of Reddy et al. [7–12]. They carried out theoretical as well as finite element analyses of the thermo-mechanical behavior of FGM cylinders, plates, and shells. Navneet Lamba [13] studied a boundary value problem for thick annular disc subjected to thermal load with radiation type boundary condition. Some features of the stress and temperature distribution are investigated by means of Integral transform technique.

In the present paper an attempt is made to determine the stress functions of a thin rotating circular disk made of functionally graded material. The direct method is used to solve the heat conduction and Navier equations. The temperature distribution is assumed to be a function of radius, inner surface of the disk is assumed to be fixed to a shaft so that isothermal conditions prevail on it and outer surface of the disk is free from any mechanical load and maintained at uniform temperature gradient. The material properties, except Poisson's ratio, are assumed to depend on variable the r and they are expressed as power functions of r.

2. DERIVATIONS

For a plane strain problems, Two Dimensional Equation of equilibrium in polar co-ordinate for a rotating disc are given by

$$\frac{\partial\sigma_r}{\partial r}+\frac{1}{r}\frac{\partial\tau_{r\theta}}{\partial\theta}+\frac{\sigma_r-\sigma_\theta}{r}+\rho\omega^2 r=0 \tag{1}$$

$$\frac{\partial\tau_{r\theta}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_\theta}{\partial\theta}+2\frac{\tau_{r\theta}}{r}=0 \tag{2}$$

Where $\rho\omega^2 r$ denotes inertia force due to rotation of the disc.

Because of symmetry the value of shear stress $\tau_{r\theta}$ is equal to zero for a rotating disc, stress components are independent of θ, thus equilibrium (1) and (2) is reduced to

$$\frac{d}{dr}(r\rho_r)-\rho_\theta+\rho\omega^2 r=0 \tag{3}$$

Where $\rho(r)$ is the density of the material and it is assumed to vary exponentially as ω is the angular frequency the component of total strain are given by

$$\varepsilon_r=e_r+\varepsilon^*,\varepsilon_\theta=e_\theta+\varepsilon^* \tag{4}$$

Where $\varepsilon^*=\alpha(r)T(r)$ denote thermal Eigen strain at a point which remains same in all direction and $T(r)$ is the change in temperature at any distance.

$\varepsilon_r =$ radial component of the total strain

$\varepsilon_\theta =$ circumferential component of total strain

$e_r =$ radial component of the Elastic strain

$e_\theta =$ circumferential component of Elastic strain

Due to symmetry consideration, the shear strains do not play any role the Hooke's law provides us

$$\varepsilon_r = \frac{1}{E}(\sigma_r - \upsilon\sigma_\theta) + \alpha(r)T(r) \quad (5)$$

$$\varepsilon_\theta = \frac{1}{E}(\sigma_\theta - \upsilon\sigma_r) + \alpha(r)T(r) \quad (6)$$

where σ_r and σ_θ are the radial and circumferential stress components respectively υ is the passion ratio of the material, $E(r) = E_i r^{\beta_1}$ is the Young's modulus, $\alpha(r) = \alpha_i r^{\beta_2}$ is the coefficient of thermal expansion, where β_1 and β_2 are material parameters.

The strain compatibility equation is

$$\varepsilon_r = \frac{d}{dr}(r\varepsilon_\theta) \quad (7)$$

The equation equilibrium (3) is satisfied by the stress function F defined as

$$\sigma_r = \frac{F}{r} \quad (8)$$

$$\sigma_\theta = \frac{dF}{dr} + \rho\omega^2 r^2 \quad (9)$$

Then the equation (5) and (6) become

$$\varepsilon_r = \frac{1}{E}\left(\frac{F}{r} - \upsilon\frac{dF}{dr}\right) - \frac{\upsilon}{E}\rho\omega^2 r^2 + \alpha(r)T(r) \quad (10)$$

$$\varepsilon_\theta = \frac{1}{E}\left(\frac{dF}{dr} - \frac{\upsilon F}{r}\right) + \frac{1}{E}\rho\omega^2 r^2 + \alpha(r)T(r) \quad (11)$$

Substituting ε_r and ε_θ from (10) and (11) into the compatibility (7), finally differential equation of the stress function is found

$$r^2\frac{d^2F}{dr^2} + (1-\beta_1)r\frac{dF}{dr} + (\beta_1\upsilon - 1)F = (\beta_1 - \beta_2 - 3 - \upsilon)\rho_i\omega^2 r^{\beta_3+3} - \beta_2\alpha_i E_i \; T(r) r^{\beta_1+\beta_2+1} - \alpha_i E_i \frac{dT}{dr} r^{\beta_1+\beta_2+2} \quad (12)$$

3. HEAT CONDUCTION PROBLEM

The heat conduction equation for a dynamic coupled thermoelastic solid is given by

$$k\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right)T - \rho c_e\frac{\partial T}{\partial t} = E\alpha T_0(e_r - e_\theta)/(1-\upsilon) \quad (13)$$

Where k is the thermal conductivity c_e is the specific heat at constant strain T_0 being uniform reference temperature. For the steady state condition $\frac{\partial}{\partial t} \cong 0$ so the equation (13) reduces to the form

$$\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}\right)T = 0 \quad (14)$$

Subjected to the condition

$$u_r|_{r=a} = 0 \quad ; \quad T|_{r=a} = 0 \quad (15)$$

$$\sigma_r|_{r=b} = 0 \quad ; \quad \left.\frac{dT}{dr}\right|_{r=a} = T_0 \quad (16)$$

On solving (14) with the help of thermal condition (15) and (16) ,we obtain

$$T(r) = bT_0 \log\left(\frac{r}{a}\right), \quad \frac{dT}{dr} = \frac{bT_0}{r} \quad (17)$$

4. SOLUTION OF THE NAVIER EQUATION

Substituting Eq. (17) into Eq. (12) yields

$$r^2\frac{d^2F}{dr^2} + (1-\beta_1)r\frac{dF}{dr} + (\beta_1\upsilon - 1)F = (\beta_1 - \beta_2 - 3 - \upsilon)\rho_i\omega^2 r^{\beta_3+3} - \beta_2\alpha_i E_i \; bT_0(\log r - \log a)r^{\beta_1+\beta_2+1} - \alpha_i E_i \frac{bT_0}{r} r^{\beta_1+\beta_2+2} \quad (18)$$

$$r^2\frac{d^2F}{dr^2} + (1-\beta_1)r\frac{dF}{dr} + (\beta_1\upsilon - 1)F = (\beta_1 - \beta_2 - 3 - \upsilon)\rho_i\omega^2 r^{\beta_3+3} - \beta_2\alpha_i E_i \; bT_0(\log r - \log a)r^{\beta_1+\beta_2+1} - \alpha_i E_i bT_0 r^{\beta_1+\beta_2+1} \quad (19)$$

On simplifying

$$r^2\frac{d^2F}{dr^2} + (1-\beta_1)r\frac{dF}{dr} + (\beta_1\upsilon - 1)F = (\beta_1 - \beta_2 - 3 - \upsilon)\rho_i\omega^2 r^{\beta_3+3} - (\beta_2\log a - 1 - \beta_2\log r)\alpha_i E_i \; bT_0 r^{\beta_1+\beta_2+1} \quad (20)$$

Equation (20) is a Cauchy Homogeneous differential equation.

Put $r = e^z \Rightarrow z = \log r$

$rD = D_1, \quad r^2D^2 = D_1^2 - D_1$. We get

$$(D_1^2 - \beta_1 D_1 + \upsilon\beta_1 - 1)F = (\beta_1 - \beta_2 - 3 - \upsilon)\rho_i\omega^2 e^{(\beta_3+3)z} - (\beta_2\log a - 1 - \beta_2 z)\alpha_i E_i \; bT_0 e^{(\beta_1+\beta_2+1)z} \quad (21)$$

Its Solution is

$$F = c_1 r^{m_1} + c_2 r^{m_2} + A r^{\beta_3+3}$$

$$+B(\beta_2 loga - 1 - \beta_2 logr)r^{\beta_1+\beta_2+1} + Cr^{\beta_1+\beta_2+1} \quad (22)$$

Where

$$A = \frac{(\beta_1-\beta_2-3-\upsilon)\rho_i\omega^2}{\beta_3^2+(6-\beta_1)\beta_3+(\upsilon-3)\beta_1+8}$$

$$B = \frac{\alpha_i E_i\ bT_0}{\beta_2^2+\beta_1+2\beta_2+\beta_1\beta_2+\upsilon\beta_1}$$

$$C = B\frac{\beta_2(\beta_1+2\beta_2+2)}{\beta_2^2+\beta_1+2\beta_2+\beta_1\beta_2+\upsilon\beta_1}$$

Substituting Equation (22) into Equations (8) and (9), the stresses are obtained as

$$\sigma_r = \frac{F}{r} = c_1 r^{m_1-1} + c_2 r^{m_2-1} + Ar^{\beta_3+2} +B(\beta_2 loga - 1 - \beta_2 logr)r^{\beta_1+\beta_2} + Cr^{\beta_1+\beta_2} \quad (23)$$

$$\sigma_\theta = \frac{dF}{dr} + \rho\omega^2 r^2 = m_1 c_1 r^{m_1-1} + m_2 c_2 r^{m_2-1} + (\beta_3+3)Ar^{\beta_3+2} + B[\beta_2 loga - 1 - \beta_2 logr - \beta_2 + \beta_2(\beta_1+2\beta_2+2)]r^{\beta_1+\beta_2} +\rho\omega^2 r^2 \quad (24)$$

Using Equations (23) , (24) and (17) into Equations (5) and (6), the strains are obtained as

$$\varepsilon_r = \frac{1}{E}(\{(1-\upsilon m_1)c_1 r^{m_1-1} +(1-\upsilon m_2)c_2 r^{m_2-1} +(1-\upsilon(\beta_3+3))Ar^{\beta_3+2}\} +(1-\upsilon)B(\beta_2 loga - 1 - \beta_2 logr)r^{\beta_1+\beta_2} +Cr^{\beta_1+\beta_2}\} -\upsilon\{B[-\beta_2+\beta_2(\beta_1+2\beta_2+2)]r^{\beta_1+\beta_2} +\rho\omega^2 r^2\}) +\alpha(r)bT_0\log\left(\frac{r}{a}\right) \quad (25)$$

$$\varepsilon_\theta = \frac{1}{E}(\{(m_1-\upsilon)c_1 r^{m_1-1} + (m_2-\upsilon)c_2 r^{m_2-1} +(\beta_3+3-\upsilon)Ar^{\beta_3+2} +(1-\upsilon)B + (\beta_2 loga - 1 - \beta_2 logr)r^{\beta_1+\beta_2} +B[\beta_2(\beta_1+2\beta_2+2)]r^{\beta_1+\beta_2} +B[\beta_2 loga - 1 - \beta_2 logr - \beta_2 + \beta_2(\beta_1+2\beta_2+2)]r^{\beta_1+\beta_2}\} -\upsilon\{Cr^{\beta_1+\beta_2}\}) + \alpha(r)bT_0\log\left(\frac{r}{a}\right) \quad (26)$$

5. CONCLUSIONS

This paper presents an analytical solution for the calculation of the ax symmetric thermal and mechanical stresses in thin rotating circular disk made of FGM. The material properties through the graded direction are assumed to be nonlinear with a power law distribution. The direct method is used to solve the heat conduction and Navier equations.

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